Halo nuclei, the exotic quantal structures observed far from stability

Pierre Capel







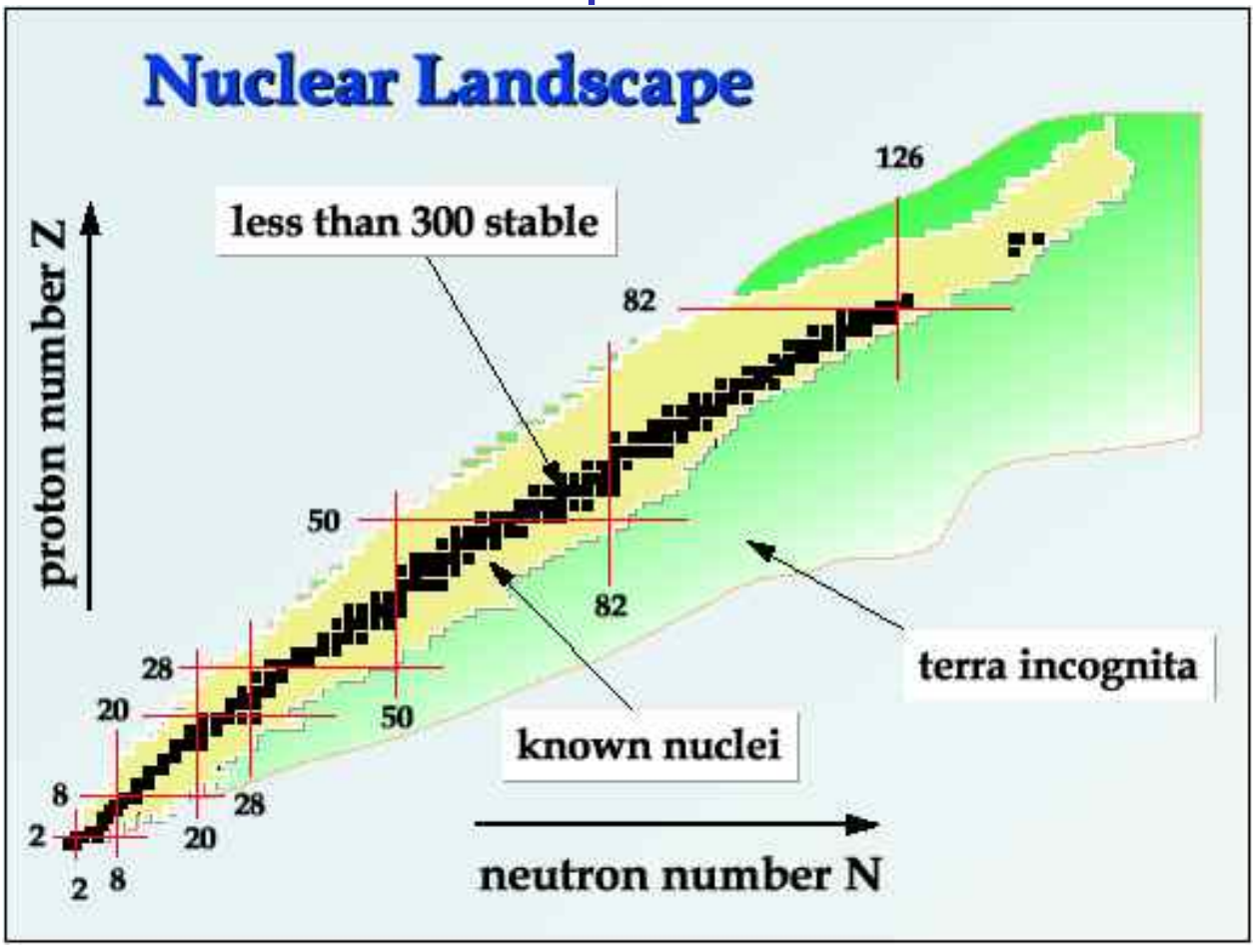




PRISMA

26 November 2026

Nuclear Landscape



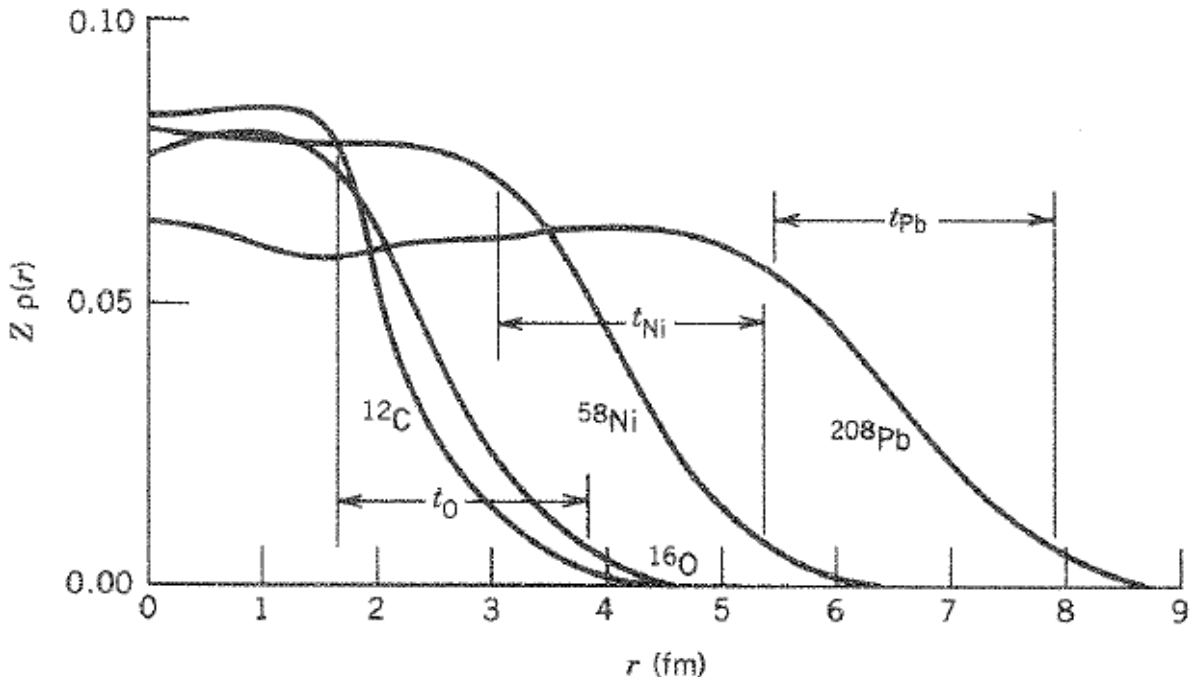
- 256 stable nuclei with
  - $N \approx Z$
  - compact
  - *magic* numbers
- What happens away from stability ?

[Figure by W. Nazarewicz]

In 1980s, Radioactive-Ion Beams (RIB) became available
$\Rightarrow$ allow to study structure and reactions with unstable nuclei

Stable nuclei

Electron scattering → charge density of stable nuclei



[K. S. Krane, *Introductory Nuclear Physics* (Wiley, NY, 1987)]

$\Rightarrow$ stable nuclei $\equiv$ liquid droplet of radius $R \propto A^{1/3}$

What happens far from stability ?

