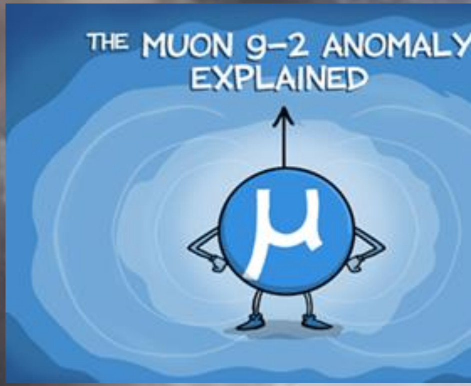
First Results – and afterthoughts -- from the Fermilab Muon g-2 Experiment

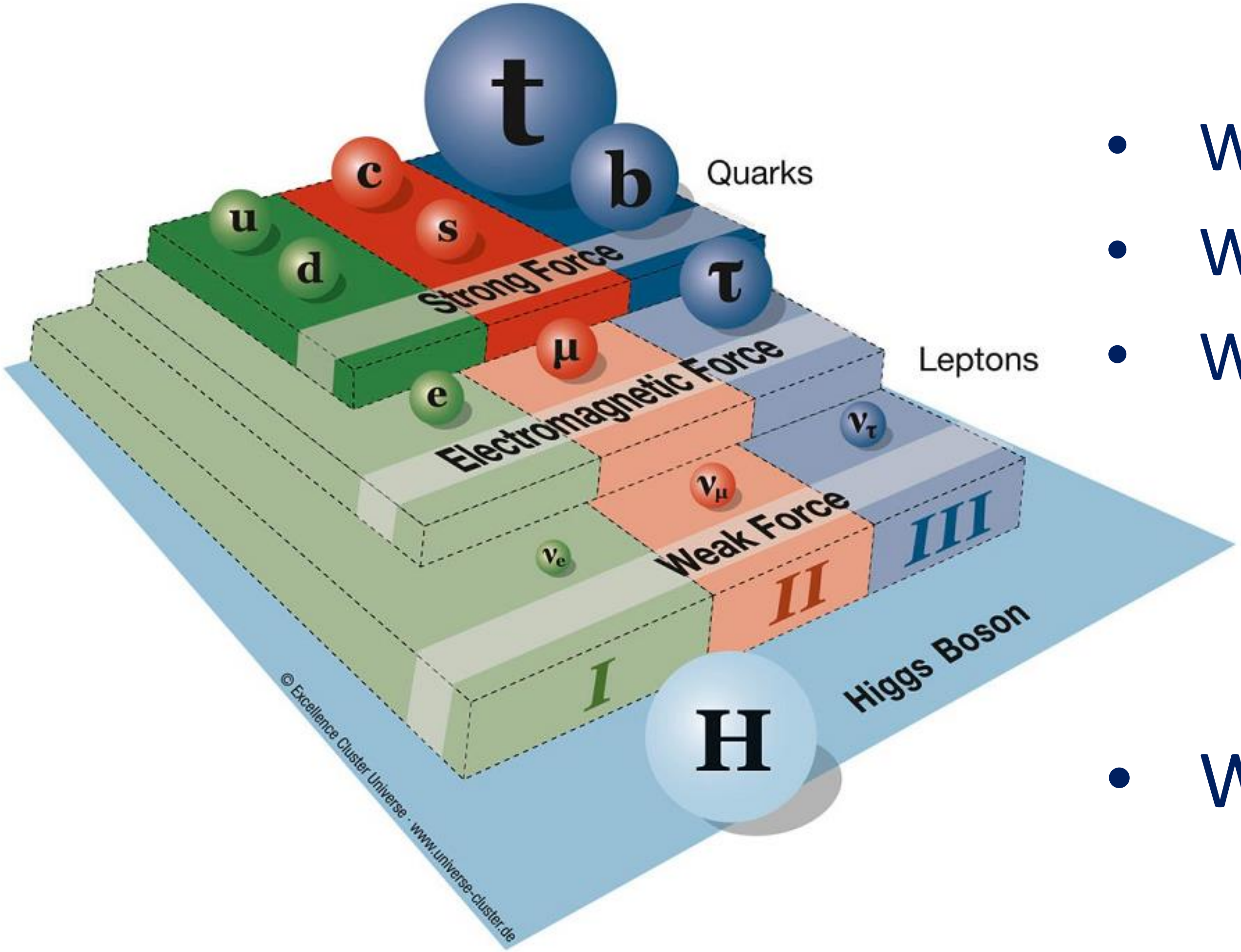


CENPA
Center for Experimental Nuclear Physics and Astrophysics

Good Morning ! ☺

David Hertzog, University of Washington
Colloquim: Paul Scherrer Institute
Sept. 23, 2021

We all know that the Standard Model of particle physics has been a Great Achievement !!



- We know the particles
- We know the forces
- We know the "rules of engagement"

- What's not to love !!

BUT! We also hear that the Standard Model is somehow "incomplete"

And, it seems like it's taking forever to figure out how to complete it.

- **Theorists** are developing creative "new physics models"
  - To "fix" one or more of the known SM problems
    - e.g, Dark Matter?, Hierarchy?, Gravity?, Matter-Antimatter Asymmetry? …
  - Or to just propose new aspects or particles
    - e.g., Dark Photons, Time-Changing Fundamental Constants, Extra Dimensions, …

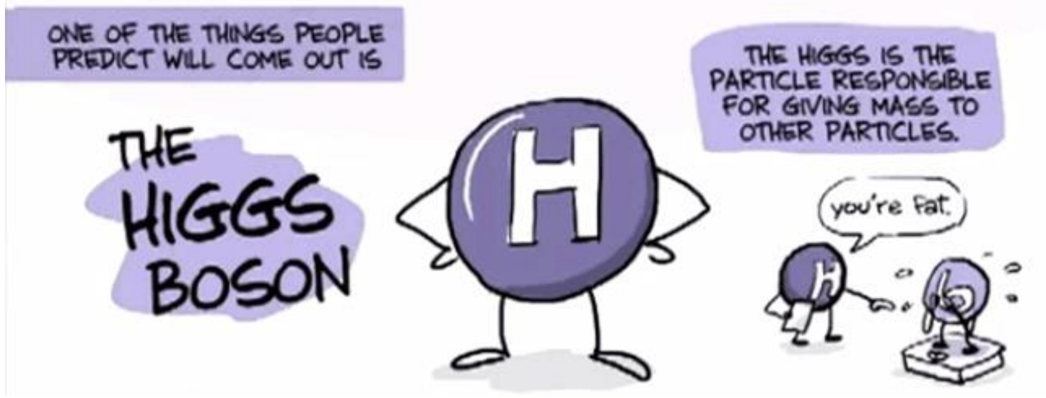
- **Experimentalists** are detectives looking for "smoking gun" evidence to support or rule out these suggestions
  - "**Negative**" results help get rid of wrong models
  - "**Positive**" results might indicate something NEW!

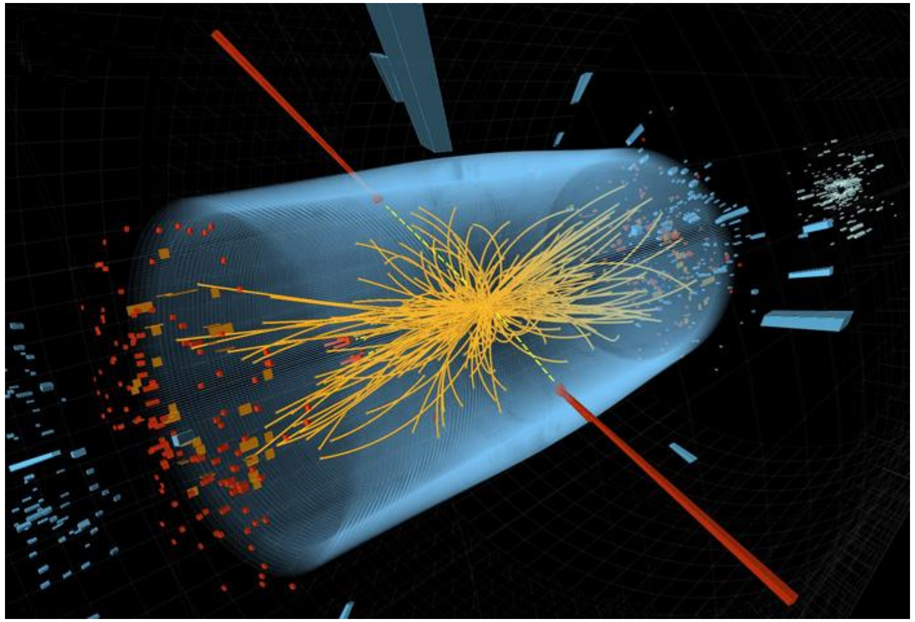
Today I will tell you a POSITIVE story ☺

How do experimentalists go about this?

Usually by smashing particles together very violently

- To reach high mass scales directly
- To be “general” in observations of interactions
- Because it “works”
- ➔Hurrah for the Higgs!





Arguably, this “completes” the Standard Model. What tools do we use to search beyond it ?

Generally, two approaches

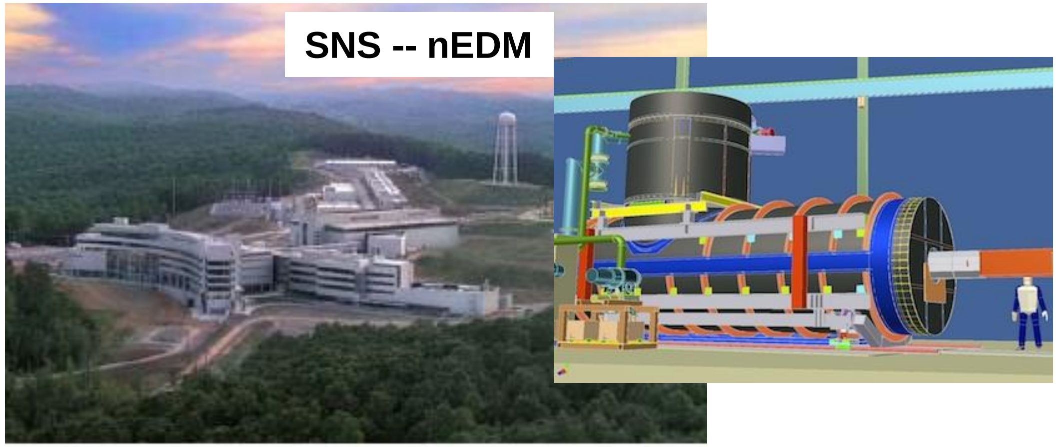
(I often use this metaphor; my apologies if you've seen it)

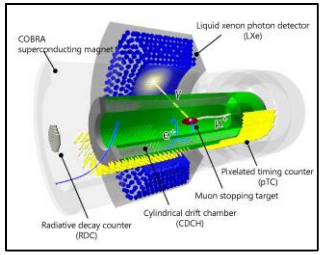
Direct approach

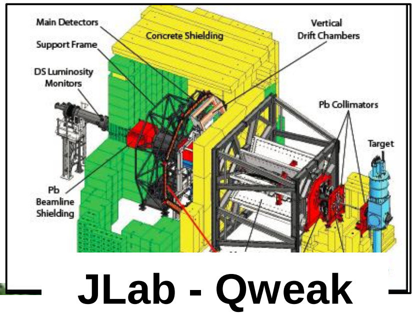
The LHC @ High Luminosity

But, there is also an indirect approach: "Quantum tunneling"

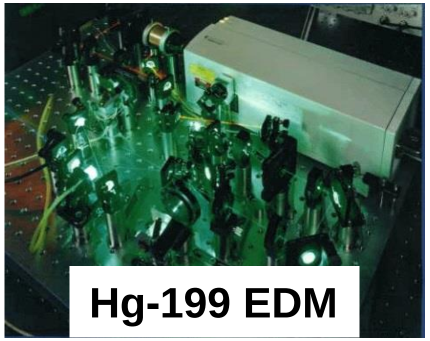
The Indirect approach using *Precision and Intensity*


SNS -- nEDM





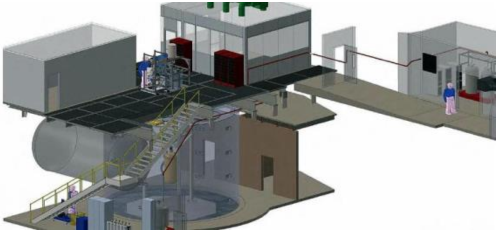
JLab - Qweak

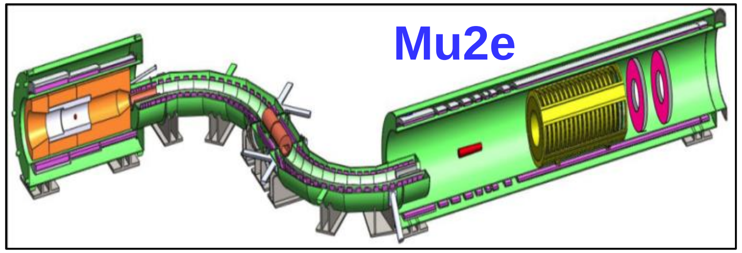

Hg-199 EDM


PSI: MEG, Mu3e, nEDM

g-2


UW - ADMX


Underground - LUX

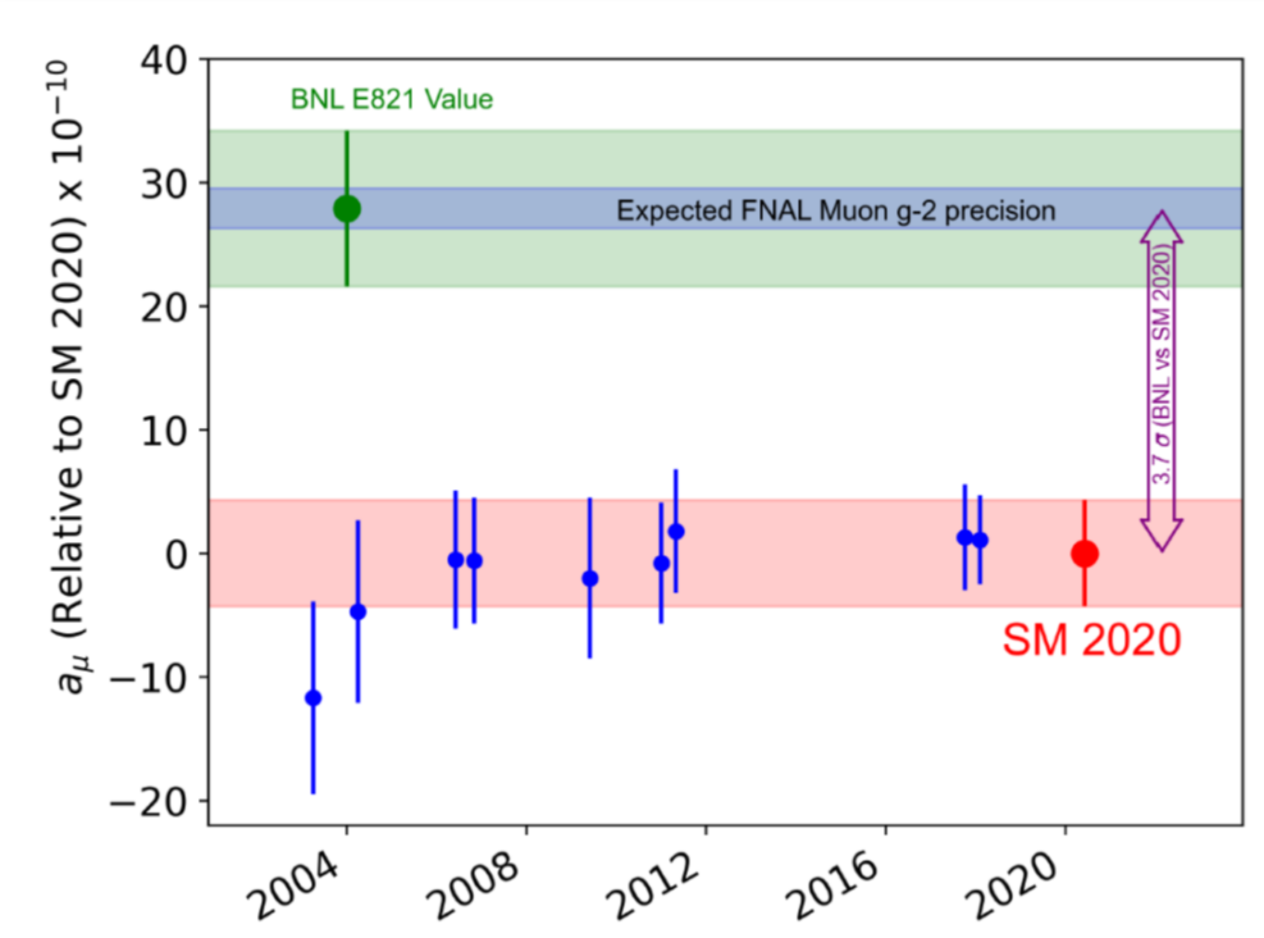

Mu2e

- **Is lepton number conserved?**
  - **MEG, Mu2e, Mu3e**
- **Origin of the Matter – Antimatter asymmetry in the universe**
  - **EDMs of neutrons, atoms, molecules …**
  - **Are neutrinos their own antiparticles? $0\nu\beta\beta$ efforts**
- **What is Dark Matter ?**
  - **WIMP searches – many clever experiments**
  - **Axion searches - ADMX**
- **Are there deviations from SM predictions?**
  - **Muon g-2**
  - **Parity Violating Electron Scattering … running of $\sin^2\theta_W$**
  - **Tests of the unitarity of the CKM mixing matrix**
- **Atomic physics tests with incredible precision (too many to list)**

For nearly 20 years, one measurement has stood out as being inconsistent with the Standard Model

Blame the experiment?

Blame the theory?

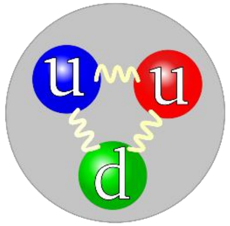


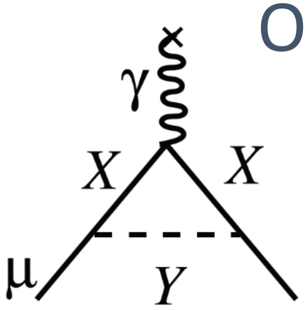
The Muon's Anomalous Magnetic Dipole Moment

$$a_\mu \equiv \frac{g-2}{2}$$

$$\vec{\mu} = \mathbf{g}\frac{Qe}{2m}\vec{S}$$

Dirac: **g = 2** for a point-like spin 1/2 fermion





One can also have an "anomalous" moment

-- from internal structures that nucleons have

-- from virtual loops that encapsulate all possible interactions with an external field

The "g-2 Test" compares a measurement to a precise calculation to investigate the completeness of the Standard Model

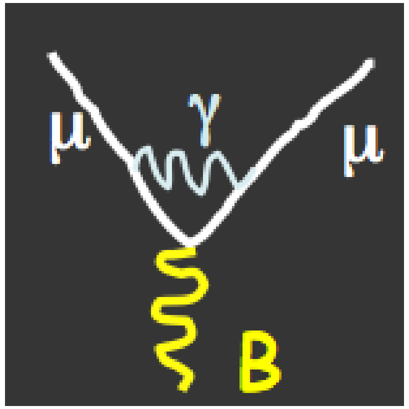
Question: How well does the Standard Model predict this quantity?