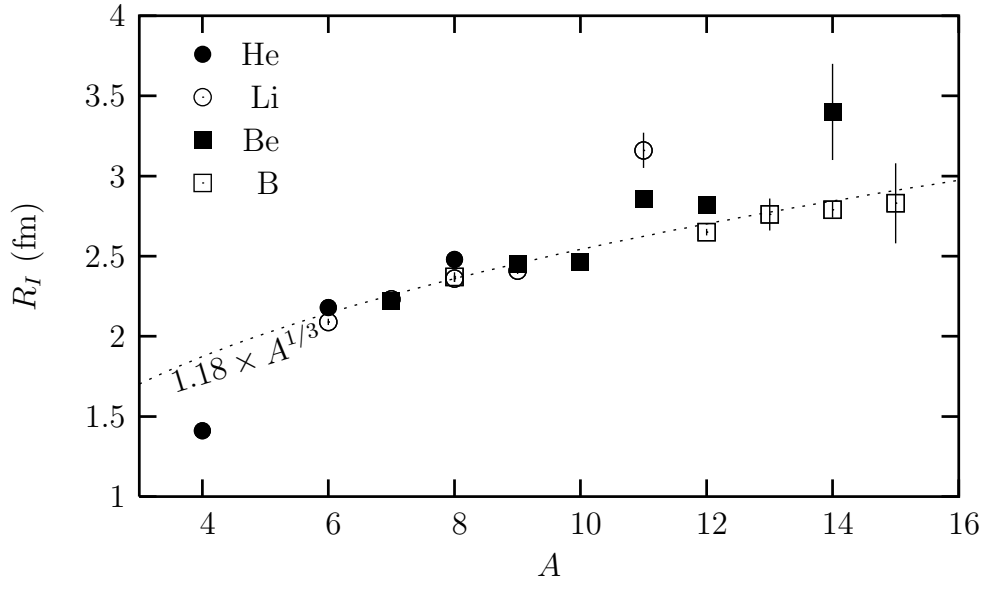
First Experiments with Unstable Nuclei...

Tanihata measures interaction cross sections σ_I of light nuclei

[Tanihata *et al.* PRL **55**, 2676 (1985)]

In a simple geometrical model

$$\sigma_I(P,T) = \pi[R_I(P) + R_I(T)]^2$$



Some nuclei appear larger : ^{11}Li, ^{11}Be...

$\Rightarrow$ large collective deformation or exotic structure ?

Role of valence neutrons

Large σ_I due to valence neutrons :

Nucleus	core	$\sigma_I - \sigma_I(c)$	$\sigma_{-\mathrm{n}}$ or $\sigma_{-2\mathrm{n}}$
^{11}Li	^{9}Li	251 ± 46 mb	213 ± 21 mb
^{11}Be	^{10}Be	129 ± 18 mb	169 ± 4 mb

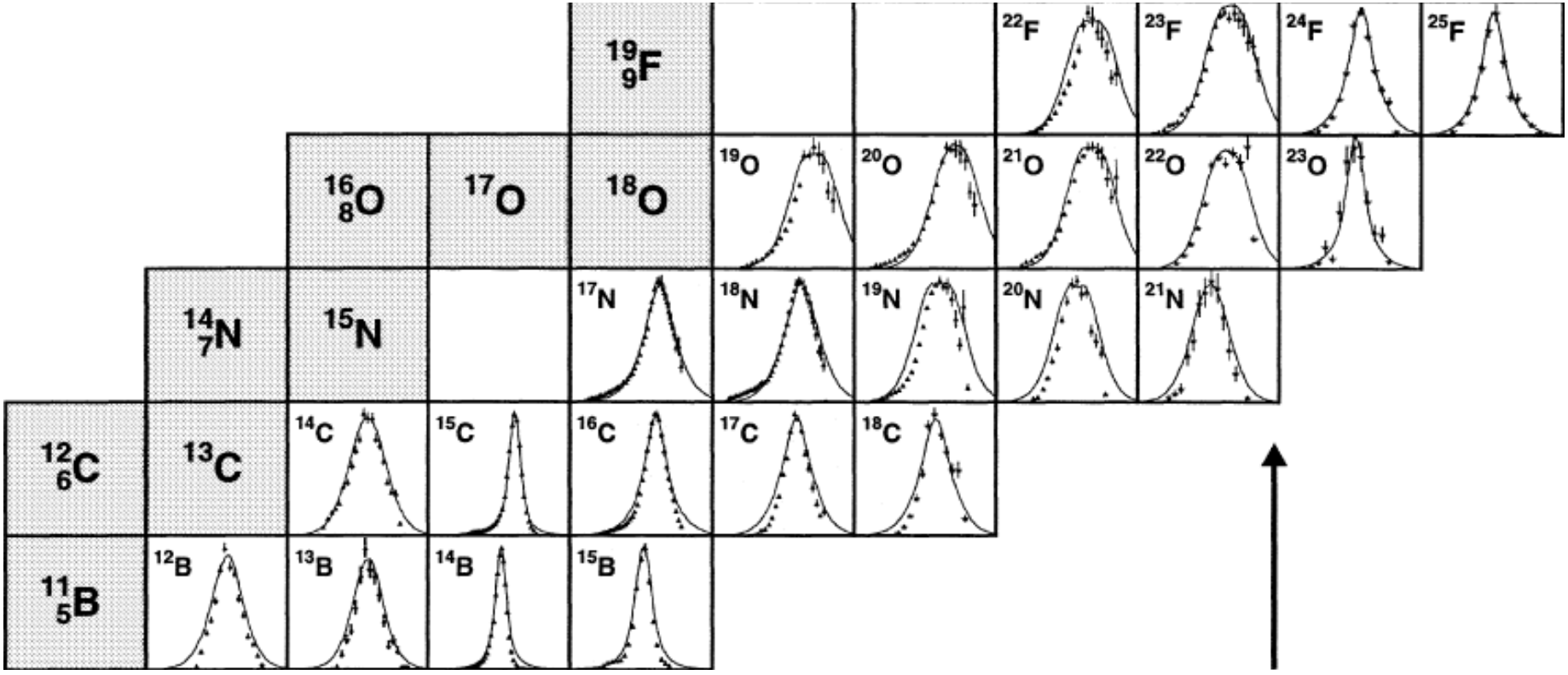
[Tanihata, JPG **22**, 157 (1996)]

These neutrons are weakly bound :

- $S_{2\mathrm{n}}(^{11}\mathrm{Li}) = 0.370\,\mathrm{MeV}$
- $S_{\mathrm{n}}(^{11}\mathrm{Be}) = 0.502\,\mathrm{MeV}$

Parallel-momentum distributions

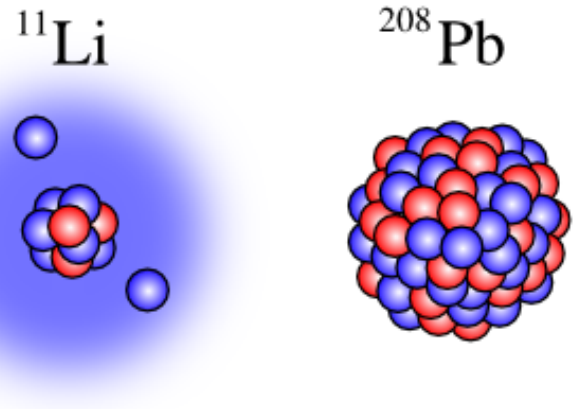
Narrow parallel-momentum distribution of core in neutron removal



[Sauvan *et al.* PLB **491**, 1 (2000)]

$\Rightarrow$ sign of extended spacial core-neutron distribution

Halo structure



Core + 1 or 2 neutrons at large distance

[Hansen & Jonson, Europhys. Lett. **4**, 409 (1987)]

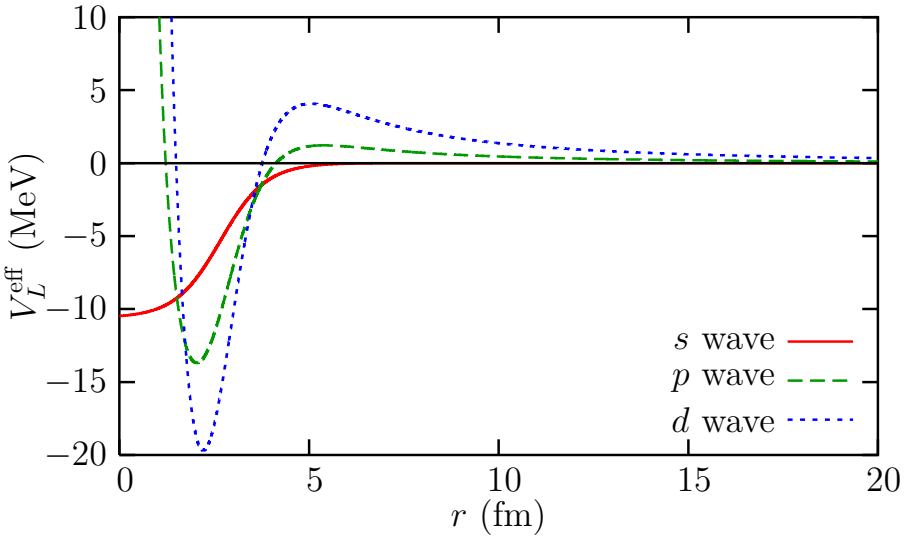
Peculiar structure due to small S_{n} or $S_{2\mathrm{n}}$

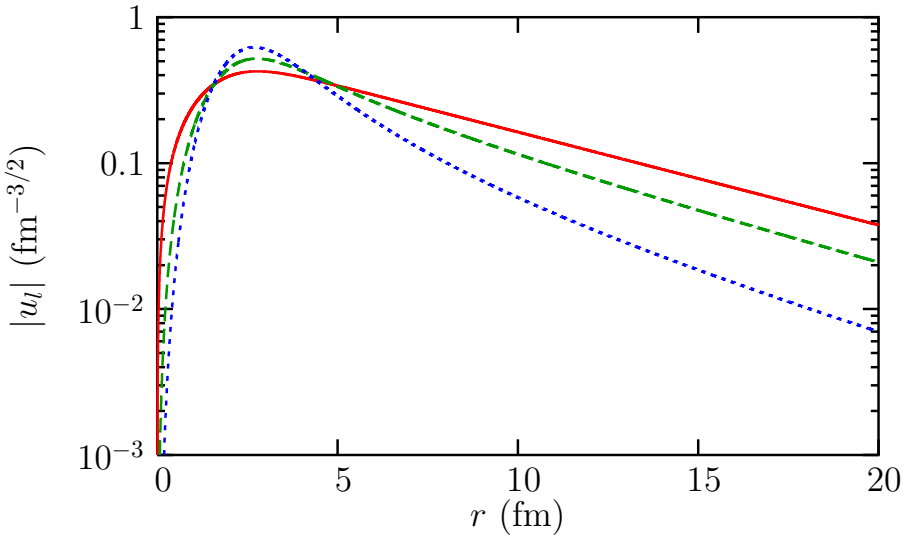
$\Rightarrow$ neutrons tunnel far from the core to form a halo

with radial wave function $u_l(r) \underset{r\to\infty}{\longrightarrow} C_l\, e^{-\kappa r}$

$\hbar^2\kappa^2/2\mu = S_{\mathrm{n}}$ and $C_l \equiv$ Asymptotic Normalisation Constant (ANC)

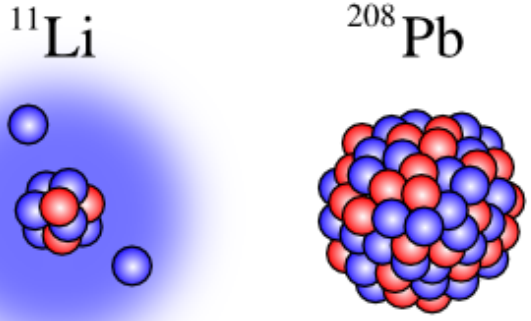
Halo appears only for low centrifugal barrier (low l)





Halo nuclei

- Light, neutron-rich nuclei
- small S_n or S_{2n}
- low-l orbital



One-neutron halo

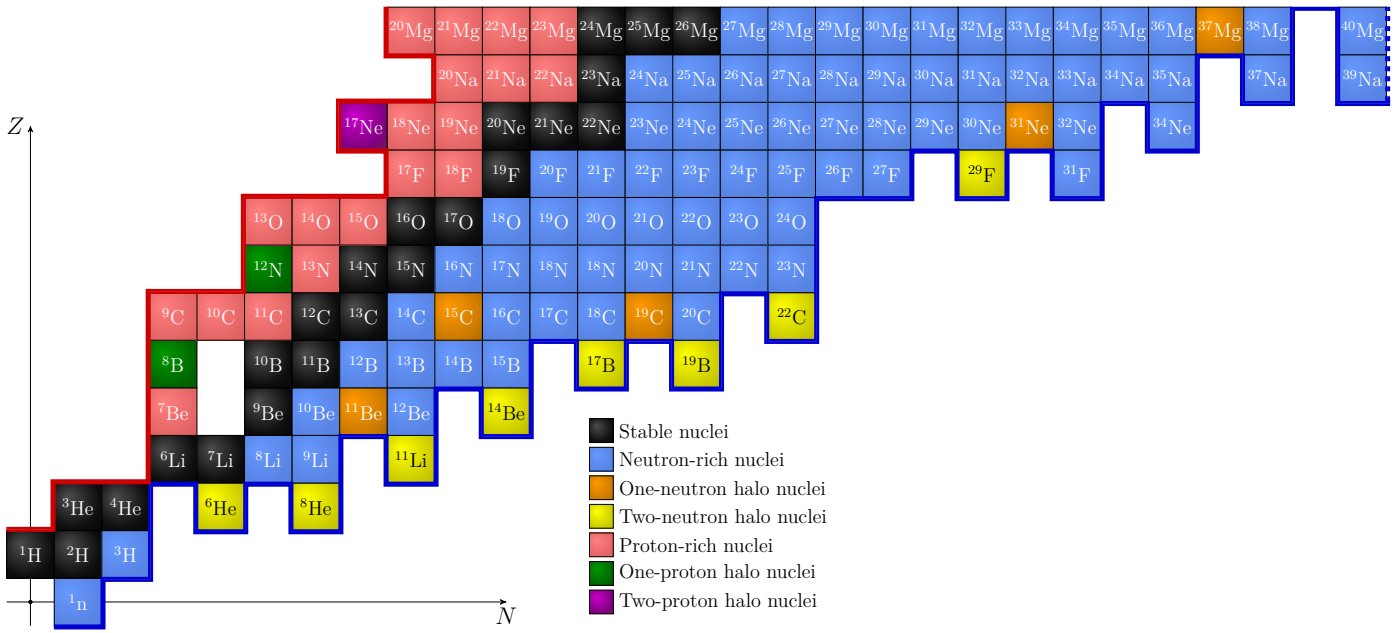
^{11}Be $\equiv$ ^{10}Be + n

^{15}C $\equiv$ ^{14}C + n

Two-neutron halo

^{6}He $\equiv$ ^{4}He + n + n

^{11}Li $\equiv$ ^{9}Li + n + n



Proton halos are possible but less probable : ^{8}B

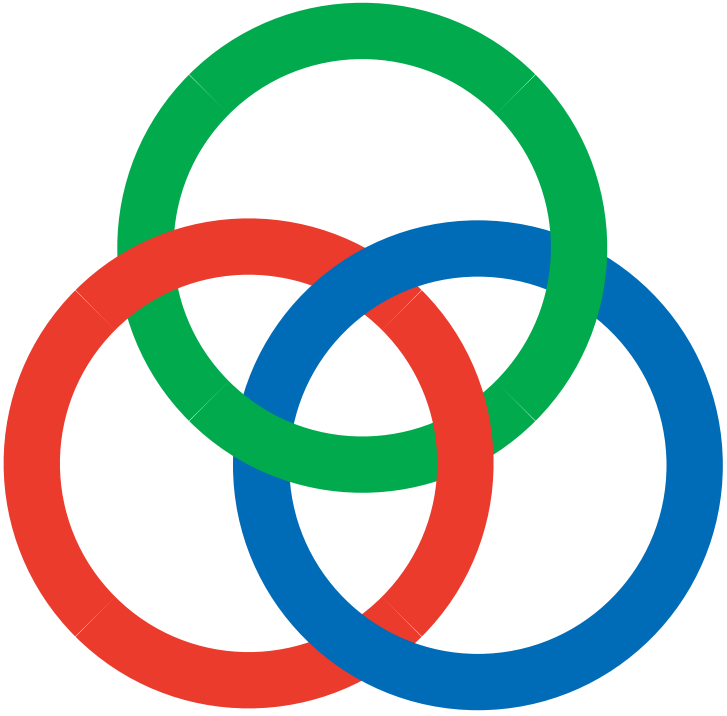
Two-neutron halo nuclei are Borromean...