The Muon g Factor Summarized (not yet including today's result)

g(exp) 2.002331

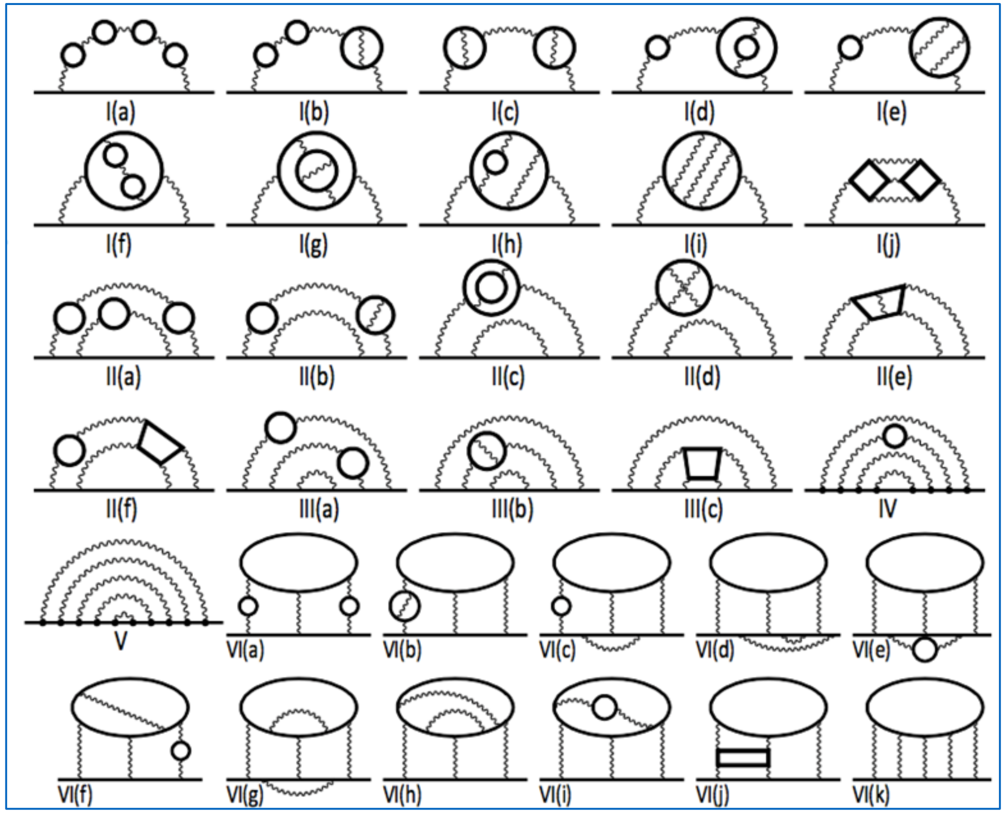
g(thy) 2.002331

QED quantized electromagnetism



99.99% of a_μ

And eventually all these:

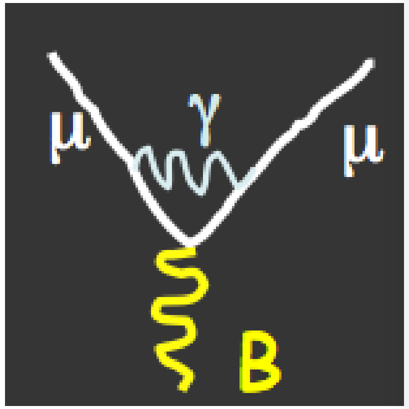


The Muon g Factor Summarized (not yet including today's result)

g(exp) 2.00233184

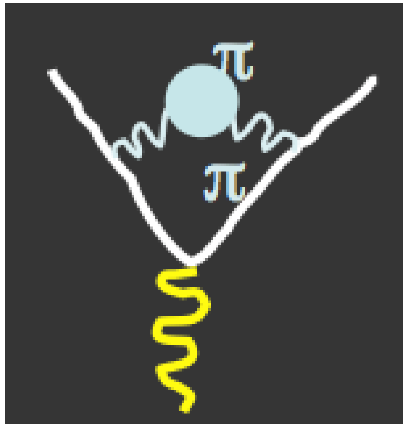
g(thy) 2.00233183

QED quantized electromagnetism



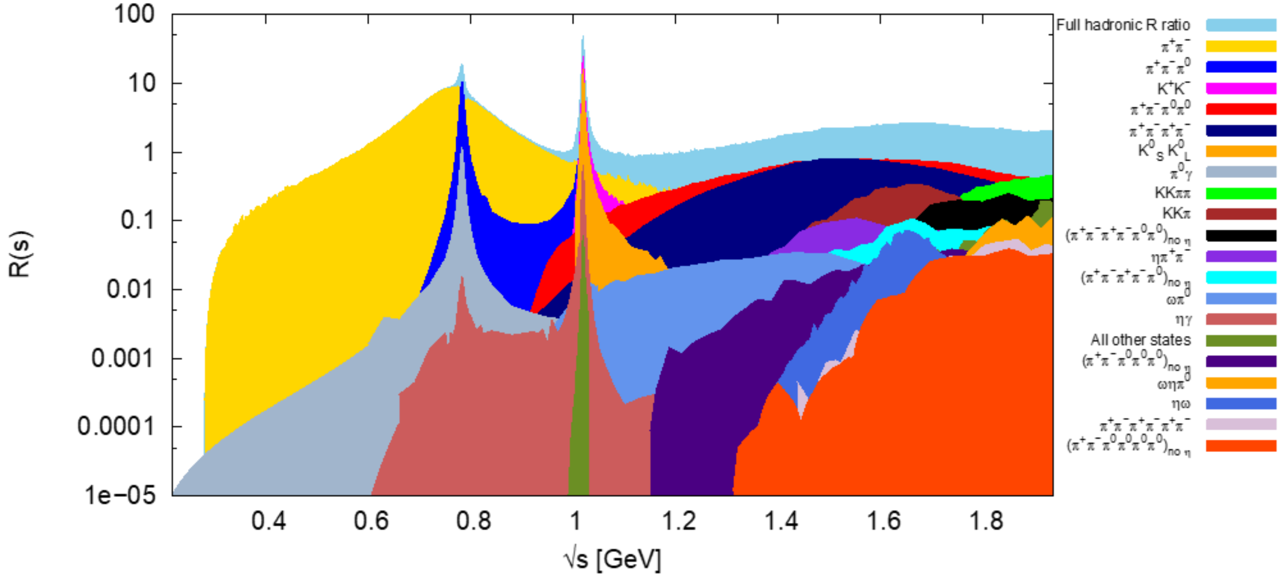
99.99% of a_μ

QCD strong force that binds nucleons



0.0061% of a_μ

Mostly "data driven" from e^+e^- cross sections:

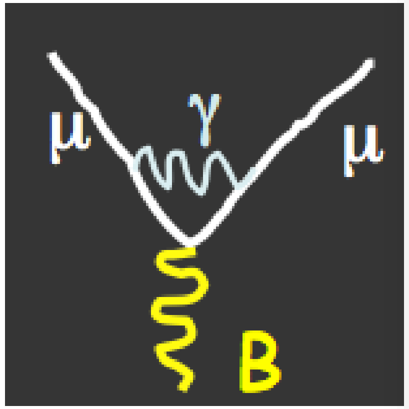


The Muon g Factor Summarized (not yet including today's result)

g(exp) 2.002331841

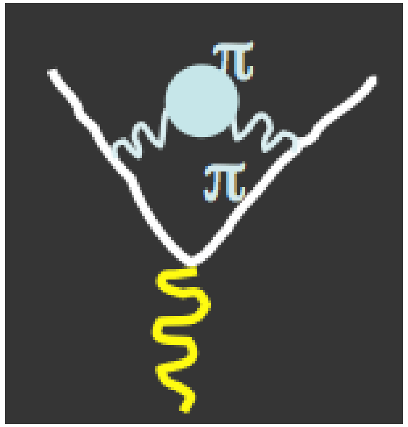
g(thy) 2.002331836

QED quantized electromagnetism



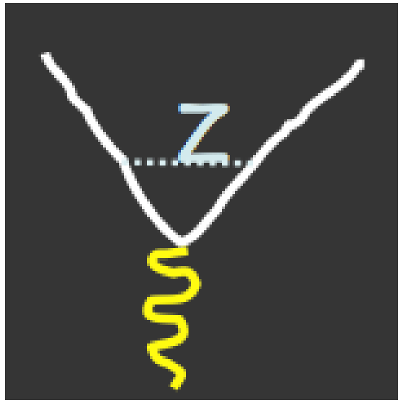
99.99% of a_μ

QCD strong force that binds nucleons



0.0061% of a_μ

Electroweak force that makes nucleons (and muons) unstable



0.0001% of a_μ

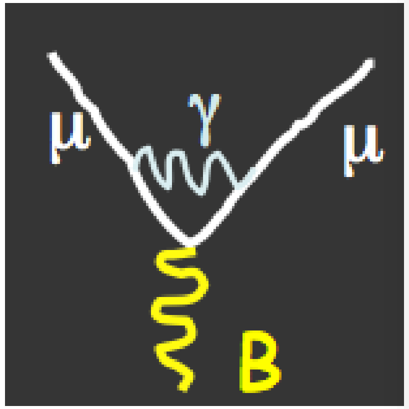
TINY effect

The Muon g Factor Summarized (not yet including today’s result)

g(exp) 2.00233184178

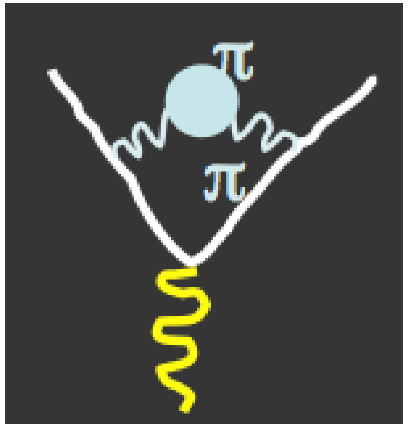
g(thy) 2.00233183620

QED quantized electromagnetism



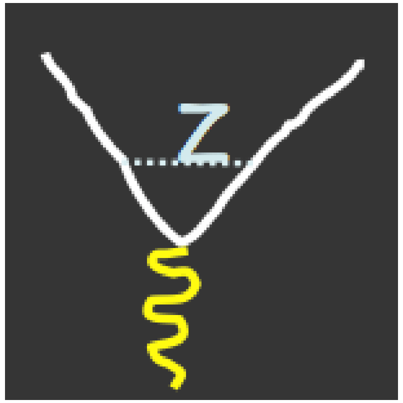
99.99% of a_μ

QCD strong force that binds nucleons



0.0061% of a_μ

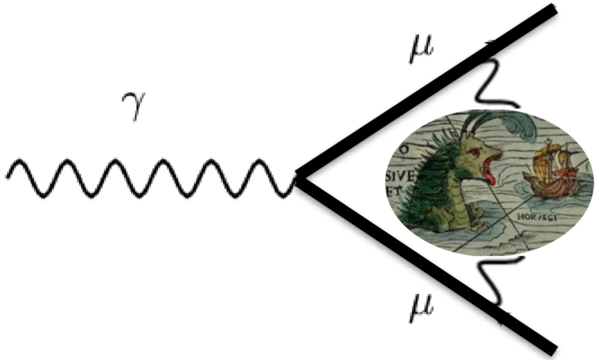
Electroweak force that makes nucleons (and muons) unstable



0.0001% of a_μ

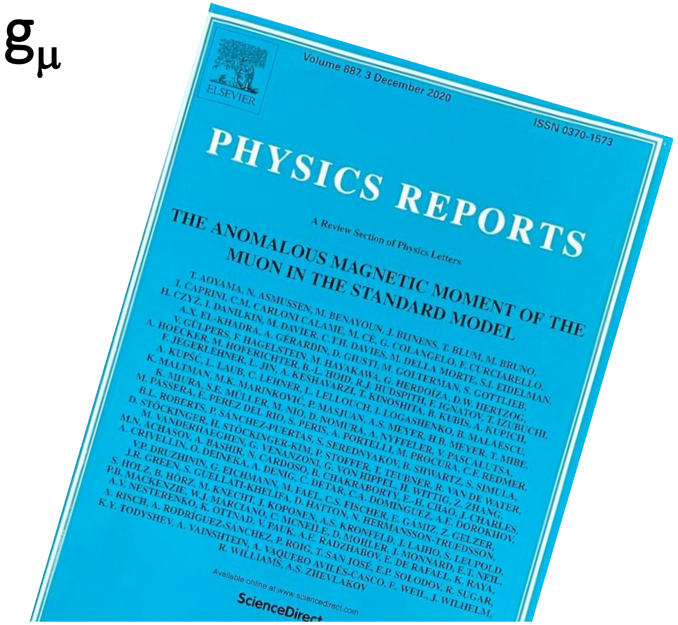
Undiscovered things?

dark matter, SUSY,…



Muon g-2 Theory Initiative defines benchmark value for g_μ
They published a global "reference value" in 2020

Group photo from the Seattle workshop in September 2019, https://indico.fnal.gov/event/21626/



Volume 887, 3 December 2020

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ELSEVIER

PHYSICS REPORTS

A Review Section of Physics Letters

THE ANOMALOUS MAGNETIC MOMENT OF THE
MUON IN THE STANDARD MODEL

T. AOYAMA, N. ASMUSSEN, M. BENAYOUN, J. BIJNENS, T. BLUM, M. BRUNO,
I. CAPRINI, C.M. CARLONI CALAME, M. CÈ, G. COLANGELO, F. CURCIARELLO,
H. CZYŻ, I. DANILKIN, M. DAVIER, C.T.H. DAVIES, M. DELLA MORTE, S.I. EIDELMAN,
A.X. EL-KHADRA, A. GÉRARDIN, D. GIUSTI, M. GOLTERMAN, S. GOTTLIEB,
V. GÜLPERS, F. HAGELSTEIN, M. HAYAKAWA, G. HERDOÍZA, D.W. HERTZOG,
A. HOECKER, M. HOFERICHTER, B.-L. HOID, R.J. HUDSPITH, F. IGNATOV, T. IZUBUCHI,
F. JEGERLEHNER, L. JIN, A. KESHAVARZI, T. KINOSHITA, B. KUBIS, A. KUPICH,
A. KUPŚĆ, L. LAUB, C. LEHNER, L. LELLOUCH, I. LOGASHENKO, B. MALAESCU,
K. MALTMAN, M.K. MARINKOVIĆ, P. MASJUAN, A.S. MEYER, H.B. MEYER, T. MIBE,
K. MIURA, S.E. MÜLLER, M. NIO, D. NOMURA, A. NYFFELER, V. PASCALUTSA,
M. PASSERA, E. PEREZ DEL RIO, S. PERIS, A. PORTELLI, M. PROCURA, C.F. REDMER,
B.L. ROBERTS, P. SÁNCHEZ-PUERTAS, S. SEREDNYAKOV, B. SHWARTZ, S. SIMULA,
D. STÖCKINGER, H. STÖCKINGER-KIM, P. STOFFER, T. TEUBNER, R. VAN DE WATER,
M. VANDERHAEGHEN, G. VENANZONI, G. VON HIPPEL, H. WITTIG, Z. ZHANG,
M.N. ACHASOV, A. BASHIR, N. CARDOSO, B. CHAKRABORTY, E.-H. CHAO, J. CHARLES,
A. CRIVELLIN, O. DEINEKA, A. DENIG, C. DETAR, C.A. DOMINGUEZ, A.E. DOROKHOV,
V.P. DRUZHININ, G. EICHMANN, M. FAEL, C.S. FISCHER, E. GÁMIZ, Z. GELZER,
J.R. GREEN, S. GUELLATI-KHELIFA, D. HATTON, N. HERMANSSON-TRUEDSSON,
S. HOLZ, B. HÖRZ, M. KNECHT, J. KOPONEN, A.S. KRONFELD, J. LAIHO, S. LEUPOLD,
P.B. MACKENZIE, W.J. MARCIANO, C. MCNEILE, D. MOHLER, J. MONNARD, E.T. NEIL,
A.V. NESTERENKO, K. OTTNAD, V. PAUK, A.E. RADZHABOV, E. DE RAFAEL, K. RAYA,
A. RISCH, A. RODRÍGUEZ-SÁNCHEZ, P. ROIG, T. SAN JOSÉ, E.P. SOLODOV, R. SUGAR,
K.Y. TODYSHEV, A. VAINSHTEIN, A. VAQUERO AVILÉS-CASCO, E. WEIL, J. WILHELM,
R. WILLIAMS, A.S. ZHEVLAKOV

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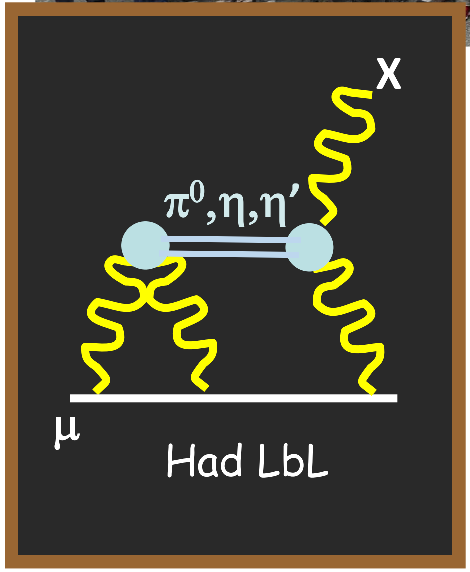


Contribution	Section	Equation	Value $\times 10^{11}$	References
Experiment (E821)		Eq. (8.13)	116 592 089(63)	Ref. [1]
HVP LO (e^+e^-)	Sec. 2.3.7	Eq. (2.33)	6931(40)	Refs. [2–7]
HVP NLO (e^+e^-)	Sec. 2.3.8	Eq. (2.34)	−98.3(7)	Ref. [7]
HVP NNLO (e^+e^-)	Sec. 2.3.8	Eq. (2.35)	12.4(1)	Ref. [8]
HVP LO (lattice, *udsc*)	Sec. 3.5.1	Eq. (3.49)	7116(184)	Refs. [9–17]
HLbL (phenomenology)	Sec. 4.9.4	Eq. (4.92)	92(19)	Refs. [18–30]
HLbL NLO (phenomenology)	Sec. 4.8	Eq. (4.91)	2(1)	Ref. [31]
HLbL (lattice, *uds*)	Sec. 5.7	Eq. (5.49)	79(35)	Ref. [32]
HLbL (phenomenology + lattice)	Sec. 8	Eq. (8.10)	90(17)	Refs. [18–30, 32]
QED	Sec. 6.5	Eq. (6.30)	116 584 718.931(104)	Refs. [33, 34]
Electroweak	Sec. 7.4	Eq. (7.16)	153.6(1.0)	Refs. [35, 36]
HVP (e^+e^-, LO + NLO + NNLO)	Sec. 8	Eq. (8.5)	6845(40)	Refs. [2–8]
HLbL (phenomenology + lattice + NLO)	Sec. 8	Eq. (8.11)	92(18)	Refs. [18–32]
Total SM Value	Sec. 8	Eq. (8.12)	116 591 810(43)	Refs. [2–8, 18–24, 31–36]
Difference: $\Delta a_\mu := a_\mu^{\mathrm{exp}} - a_\mu^{\mathrm{SM}}$	Sec. 8	Eq. (8.14)	279(76)	

The Standard Model uncertainty is 358 ppb

We determined the g-factor of the muon to be:

$$g_\mu = 2.00\underbrace{2\,331\,840\,80(1}_{\text{This piece is ``}(g_\mu - 2)\text{''}}1)$$

(540 ppt)

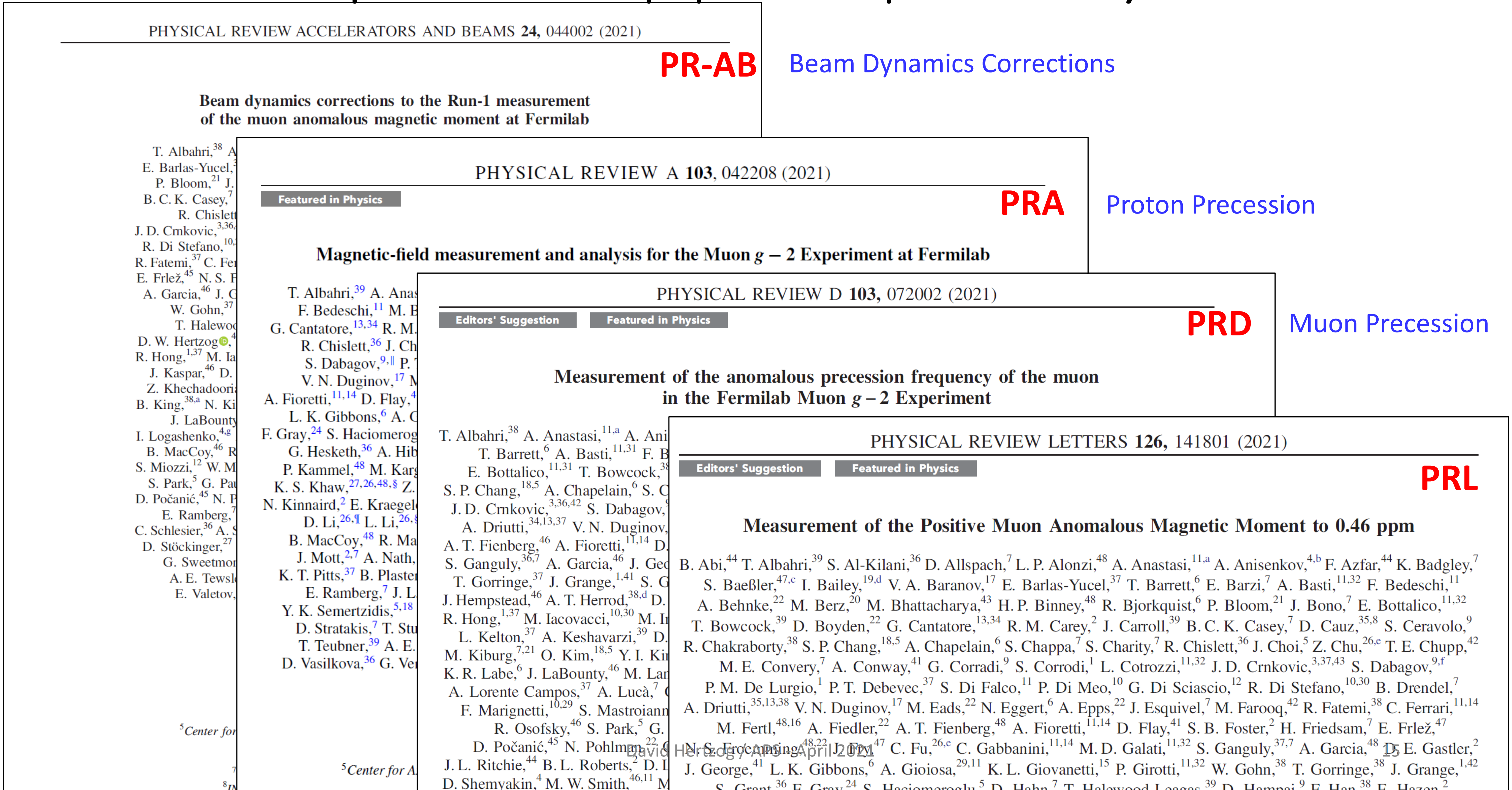
And the "anomaly" $a_\mu \equiv (g_\mu - 2)/2 = 116\,592\,040(54)$

Our final uncertainty is 460 ppb

&

The new world average experimental result is at 350 ppb

The results were published in 4 papers on April 7th … day of release



PR-AB

Beam Dynamics Corrections

PHYSICAL REVIEW ACCELERATORS AND BEAMS **24**, 044002 (2021)

Beam dynamics corrections to the Run-1 measurement of the muon anomalous magnetic moment at Fermilab

T. Albahri,[38] A
E. Barlas-Yucel,
P. Bloom,[21] J.
B. C. K. Casey,[7]
R. Chislett
J. D. Crnkovic,[3,36,
R. Di Stefano,[10,
R. Fatemi,[37] C. Fer
E. Frlež,[45] N. S. F
A. Garcia,[46] J. G
W. Gohn,[37]
T. Halewoo
D. W. Hertzog,[4
R. Hong,[1,37] M. Ia
J. Kaspar,[46] D.
Z. Khechadoori
B. King,[38,a] N. Ki
J. LaBounty
I. Logashenko,[4,g]
B. MacCoy,[46] R
S. Miozzi,[12] W. M
S. Park,[5] G. Pa
D. Počanić,[45] N. P
E. Ramberg,[7
C. Schlesier,[36] A. S
D. Stöckinger,[27]
G. Sweetmor
A. E. Tewsl
E. Valetov,

[5]*Center for*

PRA

Proton Precession

PHYSICAL REVIEW A **103**, 042208 (2021)

Featured in Physics

Magnetic-field measurement and analysis for the Muon $g-2$ Experiment at Fermilab

T. Albahri,[39] A. Anas
F. Bedeschi,[11] M. B
G. Cantatore,[13,34] R. M.
R. Chislett,[36] J. Ch
S. Dabagov,[9,‖] P.
V. N. Duginov,[17] M
A. Fioretti,[11,14] D. Flay,[4
L. K. Gibbons,[6] A. C
F. Gray,[24] S. Haciomerog
G. Hesketh,[36] A. Hib
P. Kammel,[48] M. Kar
K. S. Khaw,[27,26,48,§] Z.
N. Kinnaird,[2] E. Kraegel
D. Li,[26,¶] L. Li,[26,
B. MacCoy,[48] R. Ma
J. Mott,[2,7] A. Nath,
K. T. Pitts,[37] B. Plaster
E. Ramberg,[7] J. L
Y. K. Semertzidis,[5,18]
D. Stratakis,[7] T. Stu
T. Teubner,[39] A. E.
D. Vasilkova,[36] G. Ve

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PRD

Muon Precession

PHYSICAL REVIEW D **103**, 072002 (2021)

Editors' Suggestion Featured in Physics

Measurement of the anomalous precession frequency of the muon in the Fermilab Muon $g-2$ Experiment

T. Albahri,[38] A. Anastasi,[11,a] A. Ani
T. Barrett,[6] A. Basti,[11,31] F. B
E. Bottalico,[11,31] T. Bowcock,[38
S. P. Chang,[18,5] A. Chapelain,[6] S. C
J. D. Crnkovic,[3,36,42] S. Dabagov,
A. Driutti,[34,13,37] V. N. Duginov,
A. T. Fienberg,[46] A. Fioretti,[11,14] D.
S. Ganguly,[36,7] A. Garcia,[46] J. Geo
T. Gorringe,[37] J. Grange,[1,41] S. G
J. Hempstead,[46] A. T. Herrod,[38,d] D.
R. Hong,[1,37] M. Iacovacci,[10,30] M. I
L. Kelton,[37] A. Keshavarzi,[39] D.
M. Kiburg,[7,21] O. Kim,[18,5] Y. I. Ki
K. R. Labe,[6] J. LaBounty,[46] M. La
A. Lorente Campos,[37] A. Lucà,[7]
F. Marignetti,[10,29] S. Mastroiann
R. Osofsky,[46] S. Park,[5] G.
D. Počanić,[45] N. Pohlman,[22
J. L. Ritchie,[44] B. L. Roberts,[2] D. L
D. Shemyakin,[4] M. W. Smith,[46,11] M

PRL

PHYSICAL REVIEW LETTERS **126**, 141801 (2021)

Editors' Suggestion Featured in Physics

Measurement of the Positive Muon Anomalous Magnetic Moment to 0.46 ppm

B. Abi,[44] T. Albahri,[39] S. Al-Kilani,[36] D. Allspach,[7] L. P. Alonzi,[48] A. Anastasi,[11,a] A. Anisenkov,[4,b] F. Azfar,[44] K. Badgley,[7]
S. Baeßler,[47,c] I. Bailey,[19,d] V. A. Baranov,[17] E. Barlas-Yucel,[37] T. Barrett,[6] E. Barzi,[7] A. Basti,[11,32] F. Bedeschi,[11]
A. Behnke,[22] M. Berz,[20] M. Bhattacharya,[43] H. P. Binney,[48] R. Bjorkquist,[6] P. Bloom,[21] J. Bono,[7] E. Bottalico,[11,32]
T. Bowcock,[39] D. Boyden,[22] G. Cantatore,[13,34] R. M. Carey,[2] J. Carroll,[39] B. C. K. Casey,[7] D. Cauz,[35,8] S. Ceravolo,[9]
R. Chakraborty,[38] S. P. Chang,[18,5] A. Chapelain,[6] S. Chappa,[7] S. Charity,[7] R. Chislett,[36] J. Choi,[5] Z. Chu,[26,e] T. E. Chupp,[42]
M. E. Convery,[7] A. Conway,[41] G. Corradi,[9] S. Corrodi,[1] L. Cotrozzi,[11,32] J. D. Crnkovic,[3,37,43] S. Dabagov,[9,f]
P. M. De Lurgio,[1] P. T. Debevec,[37] S. Di Falco,[11] P. Di Meo,[10] G. Di Sciascio,[12] R. Di Stefano,[10,30] B. Drendel,[7]
A. Driutti,[35,13,38] V. N. Duginov,[17] M. Eads,[22] N. Eggert,[6] A. Epps,[22] J. Esquivel,[7] M. Farooq,[42] R. Fatemi,[38] C. Ferrari,[11,14]
M. Fertl,[48,16] A. Fiedler,[22] A. T. Fienberg,[48] A. Fioretti,[11,14] D. Flay,[41] S. B. Foster,[2] H. Friedsam,[7] E. Frlež,[47]
N. S. Froemming,[48,22] J. Fry,[47] C. Fu,[26,e] C. Gabbanini,[11,14] M. D. Galati,[11,32] S. Ganguly,[37,7] A. Garcia,[48] D. E. Gastler,[2]
J. George,[41] L. K. Gibbons,[6] A. Gioiosa,[29,11] K. L. Giovanetti,[15] P. Girotti,[11,32] W. Gohn,[38] T. Gorringe,[38] J. Grange,[1,42]

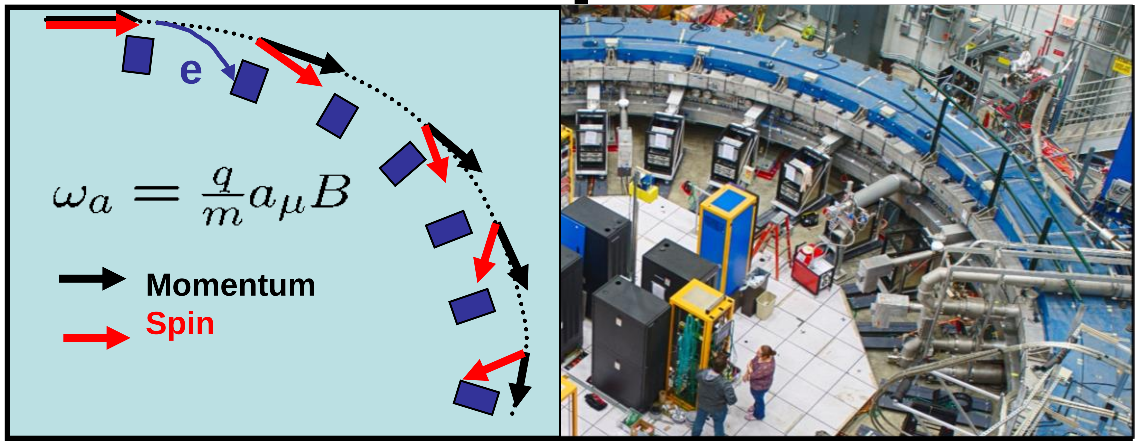
The story behind a new measurement

We include: Particle-, Nuclear-, Atomic-, Optical-, Accelerator-, and Theoretical Physicists

But, we all aim to measure g-2 to 140 ppb

(with about 20x the data obtained at BNL)

The Fundamental Experimental Principle



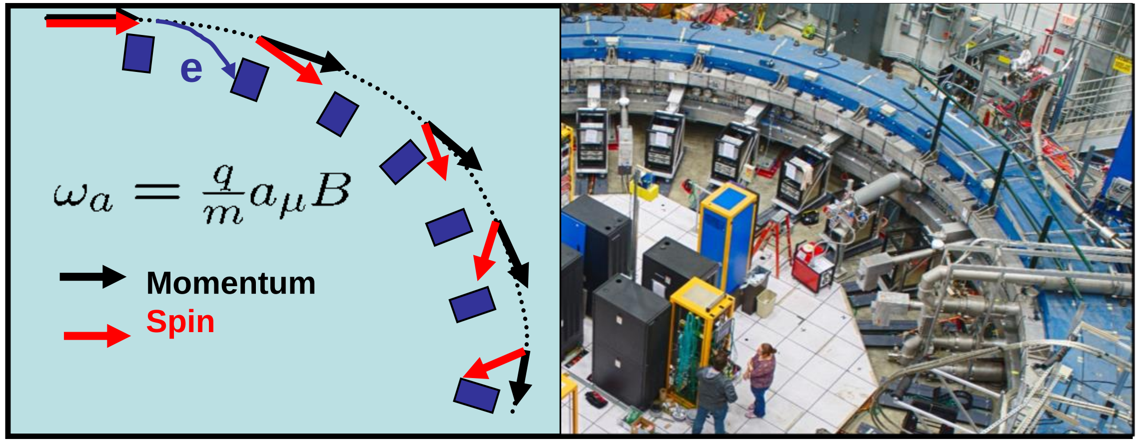
Determine difference between spin precession frequency and cyclotron frequencies for a muon moving in a magnetic field

Get a_μ

$$\vec{\omega}_s - \vec{\omega}_c \equiv \vec{\omega}_a = -\frac{q}{m} a_\mu \vec{B}$$

Measure these

The expression is more complicated when you add in *E*-field focusing and out of plane oscillations



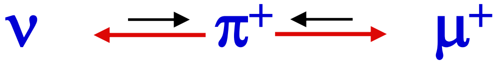
The motion is very nearly planar and the momentum is very nearly the ideal one, but both effects are not perfect and require corrections

$$\vec{\omega}_a = -\frac{q}{m}\left[a_\mu \vec{B} - a_\mu\left(\frac{\gamma}{\gamma+1}\right)\cancel{(\vec{\beta}\cdot\vec{B})}\vec{\beta} - \cancel{\left(a_\mu - \frac{1}{\gamma^2-1}\right)}\frac{\vec{\beta}\times\vec{\mathcal{E}}}{c}\right]$$

0 if “in plane”

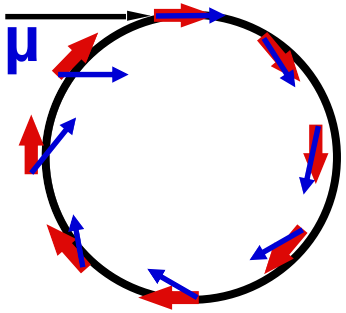
Term cancels at 3.094 GeV/c, the “Magic γ”

4 “miracles permit measurement of g-2 to sub-ppm precision



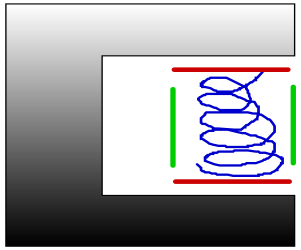
1) Polarized muons produced naturally in pion decay

~97% polarized for forward decays



2) The anomalous spin precession frequency is proportional to (g-2) … not to “g”

a factor of ~850 easier from that

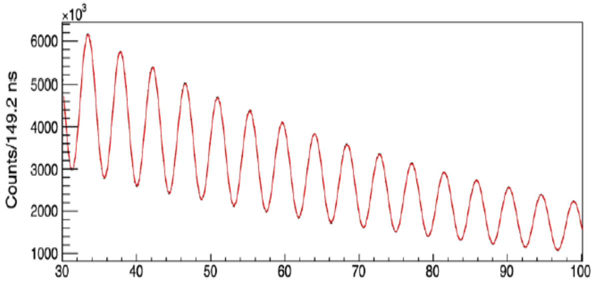


3) At the <u>magic</u> momentum, the electric holding field does not perturb the spin frequency

major breakthrough recognized in the 70’s

4) Parity violation encodes the anomalous precession frequency in the e^+ vs time spectrum

$$\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu$$



a_μ is obtained from **2 frequency measurements** we make … and well-known fundamental factors from others