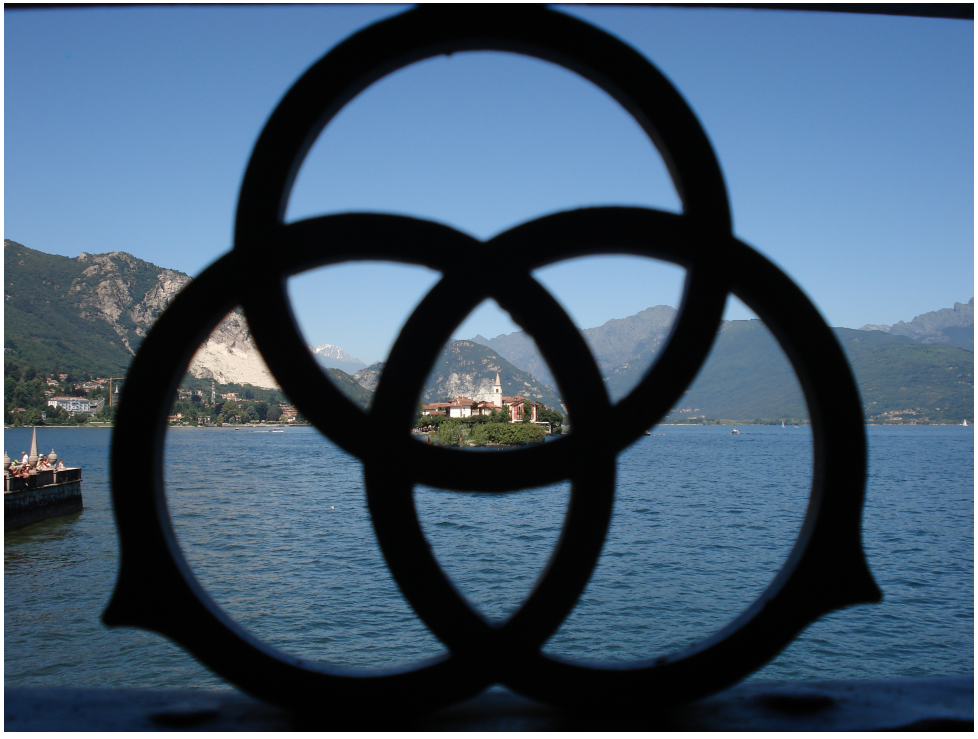
c+n+n is bound but not c+n nor n+n : ^{11}Li bound but not ^{10}Li nor ^{2}n

Borromean nuclei

Named after the Borromean rings...

[Zhukov *et al.* Phys. Rep. **231**, 151 (1993)]





Isole Borromee

sul lago Maggiore. . .



Reactions with Halo Nuclei

Halo nuclei exhibit a very exotic structure

However difficult to study because of short lifetime : $\tau_{1/2}(^{11}\mathrm{Be}) = 13$ s

$\Rightarrow$ often studied through reactions :

- interaction :

$$^{11}\mathrm{Be} + \mathrm{C} \rightarrow \cancel{^{11}\mathrm{Be}} + \mathrm{X}$$

- knockout, e.g., one-neutron removal :

$$^{11}\mathrm{Be} + \mathrm{C} \rightarrow {}^{10}\mathrm{Be} + \mathrm{X}$$

- transfer, e.g. (d,p) :

$$^{10}\mathrm{Be} + \mathrm{d} \rightarrow {}^{11}\mathrm{Be} + \mathrm{p}$$

- (elastic) breakup :

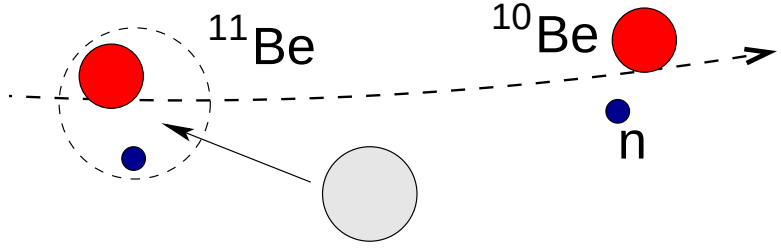
$$^{11}\mathrm{Be} + \mathrm{Pb} \rightarrow {}^{10}\mathrm{Be} + \mathrm{n} + \mathrm{Pb}$$

Breakup reaction

Breakup ≡ dissociation of projectile in constituent clusters by interaction with target

$$^{11}\mathrm{Be} + \mathrm{Pb} \quad \rightarrow \quad ^{10}\mathrm{Be} + \mathrm{n} + \mathrm{Pb}$$

The target T acts differently on projectile P contituents
$\Rightarrow$ tidal force $\rightarrow$ breakup



[Figure by A. Moro]

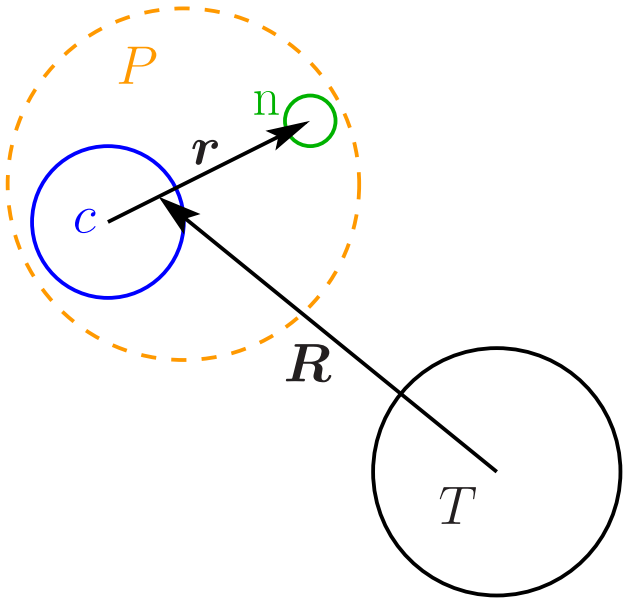
$\Rightarrow$ reveals the internal structure

Need a good understanding of the reaction mechanism
i.e. an accurate theoretical description of reaction coupled to a realistic model of projectile

Framework

Projectile (P) modelled as a two-body system :
core (c)+loosely bound neutron (n) described by

$$H_0 = T_r + V_{c\mathrm{n}}(\boldsymbol{r})$$



$V_{c\mathrm{n}}$ adjusted to reproduce
P spectrum

Target T seen as
structureless particle

P-T interaction simulated by optical potentials

$\Rightarrow$ breakup reduces to three-body scattering problem :

$$[T_R + H_0 + V_{cT} + V_{\mathrm{n}T}]\,\Psi(\boldsymbol{r},\boldsymbol{R}) = E_T\Psi(\boldsymbol{r},\boldsymbol{R})$$

with initial condition $\Psi(\boldsymbol{r},\boldsymbol{R}) \underset{Z\to-\infty}{\longrightarrow} e^{iKZ+\cdots}\Phi_0(\boldsymbol{r})$

[Baye & P.C., Lecture Notes in Phys. **848**, 121 (2012)]

Halo-EFT

Simulate c-n interaction by effective potential

Halo EFT : clear separation of scales (in energy or in distance)
$\Rightarrow$ provides an expansion parameter (small scale / large scale)
along which the low-energy behaviour is expanded

[Bertulani, Hammer, Van Kolck, NPA **712**, 37 (2002)]

[Hammer, Ji, Phillips, JPG **44**, 103002 (2017)]