We measure these 2 frequencies

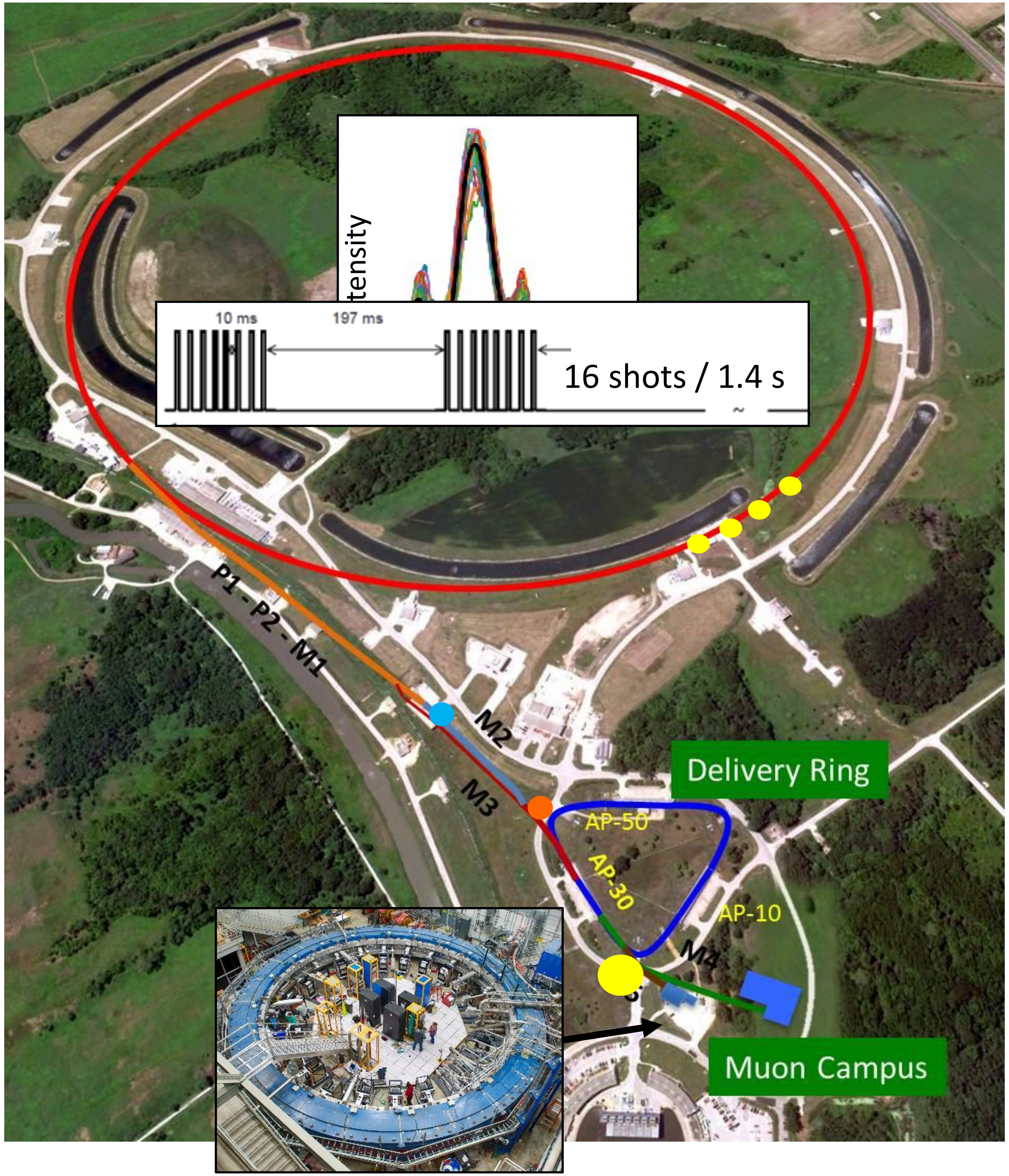
$$a_\mu = \left[\frac{\omega_a}{\tilde{\omega}'_p(T_r)}\right] \underbrace{\frac{\mu'_p(T_r)}{\mu_e(H)} \frac{\mu_e(H)}{\mu_e} \frac{m_\mu}{m_e} \frac{g_e}{2}}$$

$\frac{\mu_e(H)}{\mu'_p(T)}$ Measured to 10.5 ppb at T = 34.7℃
Metrologia **13**, 179 (1977)

$\frac{\mu_e}{\mu_e(H)}$ Bound-state QED (exact)
Rev. Mod. Phys. **88** 035009 (2016)

$\frac{m_\mu}{m_e}$ Known to 22 ppb from muonium hyperfine splitting
Phys. Rev. Lett. **82**, 711 (1999)

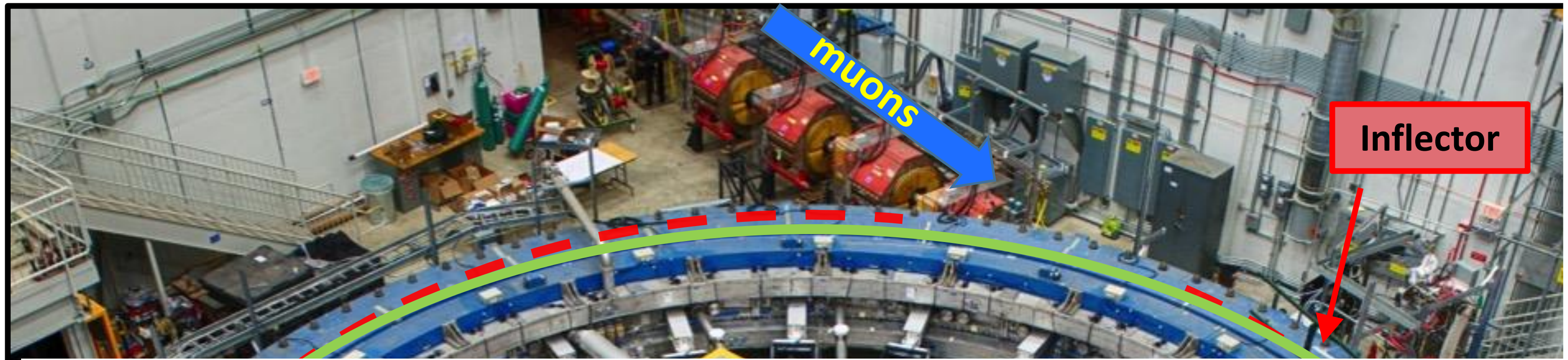
$\frac{g_e}{2}$ Measured to 0.28 ppt
Phys. Rev. A **83**, 052122 (2011)



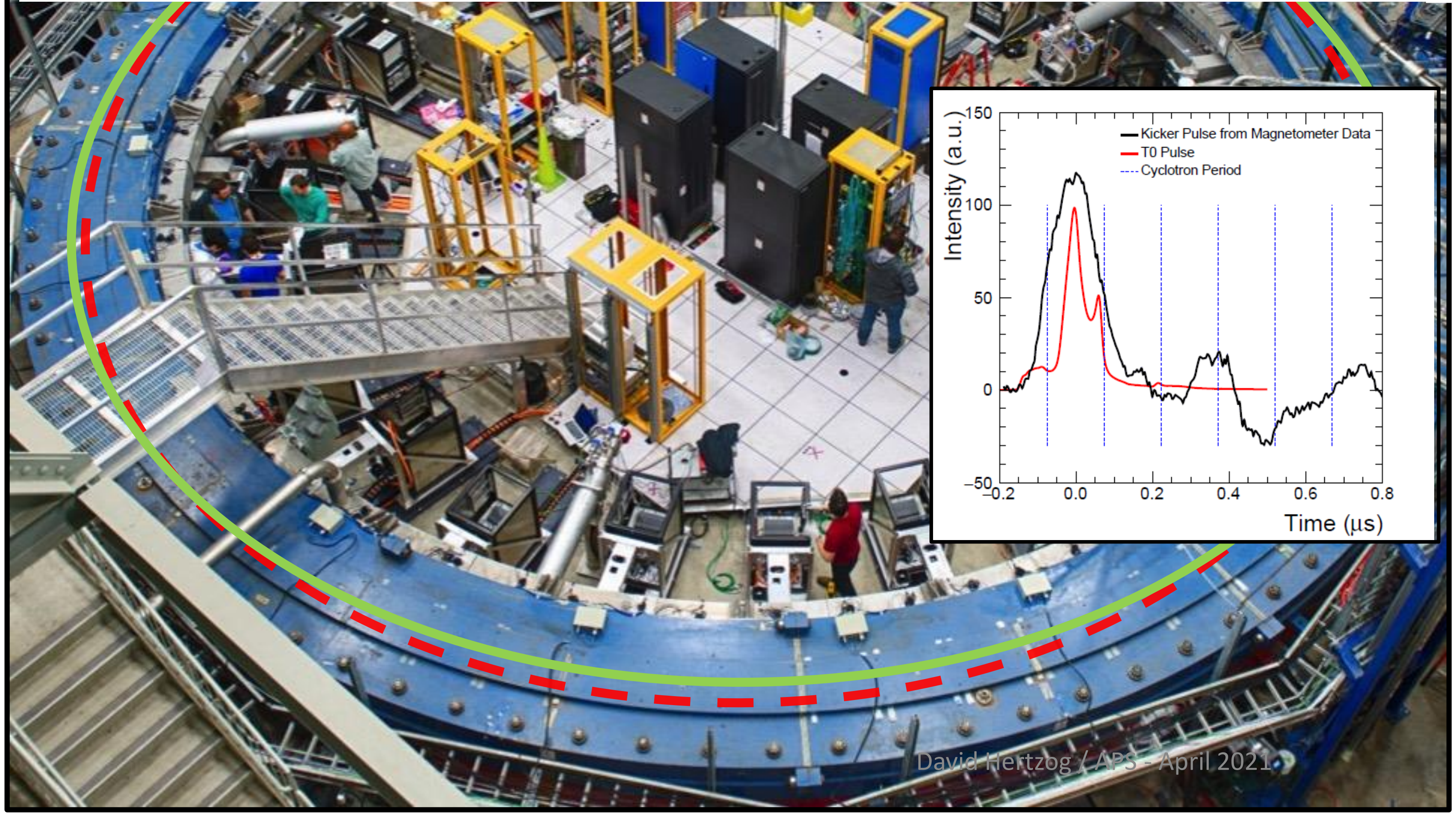
Creating the Polarized Muon Beam for g-2

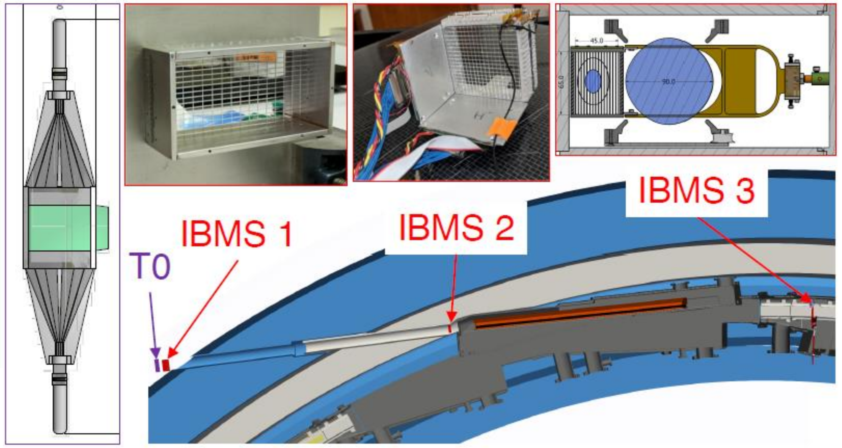
- 8 GeV protons
- Divide in 4 bunches
- Extract each to strike target
- Magnetic lenses collect $\pi \rightarrow \mu\nu$
- $p/\pi/\mu$ beam enters Delivery Ring – protons get kicked out; pions decay away
- **And only muons enter storage ring**

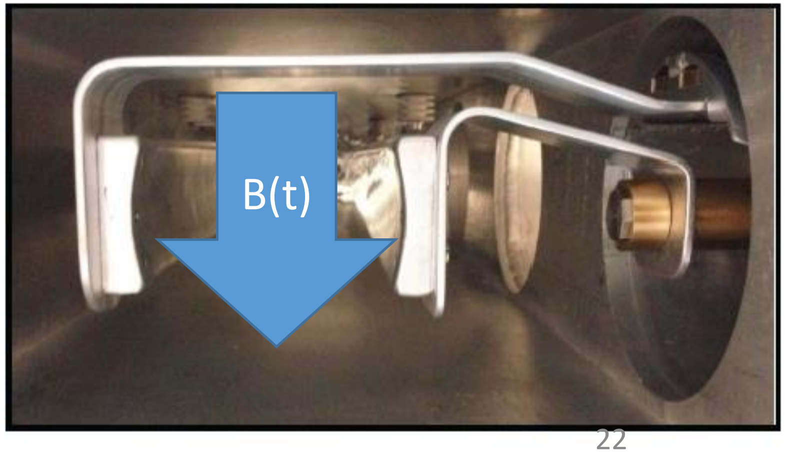
Comment on polarity flip for Mu Minus running



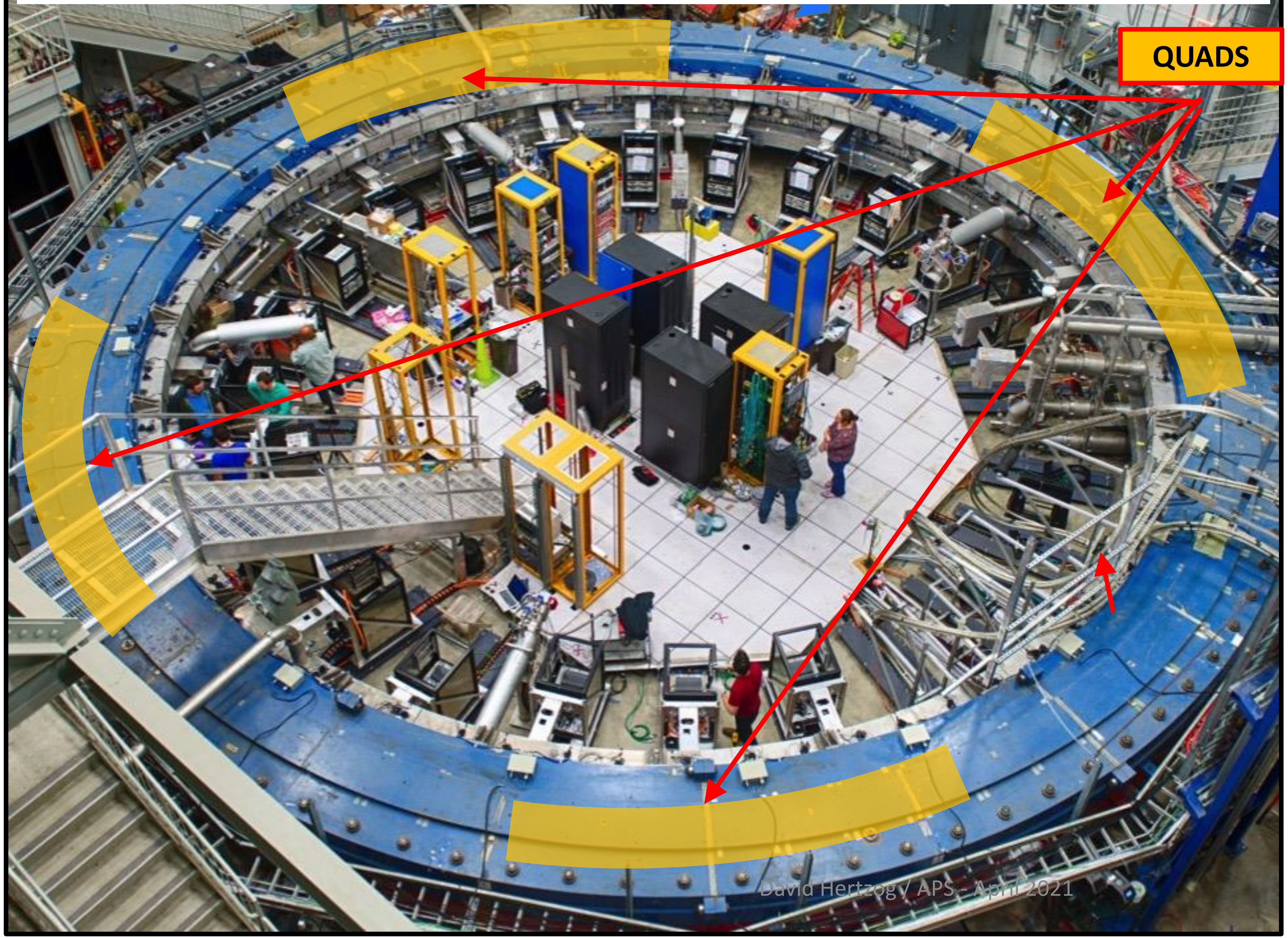
Inject muons into the ring and kick them onto a stable orbit

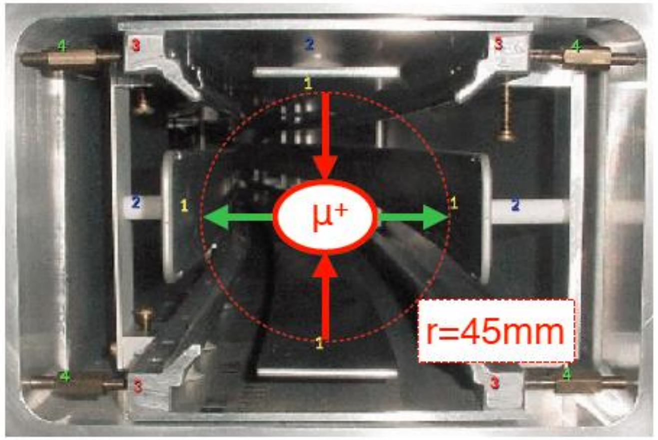


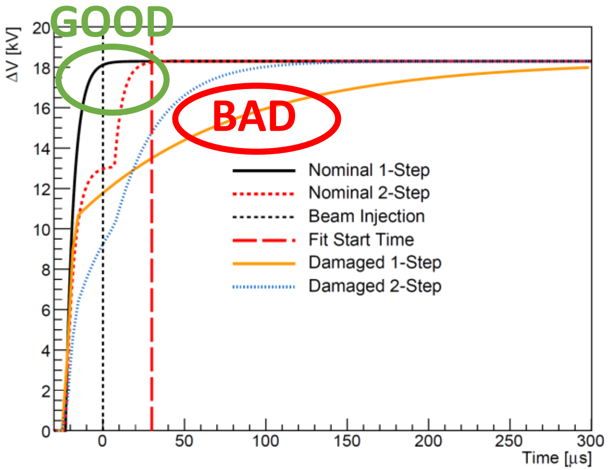




Electrostatic quadrupoles provide weak vertical focusing.
The ring is a large “Penning Trap”