Use narrow Gaussian potentials

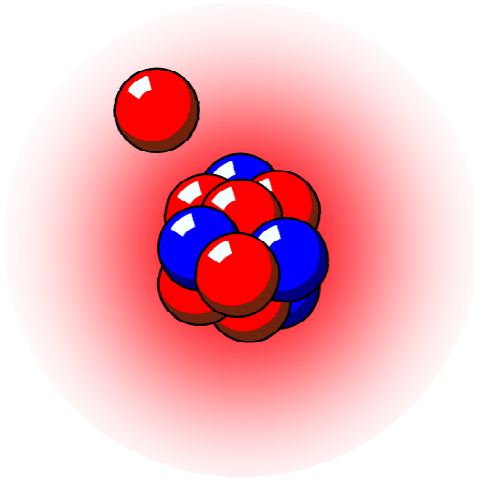
$$V_{lj}(r) = V_0^{lj}\, e^{-\frac{r^2}{2\sigma^2}} + V_2^{lj}\, r^2 e^{-\frac{r^2}{2\sigma^2}}$$

σ evaluates the sensitivity of calculations to short-range physics

@ LO :
Fit V_0^{lj} to reproduce S_{n}

@ NLO :
Fit V_2^{lj} to also reproduce Asymptotic Normalisation Constant C_{J^π}

Example : ^{11}Be



^{10}Be + n

1/2$^-$	-0.184	$0p_{1/2}$
1/2$^+$	-0.502	$1s_{1/2}$

^{11}Be bound spectrum

$$^{11}\mathrm{Be} \equiv {}^{10}\mathrm{Be}(0^+) + \mathrm{n}$$

^{10}Be assumed in 0^+ ground state

$\Rightarrow J^\pi$ of ^{11}Be states fixed by l_j of n :

- $1/2^+$ ground state in $s_{1/2}$
- $1/2^-$ excited state in $p_{1/2}$

$\Rightarrow$ fit V_{cn} in $s_{1/2}$ and $p_{1/2}$ waves

But...

- What do we take for C_{J^π} ?
- What do we do in $p_{3/2}$?
- and for higher l ?

Ab initio description of ^{11}Be

NCSM-C calculation of ^{11}Be [Calci *et al.* PRL **117**, 242501 (2016)]

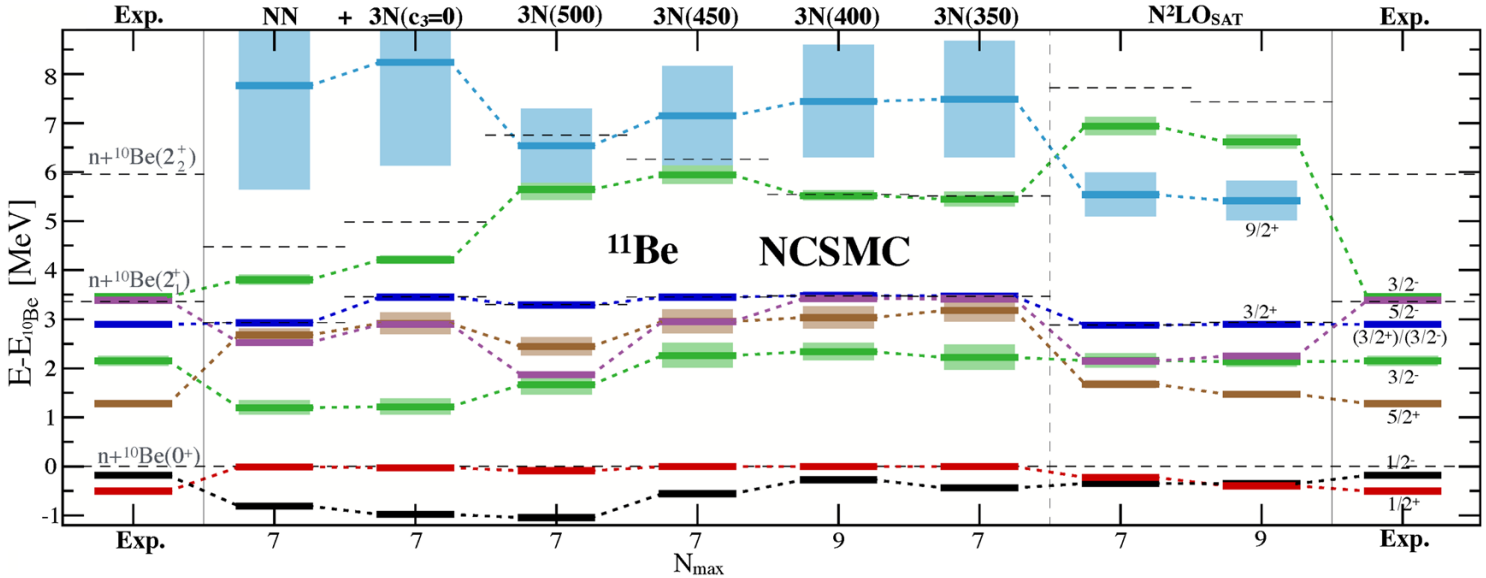


FIG. 2. NCSMC spectrum of ^{11}Be with respect to the $n + {}^{10}$Be threshold. Dashed black lines indicate the energies of the ^{10}Be states. Light boxes indicate resonance widths. Experimental energies are taken from Refs. [1,51].

- $\frac{1}{2}^+$ ground state :
  $\epsilon_{\frac{1}{2}^+} = -0.502$ MeV
  $C_{\frac{1}{2}^+} = 0.786$ fm$^{-1/2}$
- $\frac{1}{2}^-$ bound excited state :
  $\epsilon_{\frac{1}{2}^-} = -0.184$ MeV
  $C_{\frac{1}{2}^-} = 0.129$ fm$^{-1/2}$

Calci *et al.* also predict the ^{10}Be-n phaseshift

^{10}Be-n Halo-EFT potential @ NLO

Use 2 narrow Gaussian terms @ NLO

$$V_{lj}(r) = V_0^{lj}\, e^{-\frac{r^2}{2\sigma^2}} + V_2^{lj}\, r^2 e^{-\frac{r^2}{2\sigma^2}}$$

[P.C., Phillips & Hammer, PRC **98**, 034610 (2018)]

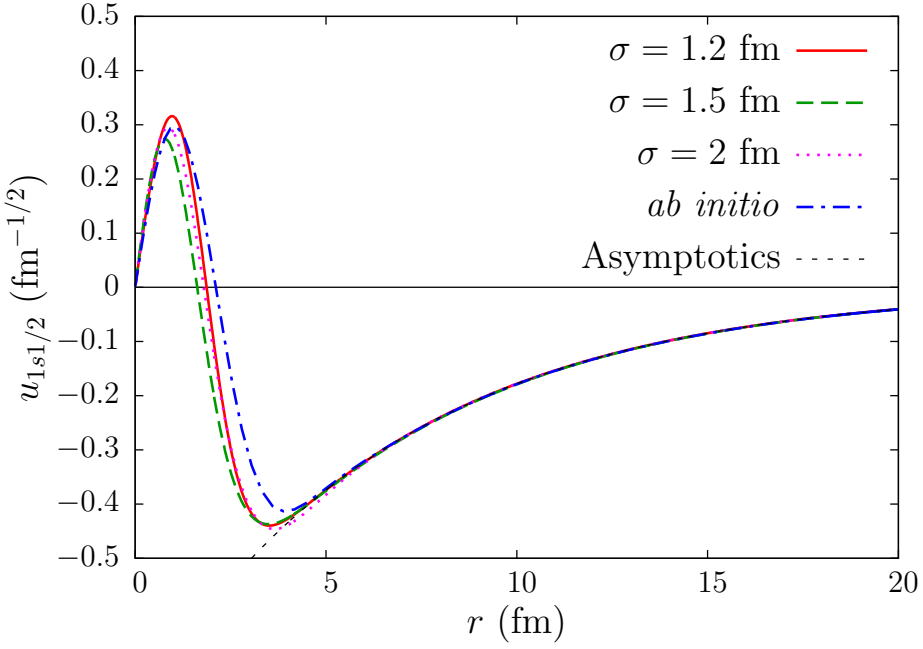
- In $s\frac{1}{2}$ and $p\frac{1}{2}$: fit V_0^{lj} and V_2^{lj} to reproduce
  - ϵ_{nlj} (known experimentally)
  - C_{nlj} (predicted *ab initio*) [Calci *et al.* PRL **117**, 242501 (2016)]
- $V_{p3/2} = 0$ to reproduce *ab initio* $\delta_{3/2^-} \sim 0$
- For $l > 1$: $V_{lj} = 0$ @ NLO