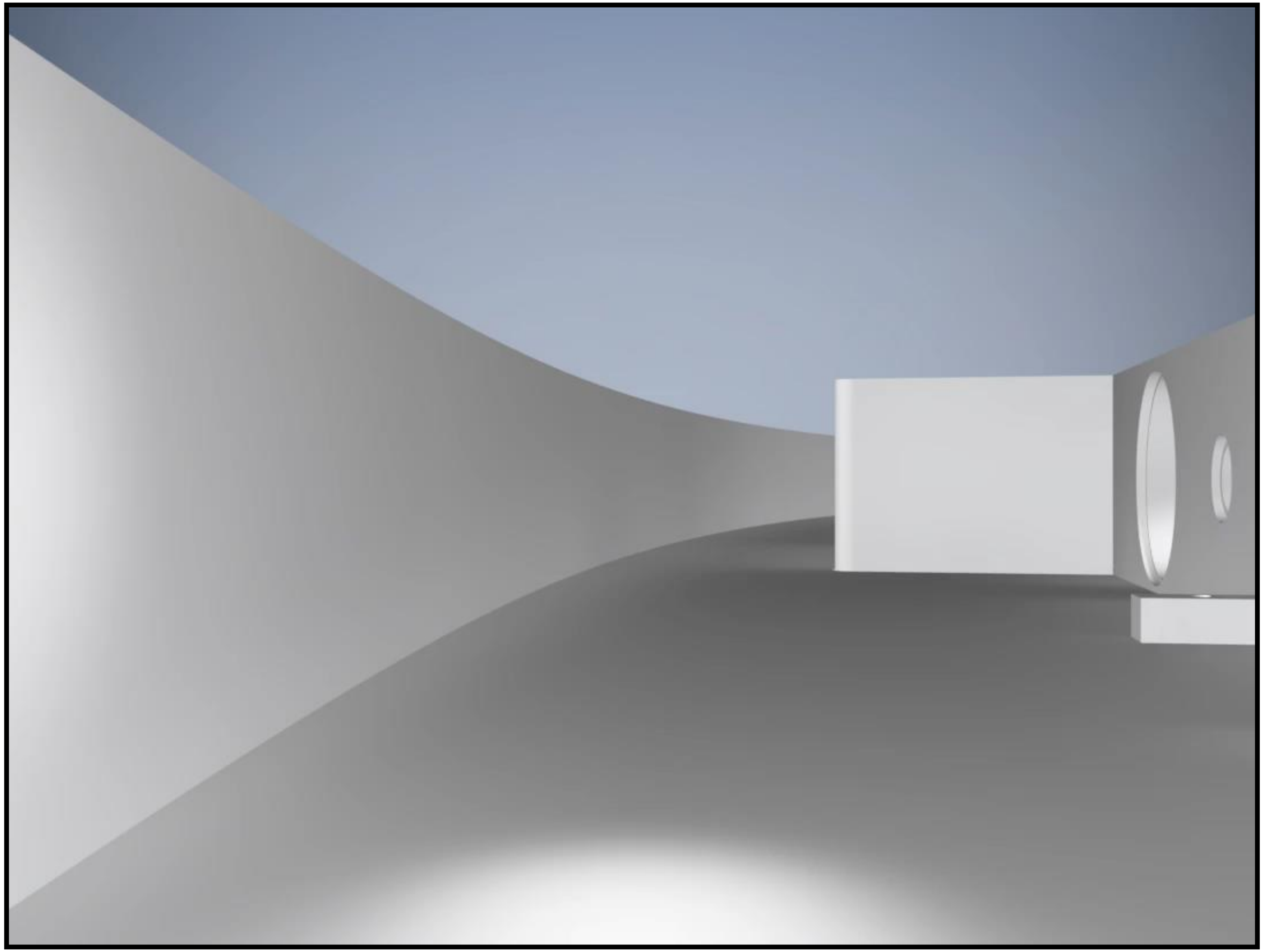




2/32 DAMAGED → $\tau_{RC} > 100\ \mu s$

UNstable during fit …. Lots of consequences will follow that cost us considerable time to understand

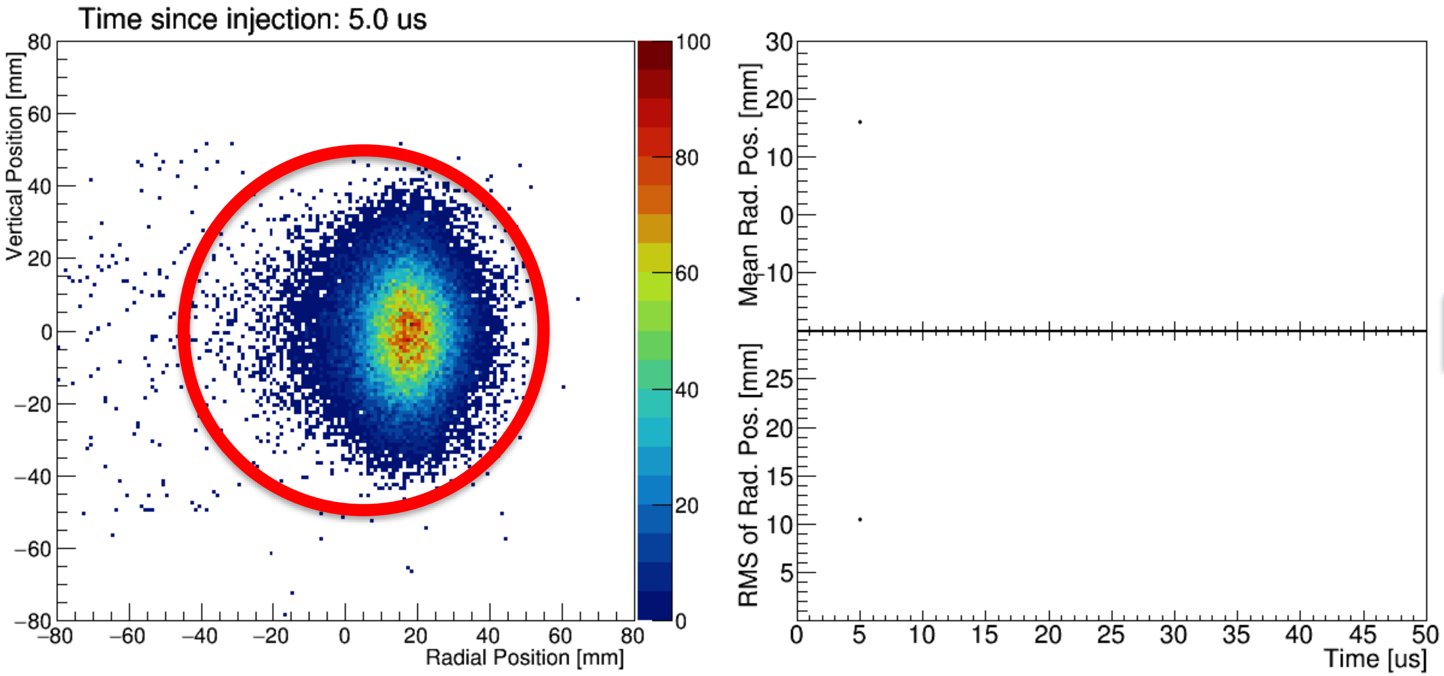
Let's pause to drive around inside the ring …

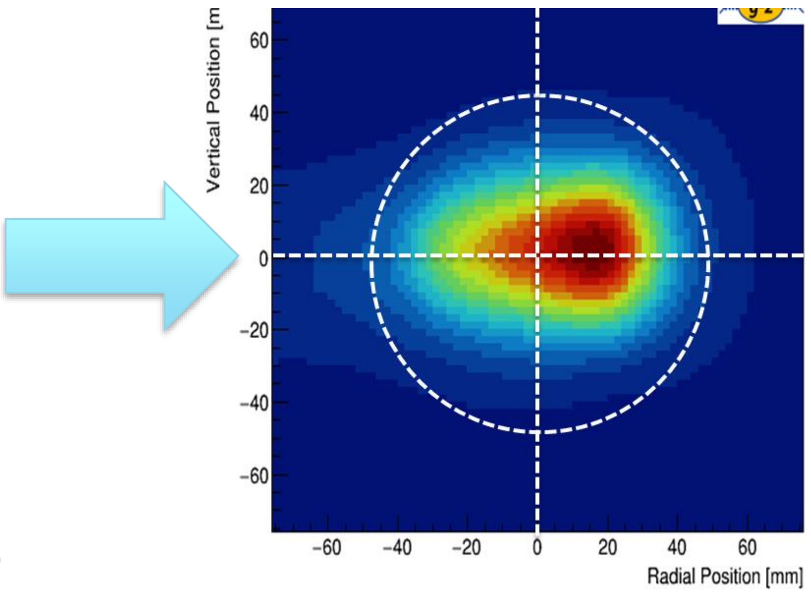


You can spot …

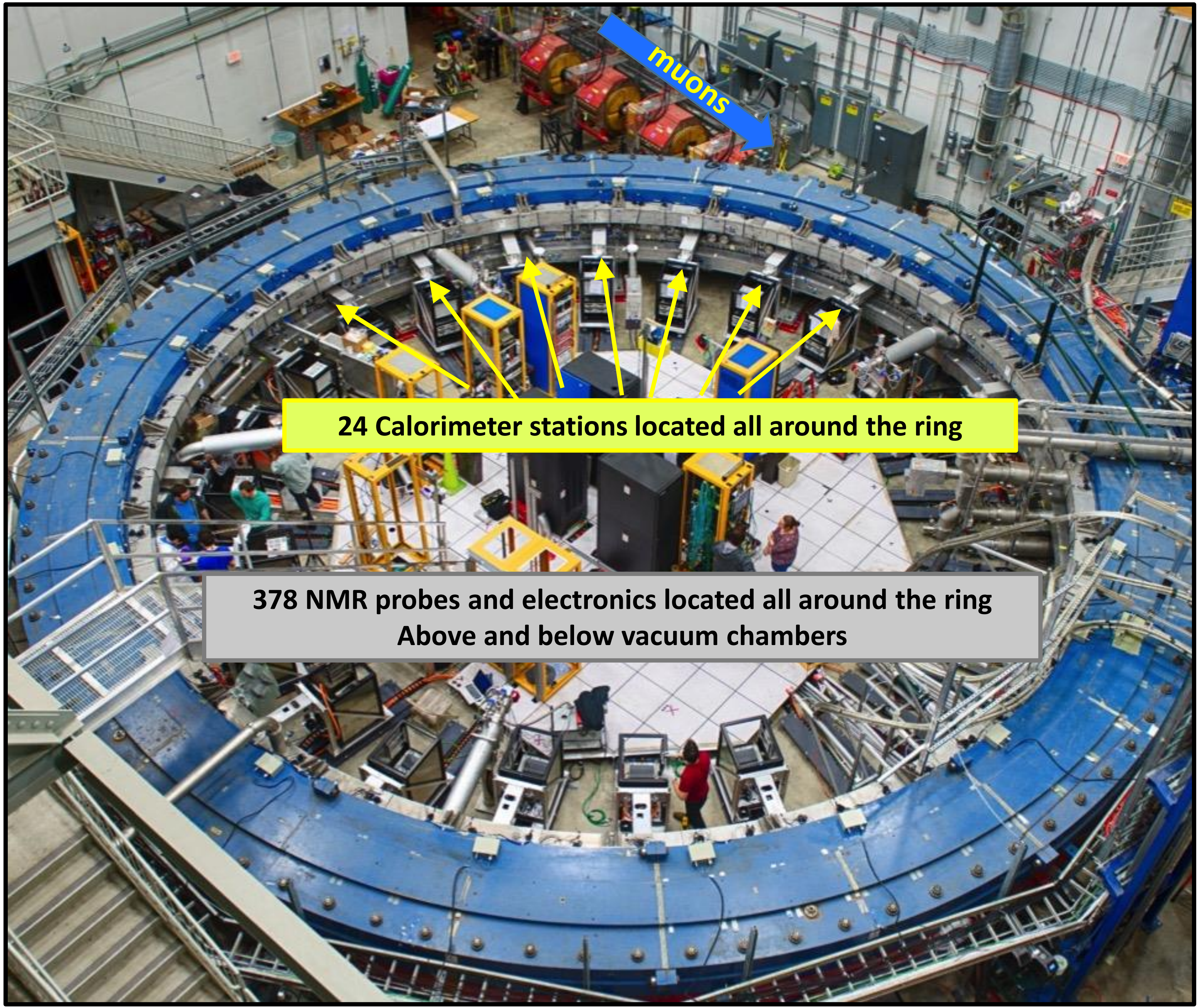
1) Quads
2) Kicker
3) Straw Trackers

We can make a movie of where the muons are as they go past one of our detectors



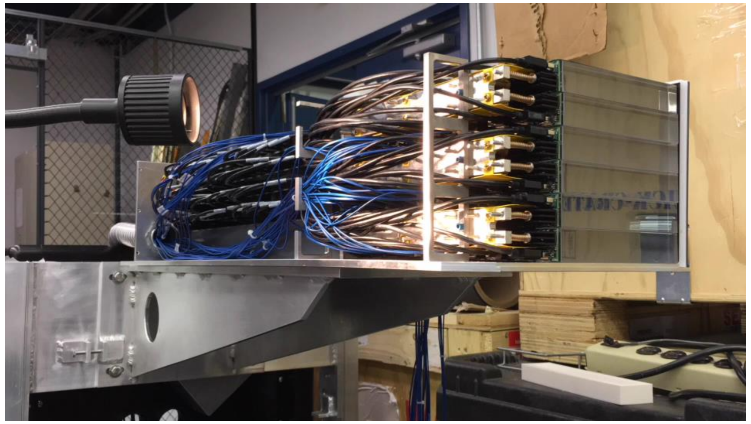


Average x-y profile around the ring



24 Calorimeter stations located all around the ring

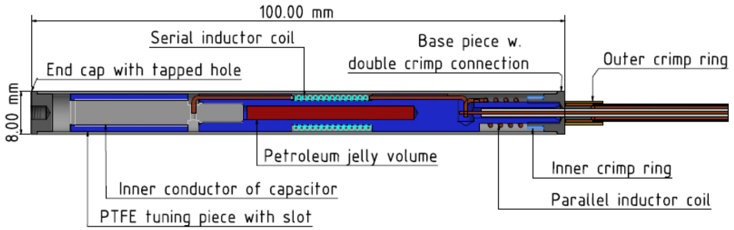
378 NMR probes and electronics located all around the ring
Above and below vacuum chambers



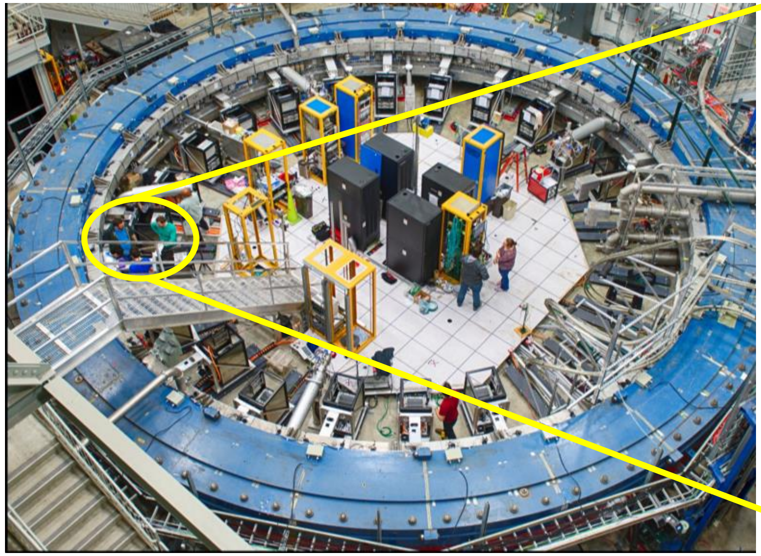
- **24 calorimeters measure the e^+ decay time and energy**

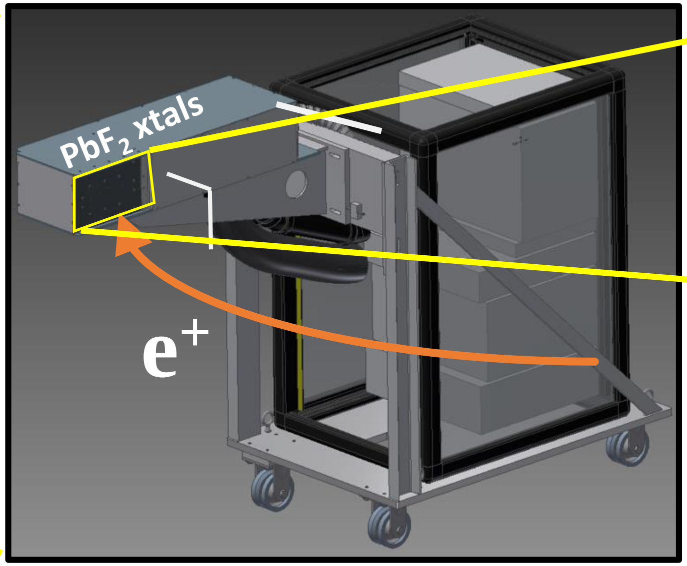
and

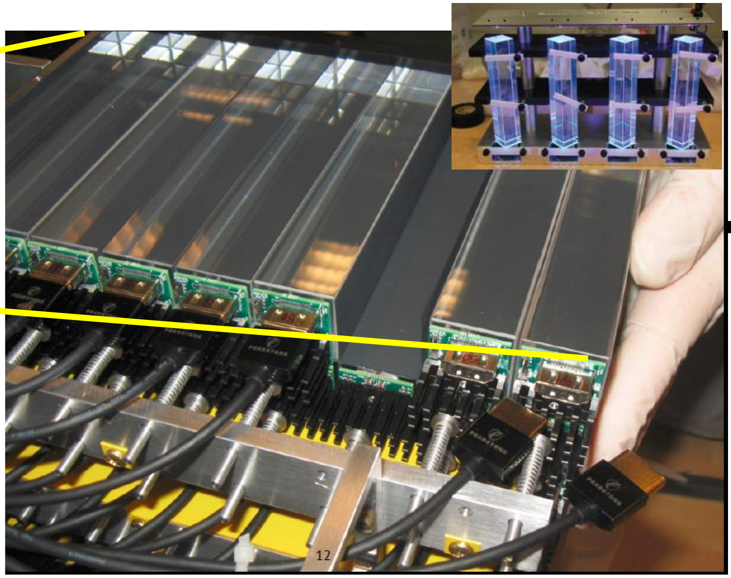
- **378 NMR probes continuously measure the magnetic field**

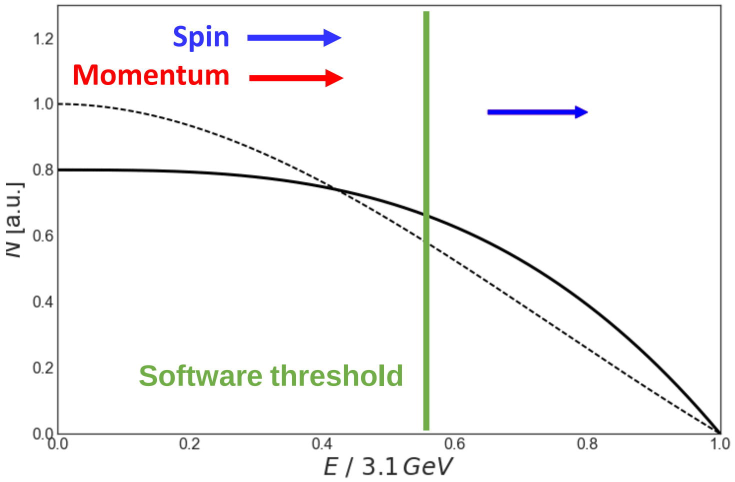


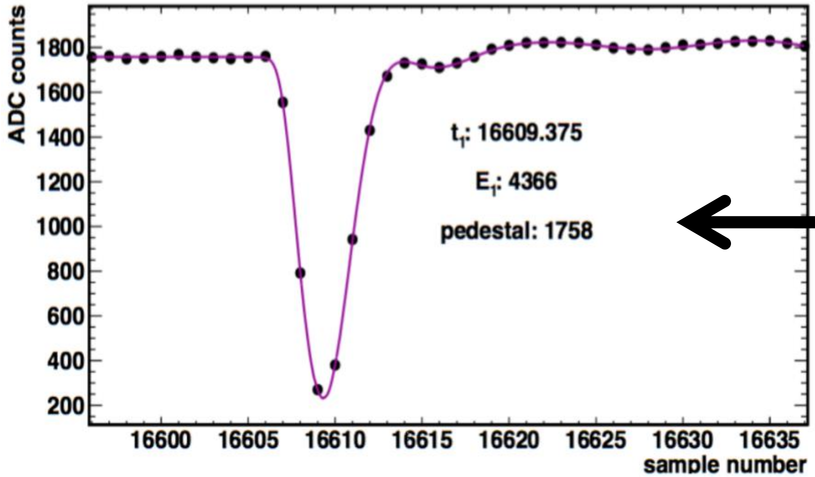
The precession frequency, ω_a is derived from a time histogram of high-energy e^+ decay events



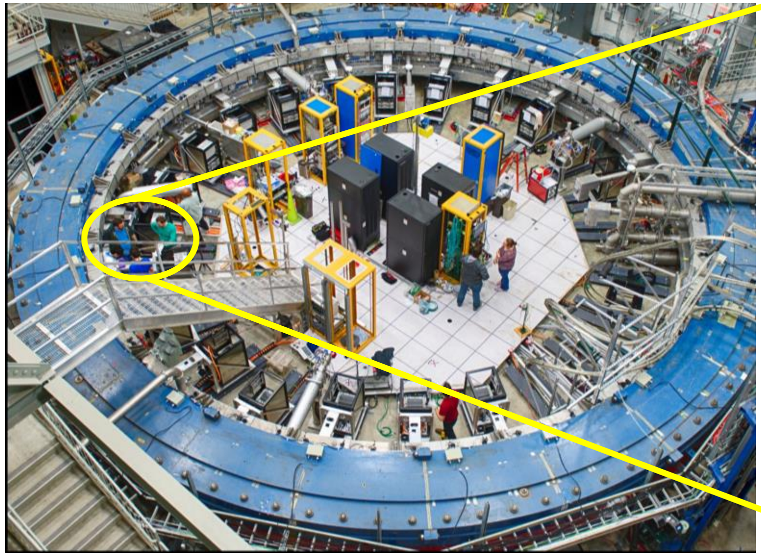


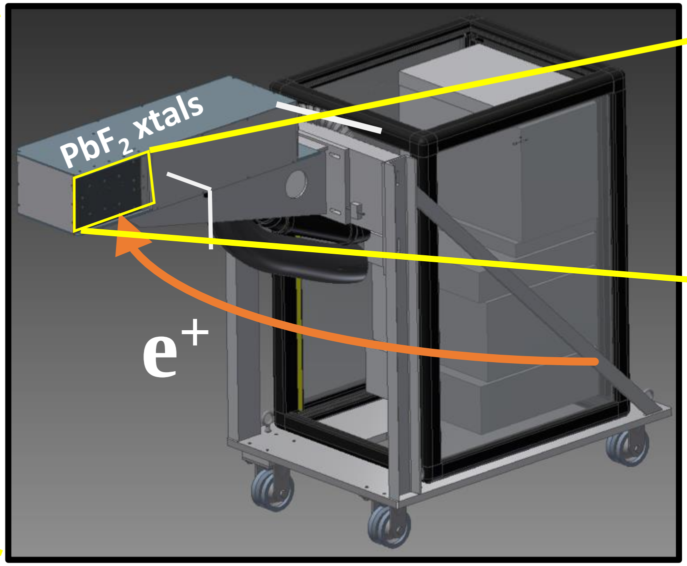


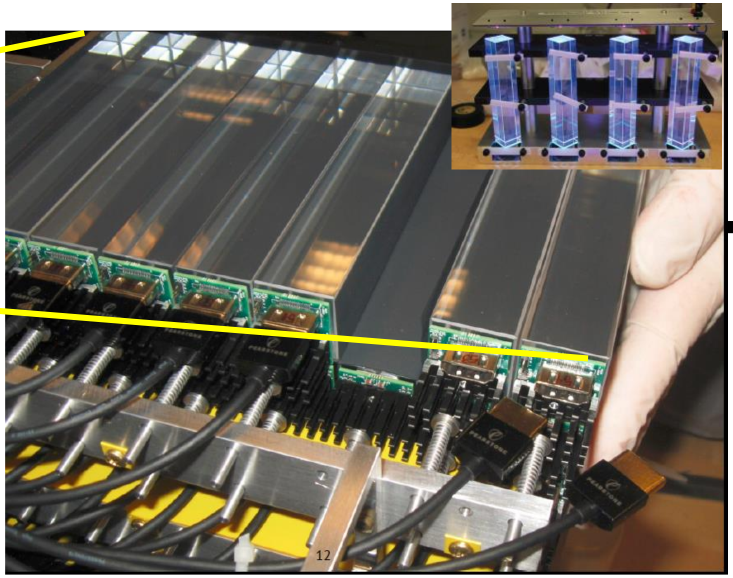




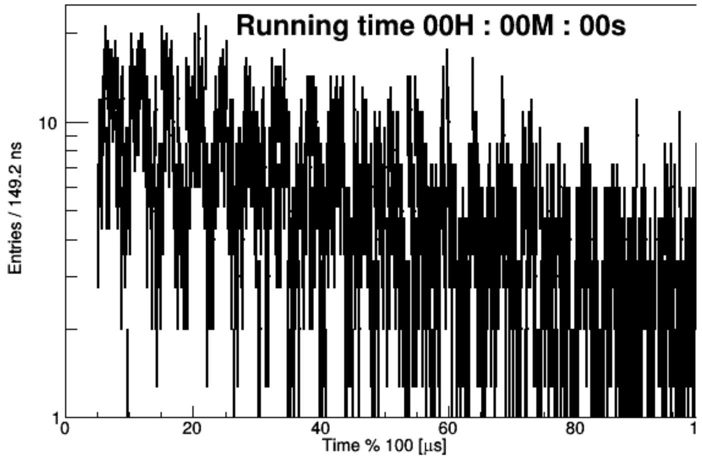
The precession frequency, ω_a is derived from a time histogram of high-energy e^+ decay events

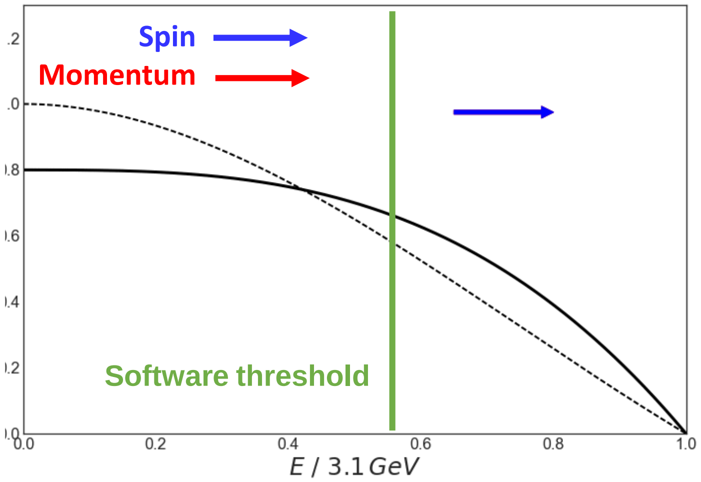


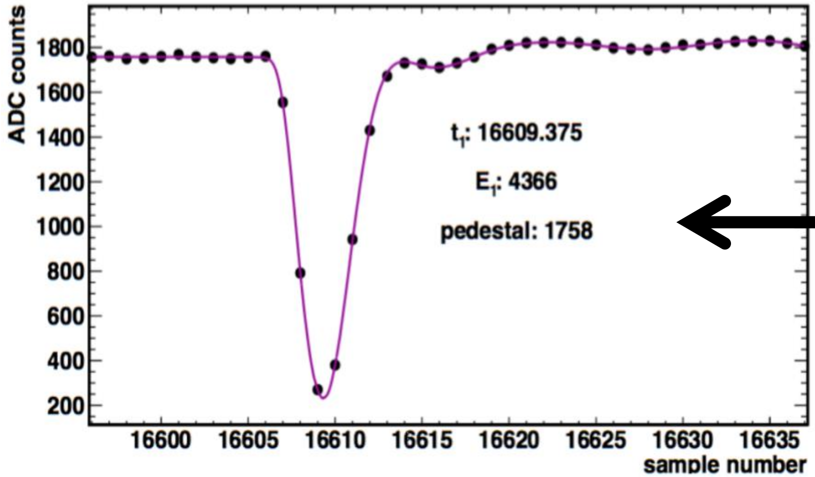




Events above threshold







The Field, ω_p begins with the BNL magnet moved to Fermilab

Yoke Iron
Aligned to sub-mil
precision

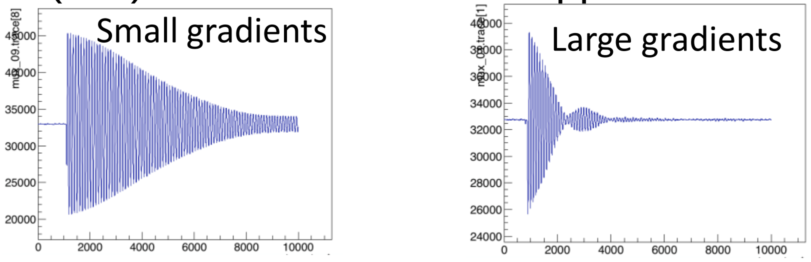
Superconducting coils
And cryostat

Magnet shimming kit

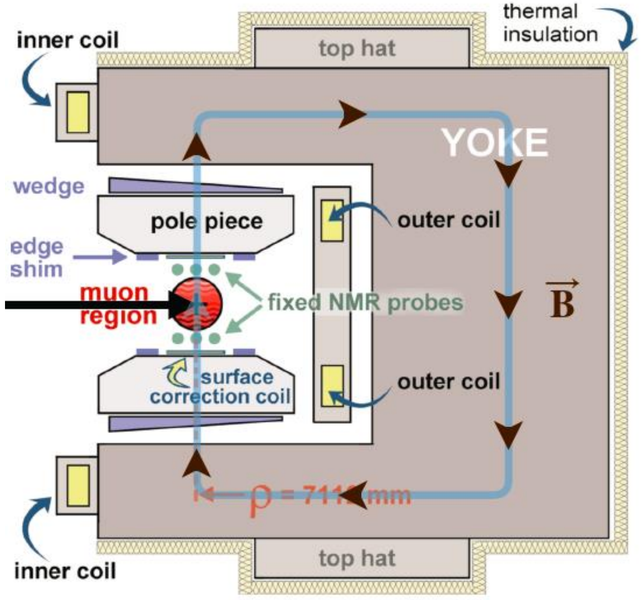
- NMR probes
- Probe Multiplexer
- Pulser-Mixer

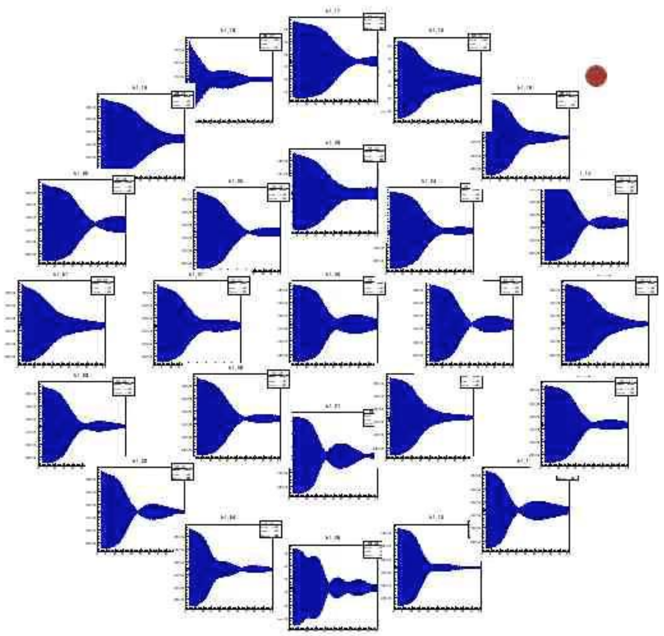
Built-in shimming tools provide many knobs to tune uniformity

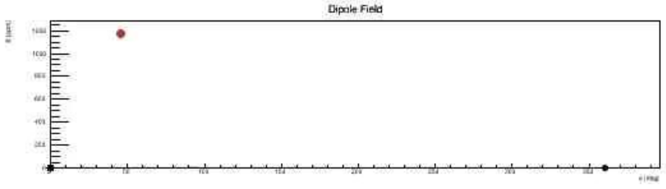
(FID) Waveforms with ~10 ppb resolution



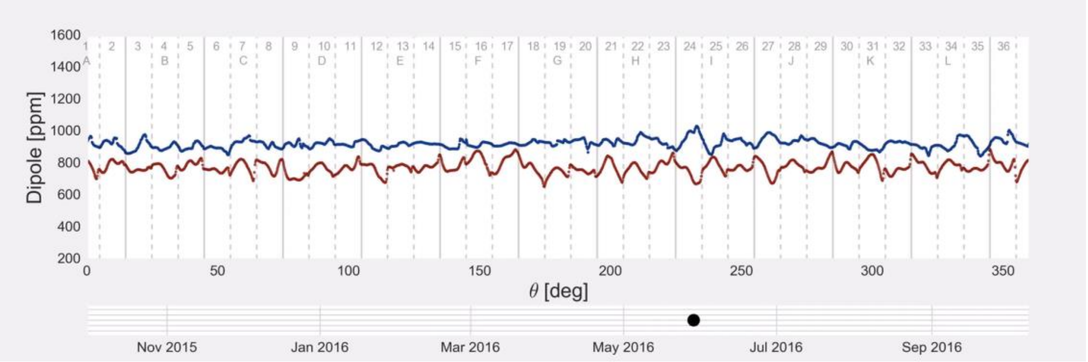
A 25-element pNMR shimming ***Trolley*** was used to map the field during a year-long shimming campaign







Innovative installation of ~8000 tiny iron laminations to minimize field inhomogeneity locally all around the ring



Final field uniformity is ~3 x finer than BNL !

Analysis of Run-1 Data

“To trust is good, not to trust is better.”

Italian Proverb

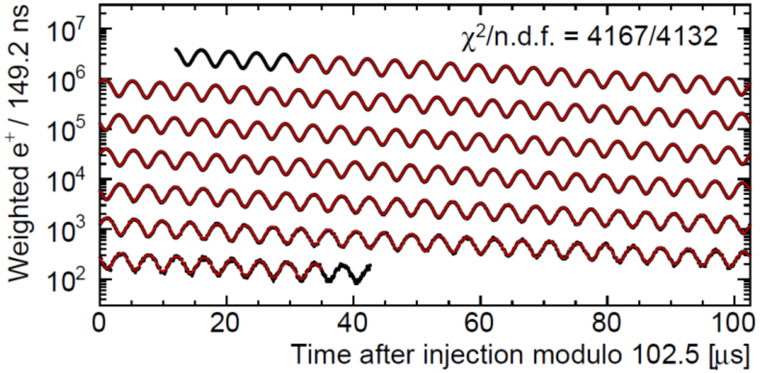
fidarsi è bene non fidarsi è meglio

Multiple analysis teams
Calibration, alignment, calibration …
Relatively blind analysis intermediate stages
Many specialized systematic measurements

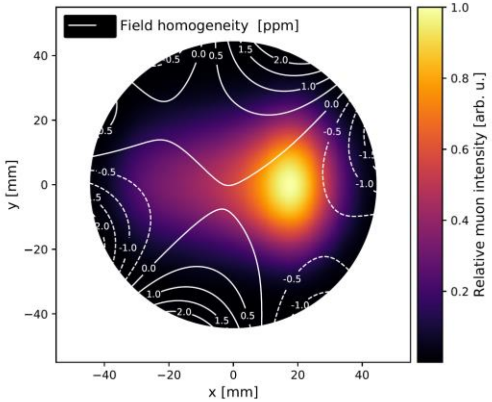
Many measurements determine a_μ. Let’s walk through a few of them so you can appreciate the multiple and parallel efforts

$$a_\mu \propto \frac{f_{\text{clock}} \;\; \omega_a^m \left(1 + C_e + C_p + C_{ml} + C_{pa}\right)}{f_{\text{calib}} \; \langle \omega_p'(x,y,\phi) \times M(x,y,\phi) \rangle \; \left(1 + B_k + B_q\right)}$$

f_{clock}	• Blinded clock
ω_a^m	• Measured precession frequency
C_e	• Electric field correction
C_p	• Pitch correction
C_{ml}	• Muon loss correction
C_{pa}	• Phase-acceptance correction



f_{calib}	• Absolute magnetic field calibration
$\omega_p'(x,y,\phi)$	• Field tracking multipole distribution
$M(x,y,\phi)$	• Muon weighted multipole distributed
B_k	• Transient field from the eddy current in kicker
Bq	• Transient field from the quad charging



The master clock is blinded until the entire analysis is complete

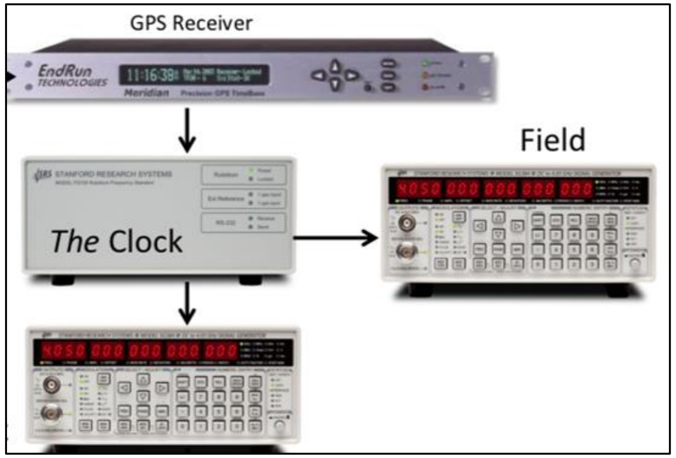
$$a_\mu \propto \frac{f_{\mathrm{clock}} \;\; \omega_a^m \, (1 + C_e + C_p + C_{ml} + C_{pa})}{f_{\mathrm{calib}} \; \langle \omega_p'(x,y,\phi) \times M(x,y,\phi) \rangle \; (1 + B_k + B_q)}$$

▶ Clock

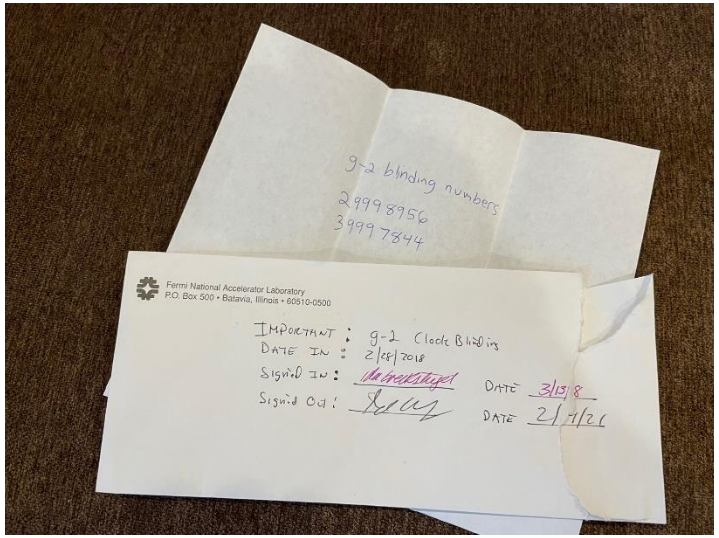
- Both ω_a and ω_p frequencies are measured by a single 10 MHz, GPS-disciplined master clock.
- Clock is hardware-blinded to have $(40 - \epsilon)$ MHz with the blinding range of ± 25 ppm.