$\sigma =$ 1.2, 1.5 or 2 fm evaluates the sensitivity to short-range physics

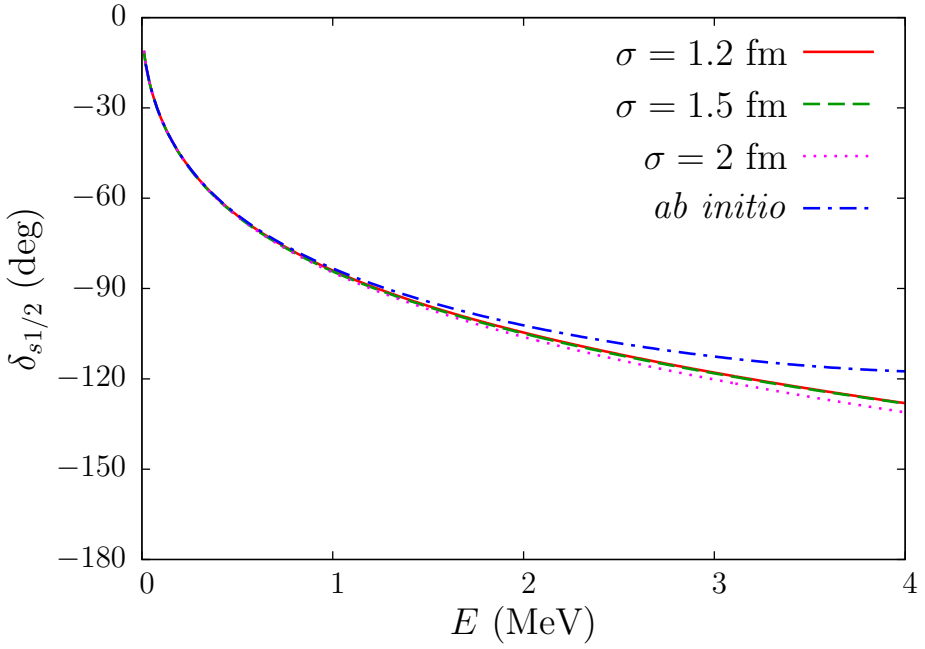
$s\frac{1}{2}$: @ NLO potentials fitted to $\epsilon_{\frac{1}{2}^+}$ and $C_{\frac{1}{2}^+}$

Potentials fitted to $\epsilon_{1s\frac{1}{2}} = -0.502$ MeV and $C_{1s\frac{1}{2}} = 0.786$ fm$^{-1/2}$

Ground-state wave function

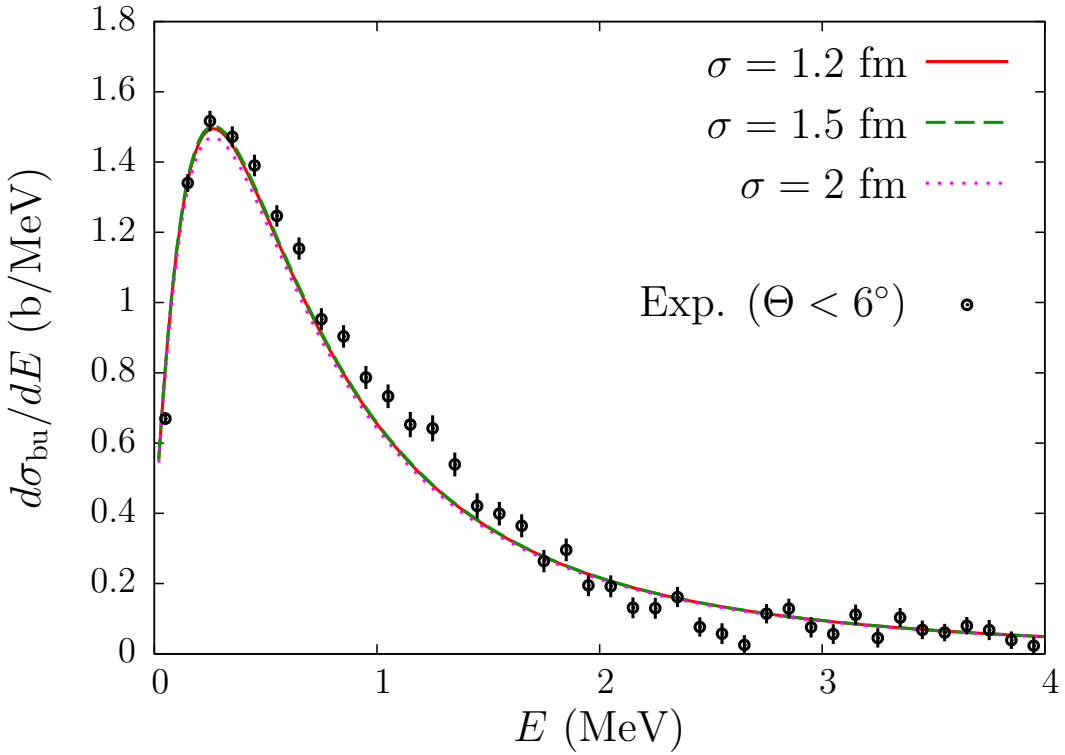


$s_{1/2}$ phaseshifts



- Wave functions : same asymptotics but different interior
- $\delta_{s\frac{1}{2}}$: all effective potentials are in good agreement with *ab initio* up to 1.5 MeV
- Similar results obtained for $p\frac{1}{2}$ (excited bound state)

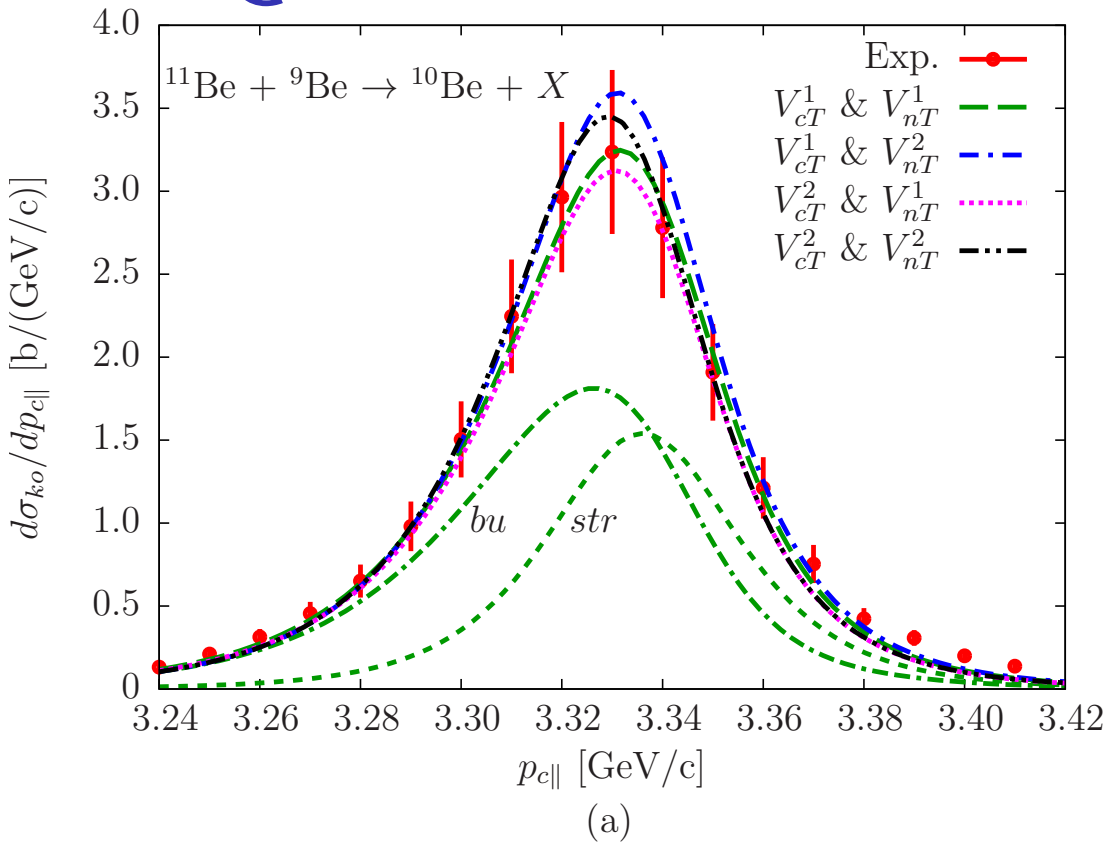
^{11}Be+Pb$\rightarrow^{10}$Be+n+Pb @ 70A MeV