▶ Blinding factor

- Set by and only known to two trusted individuals outside the collaboration.



▶ Envelopes held at UW and Fermilab for security



g-2 blinding numbers
29998956
39997844

Fermi National Accelerator Laboratory
P.O. Box 500 • Batavia, Illinois • 60510-0500

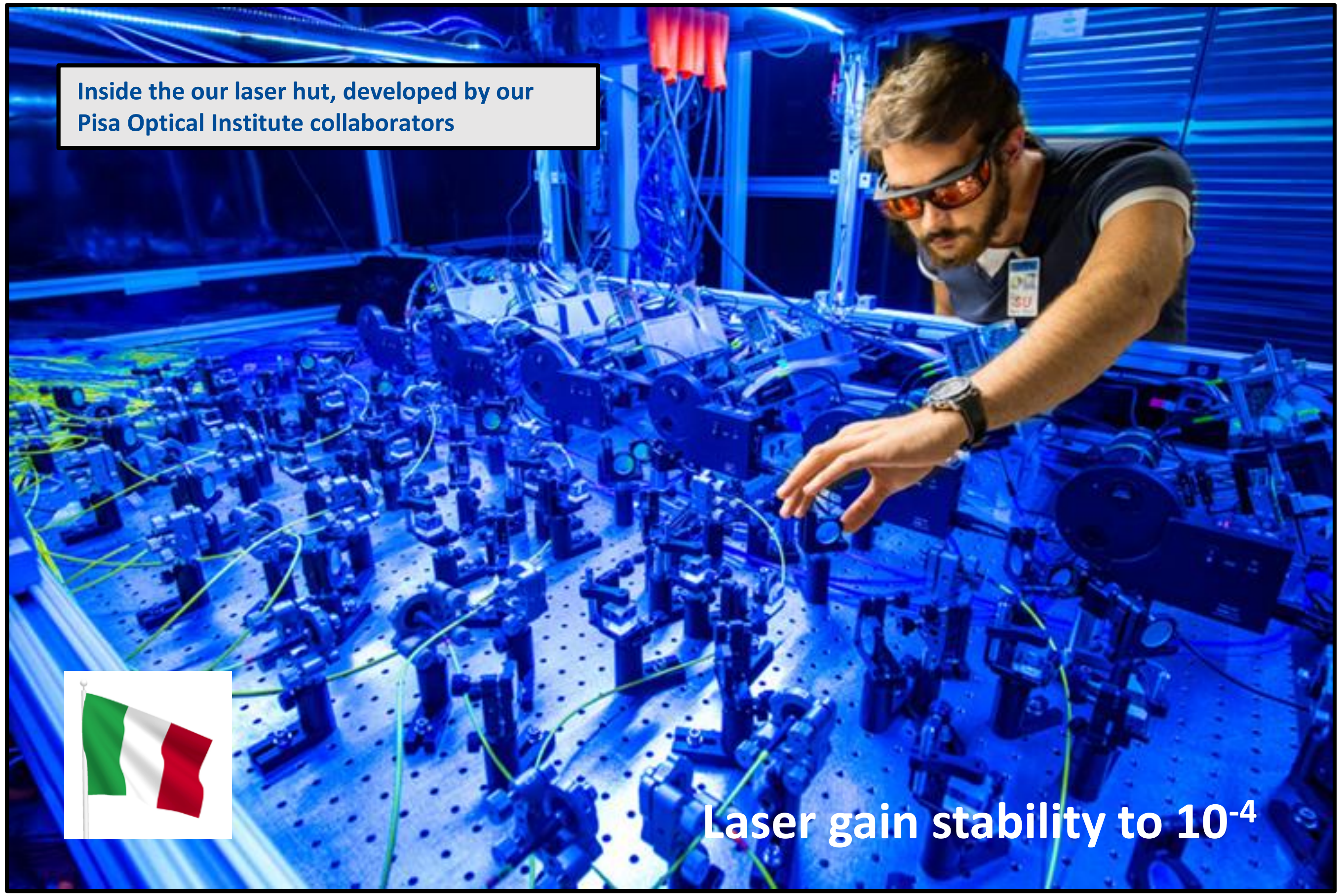
IMPORTANT : g-2 Clock Blinding
DATE IN : 2/28/2018
Signed IN : DATE 3/15/18
Signed Out : DATE 2/17/21

Takeaway: Uncertainty "0" (clocks are very stable and accurate)

The e^+ time histograms are prepared with exquisite gain (energy) control

$$a_\mu \propto \frac{f_{\text{clock}}\ \ \omega_a^m\ (1 + C_e + C_p + C_{ml} + C_{pa})}{f_{\text{calib}}\ \langle \omega'_p(x,y,\phi) \times M(x,y,\phi) \rangle\ (1 + B_k + B_q)}$$

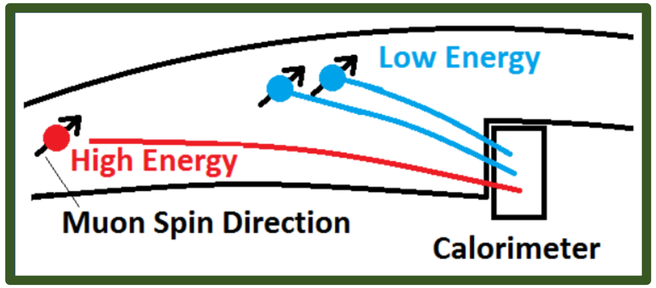
They also require pileup removal to avoid an important systematic

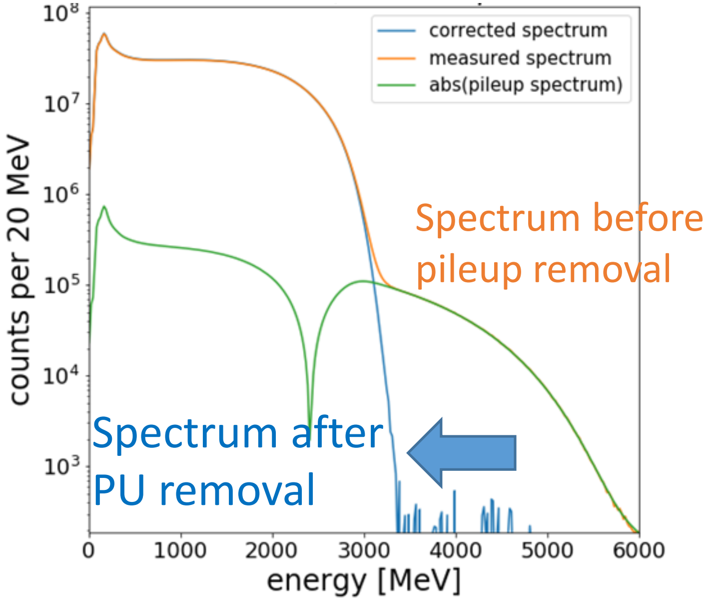


Inside the our laser hut, developed by our Pisa Optical Institute collaborators

Laser gain stability to 10^{-4}

1296 PbF_2 crystals with individual laser calibrations into each channel



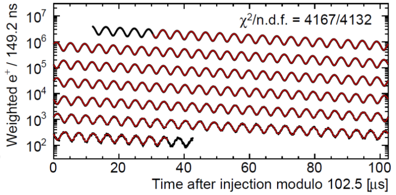


Fit to get the "measured" precession frequency

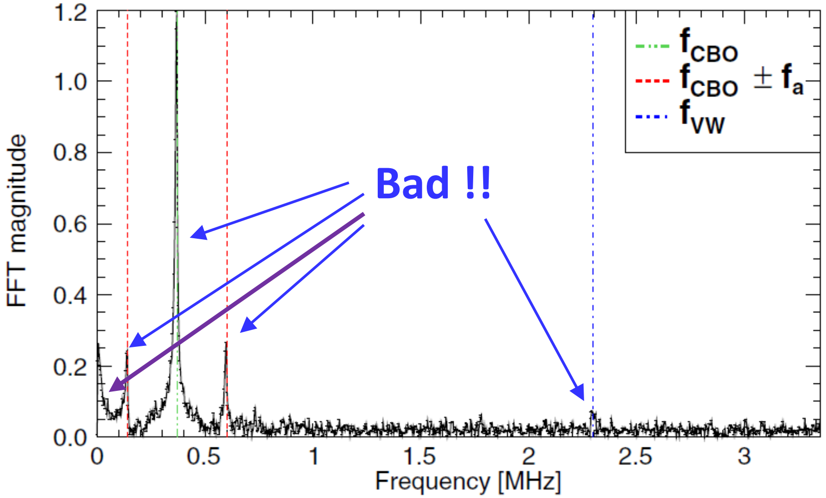
$$a_\mu \propto \frac{f_{\text{clock}}\;\omega_a^m\,(1+C_e+C_p+C_{ml}+C_{pa})}{f_{\text{calib}}\;\langle\omega_p'(x,y,\phi)\times M(x,y,\phi)\rangle\;(1+B_k+B_q)}$$

- Ideally, a simple five-parameter function makes sense

$$F(t) = N_0 e^{-t/\gamma\tau_\mu}\left[1 + A_0\cos(\omega_a^m t + \phi_0)\right]$$

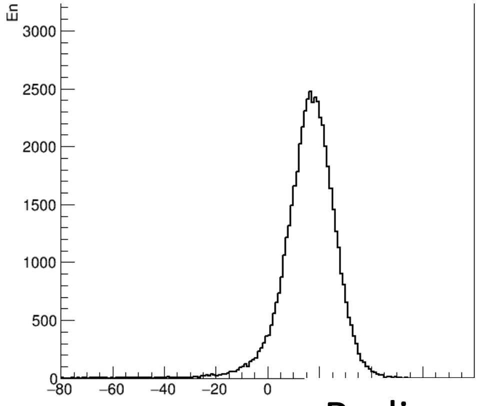


- This fit model is incomplete.
- **FFT of residuals** shows peaks at **CBO frequencies** (radial and vertical motions) and a slow component from **muon losses**
- Fit function expands to include all motions and dynamic effects



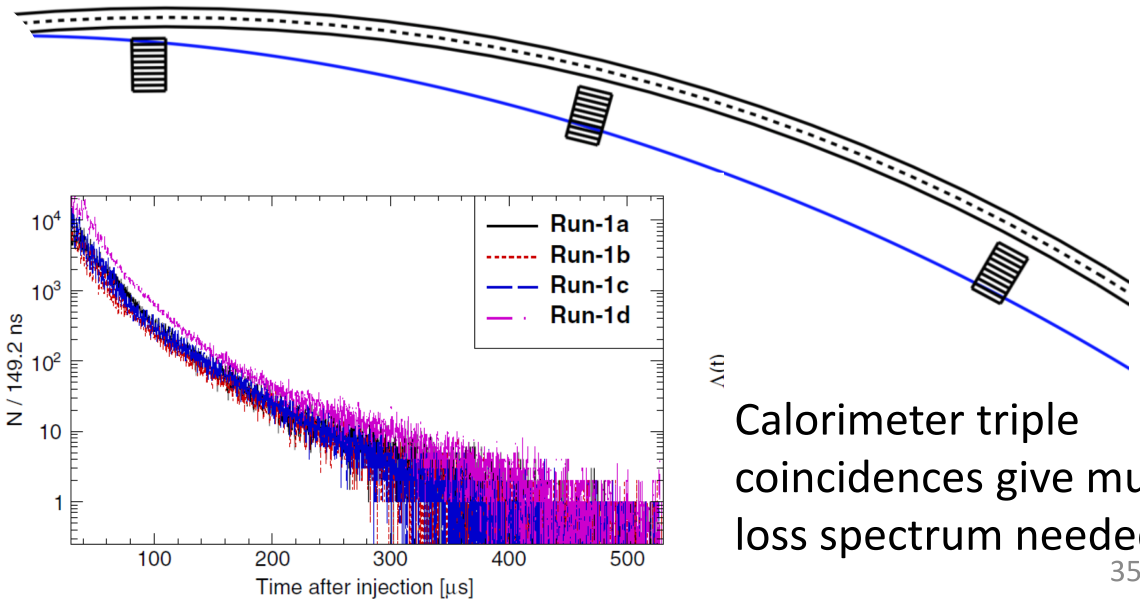
Bad !!

Coherent Betatron Oscillations



Radius

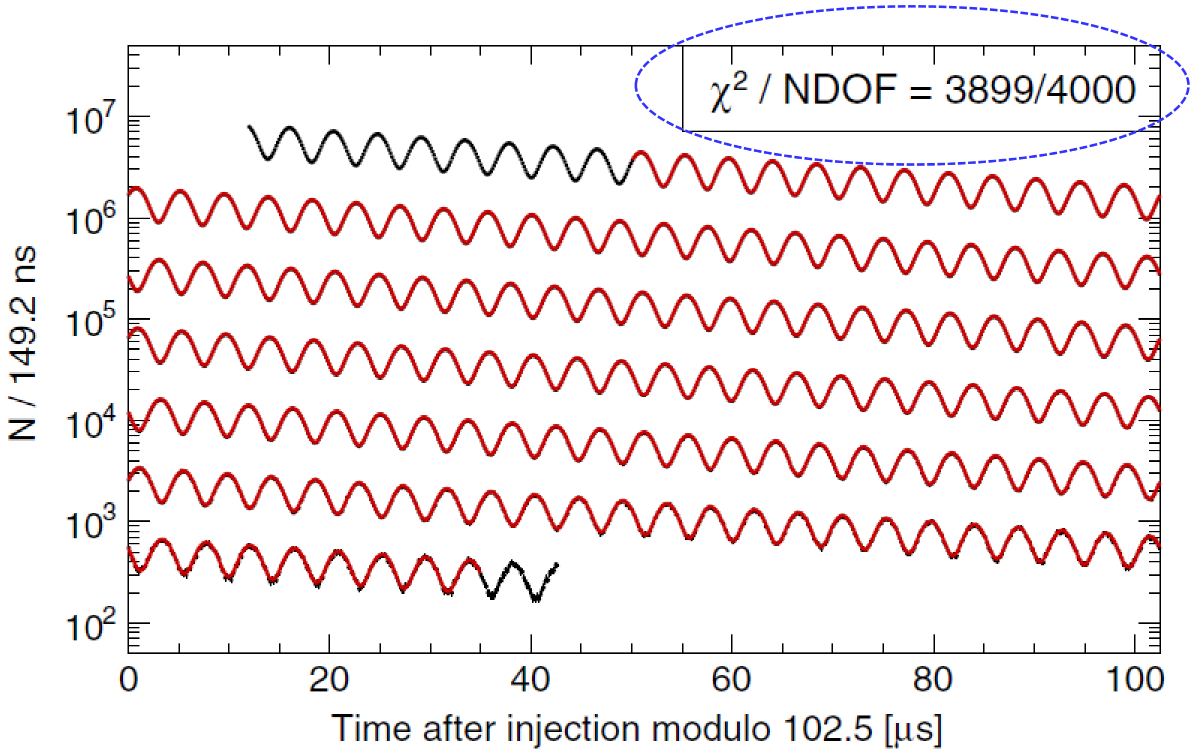
Modulates the spectrum

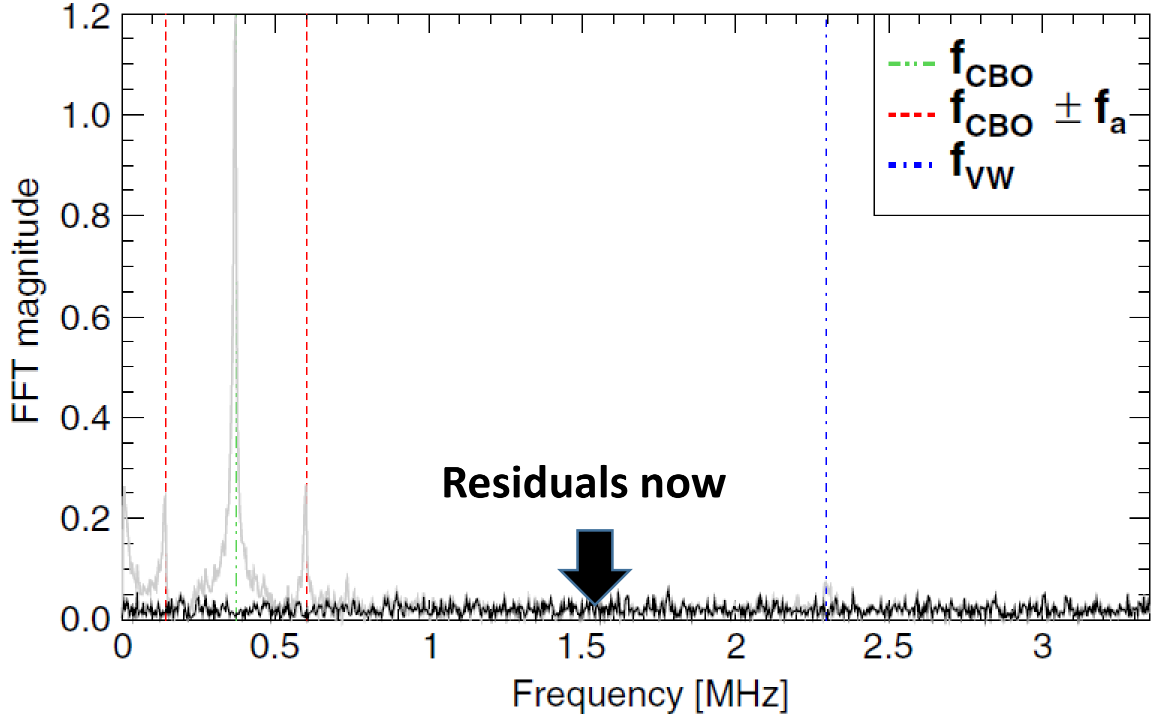


Calorimeter triple coincidences give muon loss spectrum needed in fit

$$a_\mu \propto \frac{f_{\mathrm{clock}}\ \omega_a^m\,(1+C_e+C_p+C_{ml}+C_{pa})}{f_{\mathrm{calib}}\ \langle \omega'_p(x,y,\phi)\times M(x,y,\phi)\rangle\,(1+B_k+B_q)}$$