Exp : [Fukuda *et al.* PRC **70**, 054606 (2004)]
Th. : [P.C., Phillips & Hammer, PRC **98**, 034610 (2018)]

- All calculations provide very similar results $\forall\sigma$ despite differences in internal wave function
  $\Rightarrow$ reaction is peripheral [P.C. & Nunes PRC **75**, 054609 (2007)]
- Excellent agreement with data on Pb (no fitting parameter)
  $\Rightarrow$ confirms *ab initio* ANC and phaseshift

^{11}Be+Be→^{10}Be+X @ 60A MeV



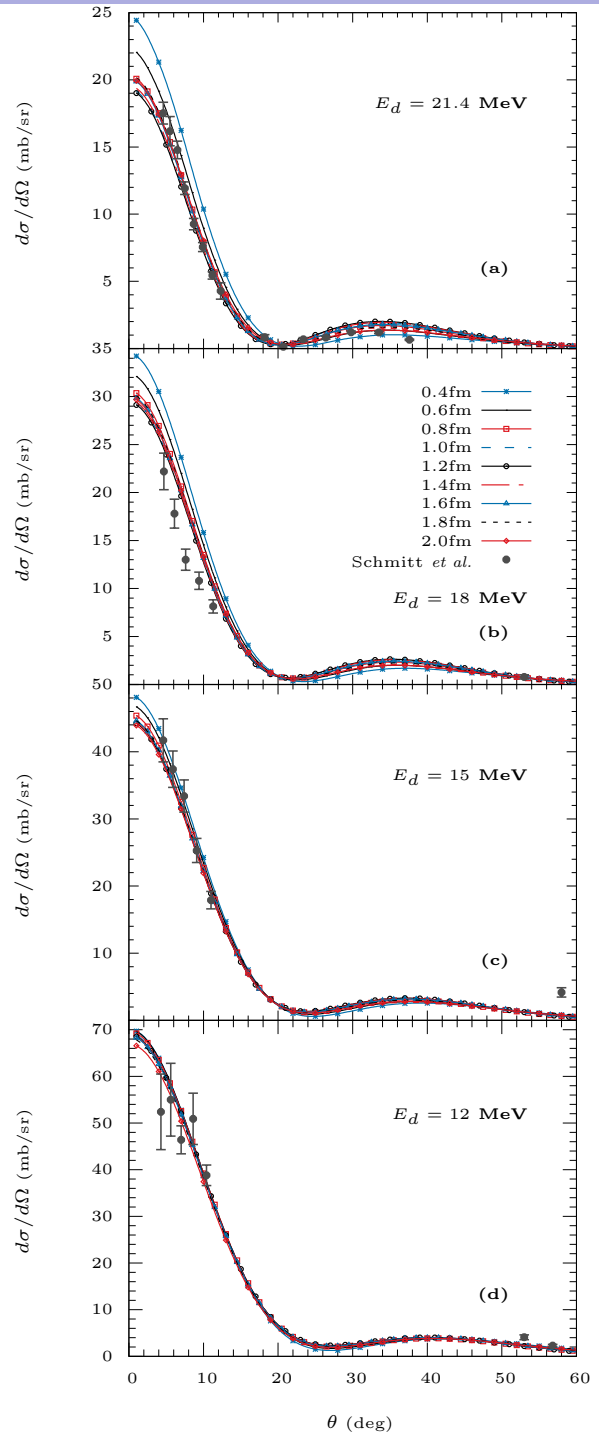
(a)

Exp : [Aumann *et al.* PRL **84**, 35 (2000)]
Th. : [Hebborn & P.C. PRC **104**, 024616 (2021)]

- Excellent agreement with KO experiment (no fitting parameter)
  Though strong sensitivity to optical potentials
- Very similar results $\forall\sigma \Rightarrow$ peripheral [PRC **100**, 054607 (2019)]
  $\Rightarrow$ confirms *ab initio* ANC and phaseshift

^{10}Be(d,p)^{11}Be

- This idea can be extended to transfer
  [Yang & P.C. PRC **98**, 054602 (2018)]
- Various descriptions of ^{11}Be
  with $\sigma = 0.4 - 2.0$ fm
- Excellent agreement with data
  [Schmidt *et al.* PRL **108**, 192701 (2012)]
  $\Rightarrow$ confirms *ab initio* ANC



What about Spectroscopic Factors ?

So far, we have assumed a single-particle description
But reality is more complex :

$$\Psi^{1/2^+}\left(^{11}\mathrm{Be}\right) = \varphi_{1s1/2} \otimes \phi^{0^+}\left(^{10}\mathrm{Be}\right) + \varphi_{0d5/2} \otimes \phi^{2^+_1}\left(^{10}\mathrm{Be}\right) + \ldots$$

with Spectroscopic Factors $\mathcal{S}_{1s1/2\otimes 0^+} = \|\varphi_{1s1/2}\|^2 = 1$with
Spectroscopic Factors $\mathcal{S}_{1s1/2\otimes 0^+} = \|\varphi_{1s1/2}\|^2 < 1$

Usually, $\mathcal{S}_{1s1/2\otimes 0^+}$ inferred from experiment

$$\mathcal{S}_{1s1/2\otimes 0^+} = \frac{\sigma^{\mathrm{exp}}}{\sigma^{\mathrm{th}}}$$

where σ^{th} computed assuming single-particle structure normed to 1

Is this correct ?
How reliable are these SFs ?

Including core excitation within Halo-EFT

Effective c-n Hamiltonian :

$$H(\boldsymbol{r},\xi) = T_r + V_{\rm cn}(\boldsymbol{r},\xi) + h_c(\xi)$$

where $h_c \equiv$ core Hamiltonian with eigenstates $\phi^{I_c^{\pi_c}}_{M_c}$

We consider a particle-rotor model *à la* Bohr and Mottelson

$$V_{cn}(\boldsymbol{r},\xi) = V(r) + \beta\,\sigma\,Y^0_2(\hat{r}')\frac{d}{d\sigma}V(r)$$

The wave function is expanded on the core eigenstates

$$\Psi^{J^\pi M}(\boldsymbol{r},\xi) = \sum_\alpha i^l \frac{u_\alpha(r)}{r}[\mathcal{Y}_{lj}(\hat{r}) \otimes \phi^{I_c^{\pi_c}}_{M_c}(\xi)]^{JM}$$

Leads to set of coupled equations

$$\left\{\frac{\hbar^2}{2\mu}\left[-\frac{d^2}{dr^2} + \frac{l(l+1)}{r^2}\right] + V_{\alpha\alpha}(r) + \epsilon_\alpha - E\right\} u_\alpha(r) = -\sum_{\alpha'\neq\alpha} V_{\alpha\alpha'}(r)u_{\alpha'}(r),$$

[Kubushishi, P.C. arXiv:2507.13585 (2025)]

^{11}Be $\frac{1}{2}^+$ ground state

We include only the 2^+_1 excited state of ^{10}Be in the model ^{11}Be $\frac{1}{2}^+$ g.s. :

$$\left|\frac{1}{2}^+\right\rangle = \left|1s_{1/2} \otimes 0^+\right\rangle + \left|0d_{5/2} \otimes 2^+\right\rangle + \left|0d_{3/2} \otimes 2^+\right\rangle$$

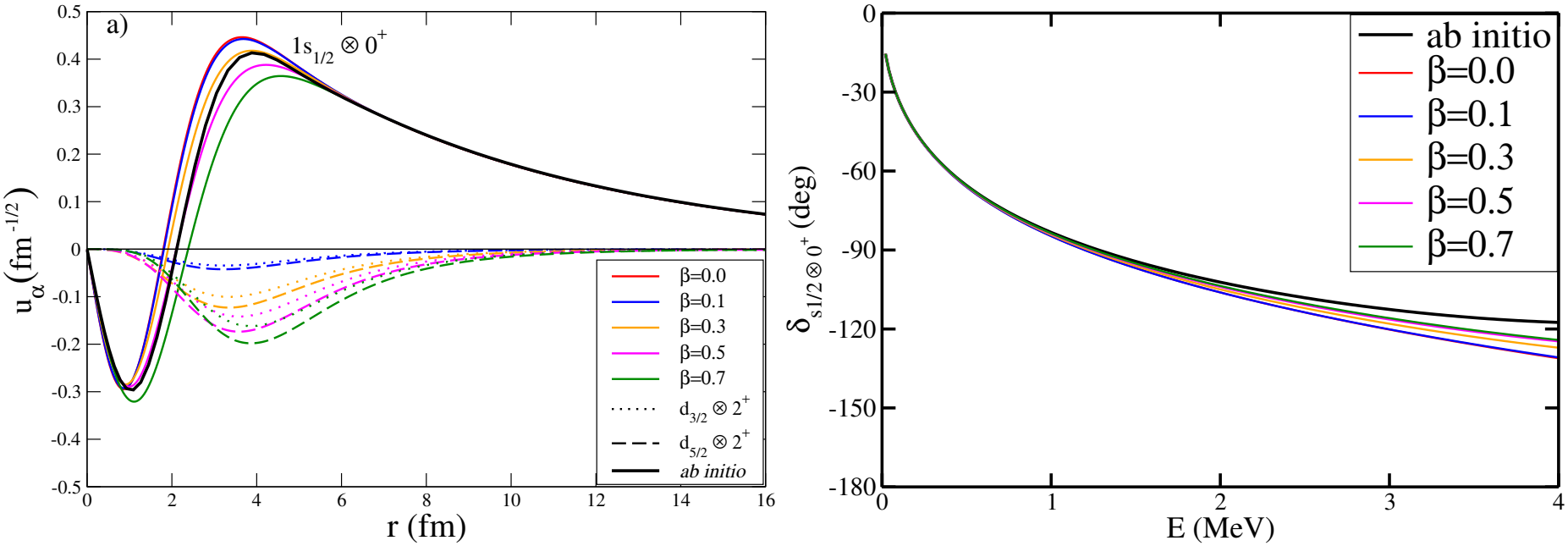
As in Halo-EFT, c-n potential V is Gaussian + derivative

Both LECs fitted to experimental S_n and *ab initio* ANC $C_{1/2^+}$

[Calci *et al.* PRL **117**, 242501 (2016)]

β varied to see the effect of coupling on calculations

Effect of coupling in ^{11}Be $\frac{1}{2}^+$ ground state



[Kubushishi, P.C. arXiv:2507.13585 (2025)]

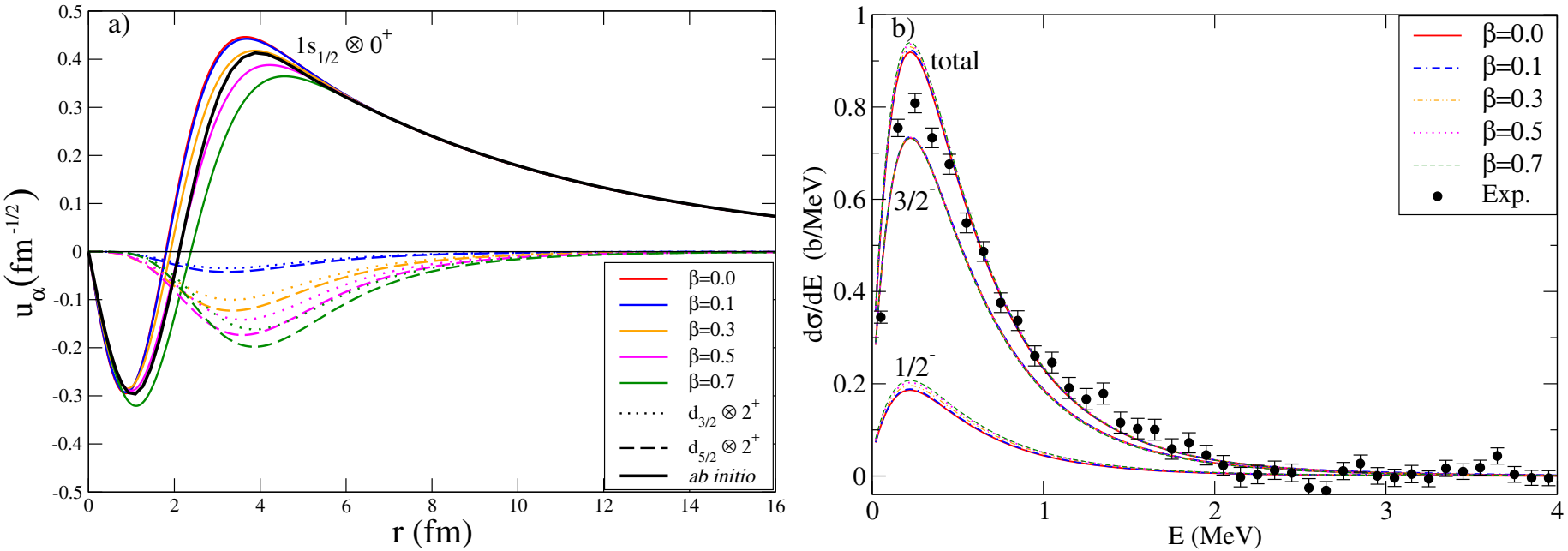
Coupling moves probability strength from $1s_{1/2} \otimes 0^+ \rightarrow 0d_j \otimes 2^+$

For $\beta = 0.3$–0.5, good agreement with *ab initio*

- radial wave function
- phaseshift

$\Rightarrow$ provides efficient emulator of *ab initio* for nuclear-reaction models and coupled-channel model of ^{11}Be with different SFs

^{11}Be Coulomb breakup on Pb @ $70A$ MeV



Th. : [Kubushishi, P.C. PLB **879**, 140694 (2026)] Exp. : [Fukuda *et al.* PRC **70**, 054606 (2004)]

Coupling leads to $\mathcal{S}_{1s_{1/2}\otimes 0^+} = 1$ ($\beta = 0$)–0.8 ($\beta = 0.7$)
keeping $C_{1/2^+}$ predicted *ab initio*
$\Rightarrow$ we should see a decrease of $d\sigma_{\rm bu}/dE$ of 20%
BUT we see

- no change in $d\sigma_{\rm bu}/dE \Rightarrow$ not sensitive to SF
- good agreement with experiment $\Rightarrow$ validates *ab initio* $C_{1/2^+}$

Summary and prospect

Halo nuclei are exotic quantal structures found far from stability
Seen as a core + 1 or 2 loosely-bound neutrons

Studied mostly through reactions : KO, breakup...

Halo EFT very efficient to describe halo nuclei in reaction models

- Ideal to test *ab initio* predictions on reaction measurements
  [P.C., Phillips & Hammer, PRC **98**, 034610 (2018)]
  $\Rightarrow$ confirms ^{11}Be calculations [Calci *et al.* PRL **117**, 242501 (2016)]
- Core excitation included within Halo EFT
  - Applied to ^{11}Be [Kubushishi, P.C. PLB **879**, 140694 (2026)]
  - Better fits $\frac{1}{2}^+$ g.s. to *ab initio*
  - Coulomb breakup shows no sensitivity to SF
- Great for a Bayesian analysis of reactions
  [Bozet, P.C., Svensson, arXiv:2610.32331]

Thank you for your attention. . .

and thanks to my collaborators :

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Daniel Phillips

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Isak Svensson

Filomena Nunes

Chloë Hebborn