When the fit is complete, it must look like this before being considered in the averaging





Residuals now

Lots and lots of consistency checks … ask if you want to learn more

$$a_\mu \propto \frac{f_{\mathrm{clock}}\ \ \omega_a^m\,(1+C_e+C_p+C_{ml}+C_{pa})}{\underbrace{f_{\mathrm{calib}}\ \langle \omega'_p(x,y,\phi)\times M(x,y,\phi)\rangle}_{\tilde{\omega}'_p(T_r)}\ (1+B_k+B_q)}$$

The prime: proton NMR, calibrated in terms of the equivalent precession frequency $\omega_p(T_r)$ of a proton shielded in a spherical sample of water at 34.7 °C

The tilde: The magnetic field multipoles folded with the muon distribution around the ring and throughout the run

- **Steps to obtain this quantity:**
  1) Absolute field calibration f_{calib}
  2) Periodic in-ring mapping of field multipoles $\omega'_p(x,y,\phi)$
  3) Continuous monitoring of field while muons are in the ring
  4) Continuous measurement of muon spatial profile $M(x,y,\phi)$ **in ring**
  5) Folding field multipoles with e^+ weighted muon distribution

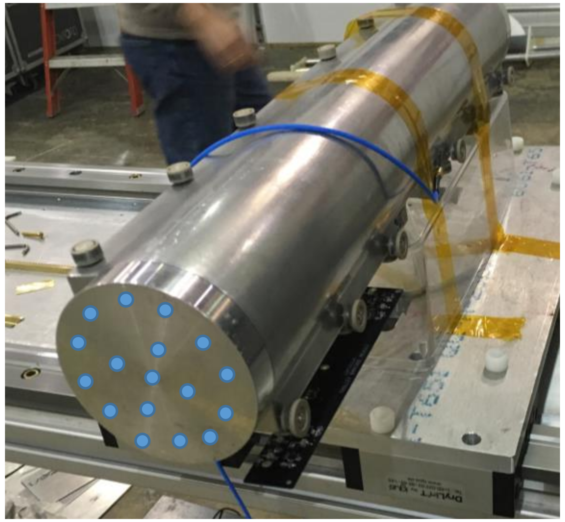
$$\omega_p^{\mathrm{meas}} = \omega'_p\left[1 - \frac{1}{\sigma}\frac{d\sigma\,(\mathrm{H_2O})}{dt}(34.7-T) - \delta_b\,(\mathrm{H_2O},T) - \delta_s - \delta_p - \delta_{\mathrm{RD}} - \delta_d\right]$$

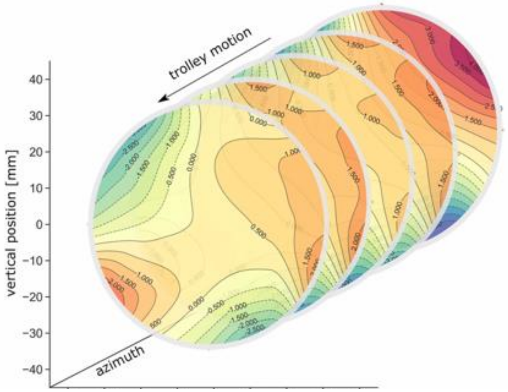
Design Goal: 35 ppb. Achieved: 15 ppb

Measure the field moments vs time

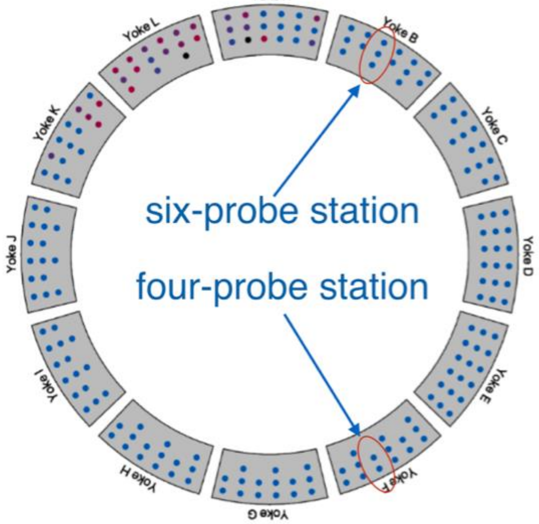
$$a_\mu \propto \frac{f_{\text{clock}} \quad \omega_a^m \left(1 + C_e + C_p + C_{ml} + C_{pa}\right)}{f_{\text{calib}} \; \langle \omega_p'(x, y, \phi) \times M(x, y, \phi)\rangle \; (1 + B_k + B_q)}$$

- 17-element Trolley maps full azimuth every few days (muons not present)
- 378 Fixed probes monitor between trolley runs (during muon data collection)
- Field map is interpolated between trolley runs using fixed probe information
- Fold with Muon Spatial Distribution





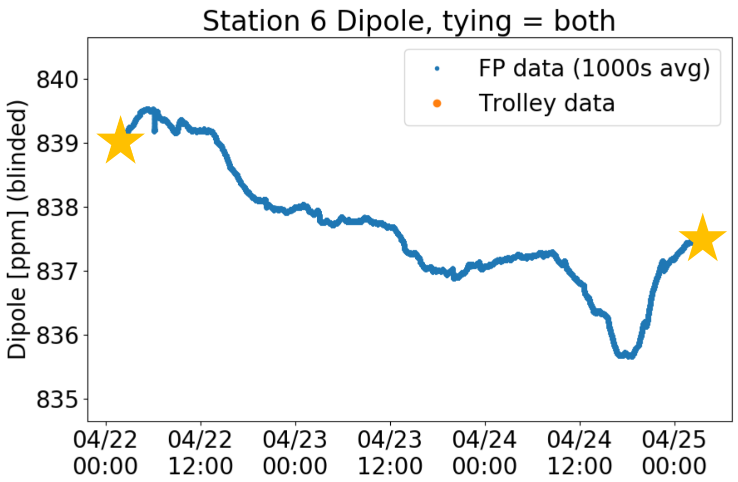
Sequence of field 2D field slices as trolley moves

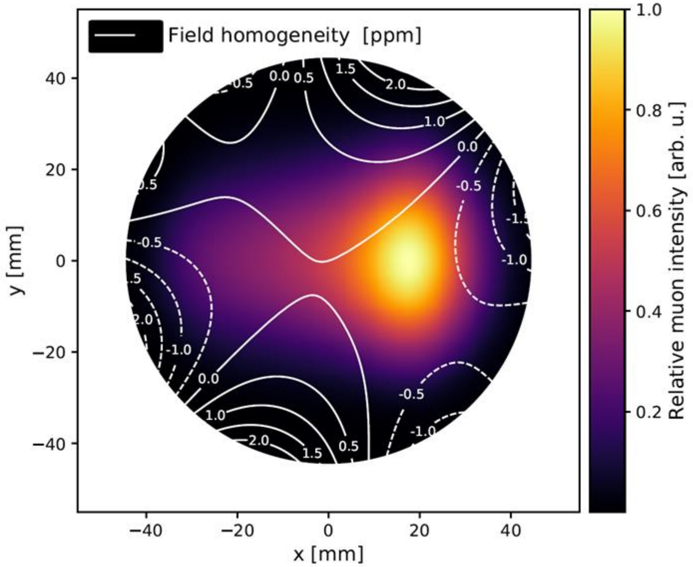


six-probe station

four-probe station

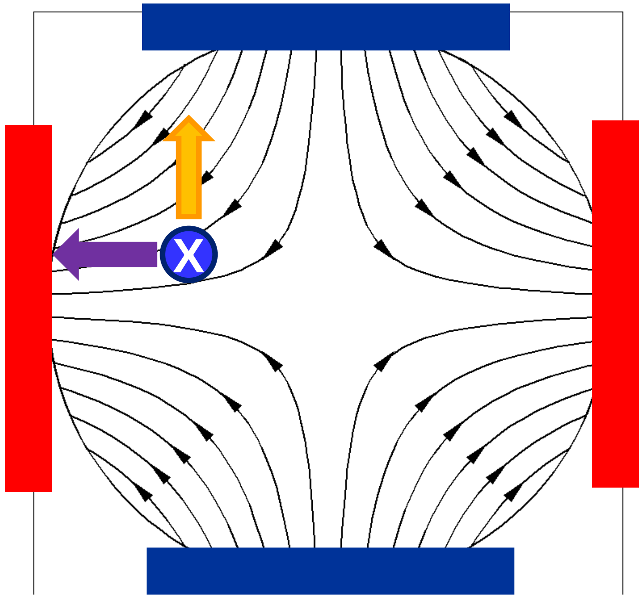
Fixed probes



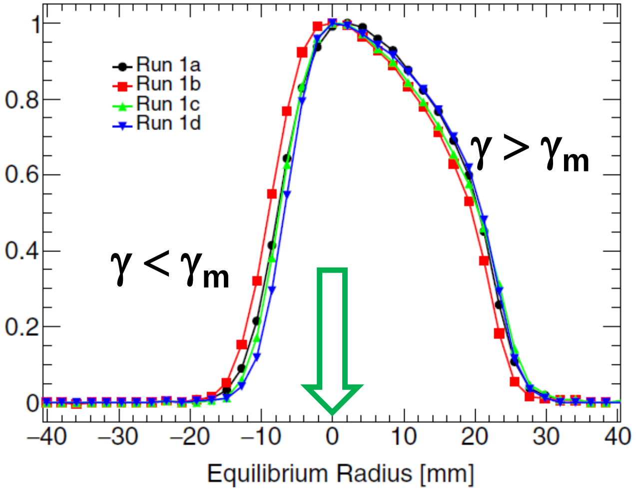


Electric field correction compensates for motional magnetic field "(v x E)" for off-momentum muons

$$a_\mu \propto \frac{f_{\text{clock}}\ \ \omega_a^m\,(1+C_e+C_p+C_{ml}+C_{pa})}{f_{\text{calib}}\ \ \langle\omega_p'(x,y,\phi)\times M(x,y,\phi)\rangle\ (1+B_k+B_q)}$$



$$\boxed{\vec{\omega}_a = -\frac{q}{m}\left[a_\mu\vec{B} - \left(a_\mu - \frac{1}{\gamma^2-1}\right)\frac{\vec{\beta}\times\vec{\mathcal{E}}}{c}\right]}$$



Magic Momentum

$\gamma = \gamma_m$

$$C_e \approx 2n(1-n)\beta_0^2\frac{\langle x_e^2\rangle}{R_0^2}$$

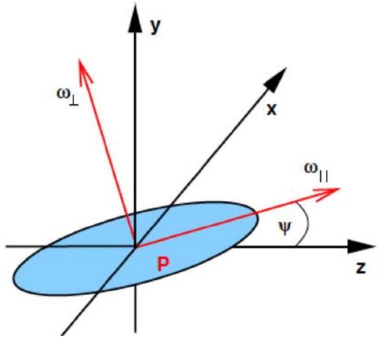
$$\langle x_e^2\rangle = \sigma_{x_e}^2 + \langle x_e\rangle^2$$

$$\boxed{C_e = 489\ \text{ppb},\ \delta_{C_e} = 53\ \text{ppb}}$$

Note: For Run-3/4 Kicker, beam is centered and C_e is smaller

The pitch correction compensates for the average vertical angle muons travel in vertical B field

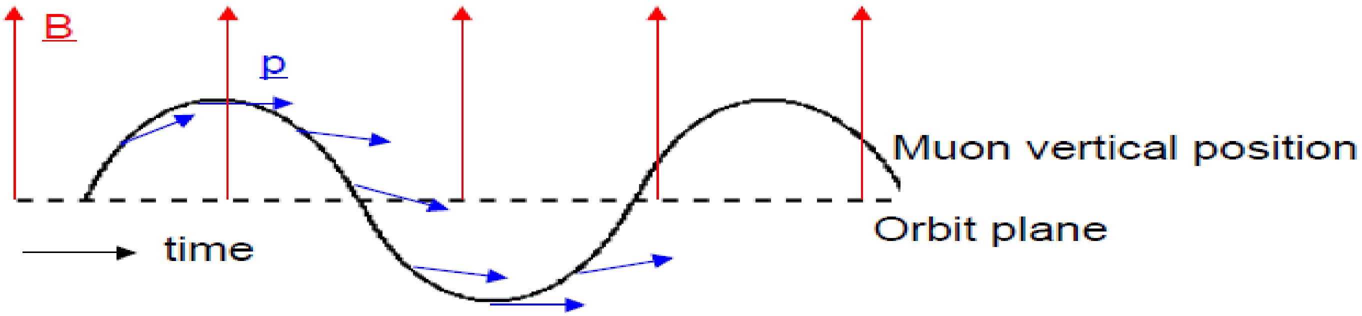
$$a_\mu \propto \frac{f_{\mathrm{clock}}\;\; \omega_a^m\,(1+C_e+C_p+C_{ml}+C_{pa})}{f_{\mathrm{calib}}\;\langle \omega_p'(x,y,\phi)\times M(x,y,\phi)\rangle\;(1+B_k+B_q)}$$



$$\boxed{\vec{\omega}_a = -\frac{q}{m}\left[a_\mu \vec{B} - a_\mu\left(\frac{\gamma}{\gamma+1}\right)(\vec{\beta}\cdot\vec{B})\vec{\beta}\right]}$$

On average, non-zero A_y measured by trackers

$$C_p = n\langle A_y^2\rangle/4R_0^2$$



$$\boxed{C_p = 180\ \mathrm{ppb}, \delta_{C_p} = 13\ \mathrm{ppb}}$$

Two corrections involve a time dependence to the average ensemble phase constant if measured vs. time-in-fill

$$a_\mu \propto \frac{f_{\text{clock}} \;\; \omega_a^m \left(1 + C_e + C_p + C_{ml} + C_{pa}\right)}{f_{\text{calib}} \; \langle \omega_p'(x,y,\phi) \times M(x,y,\phi)\rangle \; (1 + B_k + B_q)}$$

$$N(t) = N_0(t) e^{-t/\gamma\tau_\mu} \left[1 + A\cos\left(\omega_a t + \varphi_0\right)\right]$$

What if phase is not a constant? →

$$\cos\left(\omega_a t + \varphi_0(t)\right) \rightarrow \cos\left(\omega_a t + \varphi_0 + \varphi' t + \ldots\right)$$

$$\cos\left(\underbrace{(\omega_a + \varphi')}_{\omega_a' \neq \omega_a} t + \varphi_0 + \ldots\right)\Big)$$

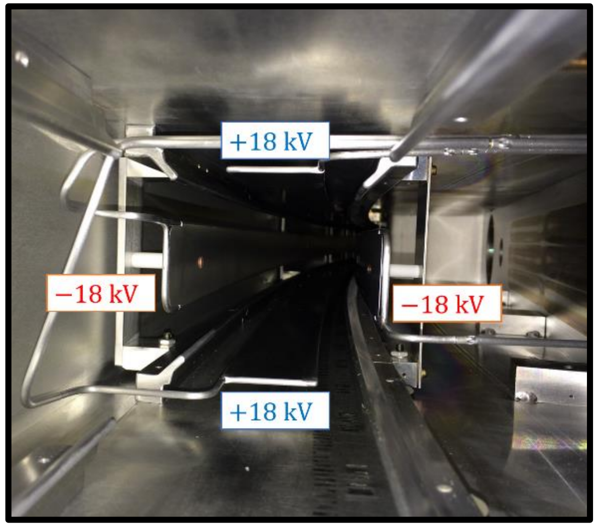
- The C_{ml} correction accounts for muons that escape the ring before they decay
- The C_{pa} "**phase-acceptance correlation**" correction accounts for the Run-1 quadrupole malfunction that allowed the stored beam to move vertically and shrink in vertical width
  - This held us up for a very long time until we understood it fully
  - ASK me about it if you wish at Q&A time… it's a bit technical, but interesting

$$\boxed{\boldsymbol{C_{ml} = -11\ \text{ppb}, \delta_{C_{ml}} = 5\ \text{ppb}}}$$

$$\boxed{\boldsymbol{C_{pa} = -158\ \text{ppb}, \delta_{C_{pa}} = 75\ \text{ppb}}}$$

Two transients effects perturbed *B* within the kicker and quadrupole plates at injection

$$a_\mu \propto \frac{f_{\mathrm{clock}} \quad \omega_a^m \left(1 + C_e + C_p + C_{ml} + C_{pa}\right)}{f_{\mathrm{calib}} \ \langle \omega_p'(x,y,\phi) \times M(x,y,\phi) \rangle \ \left(1 + B_k + B_q\right)}$$



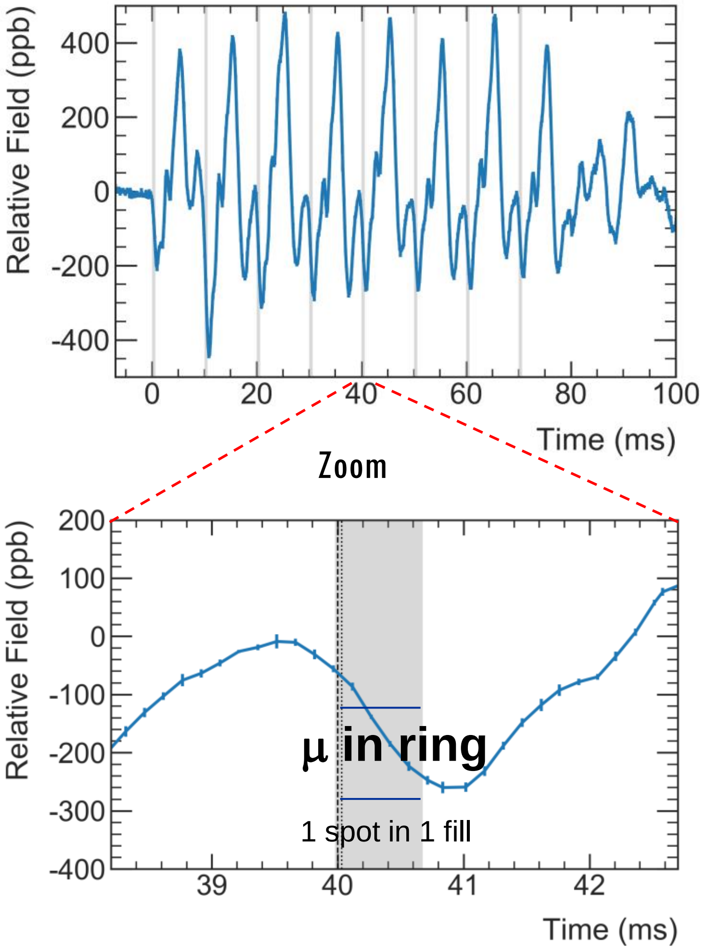
Quads pulsed on every fill

→ induces mechanical vibrations

→ oscillating B field

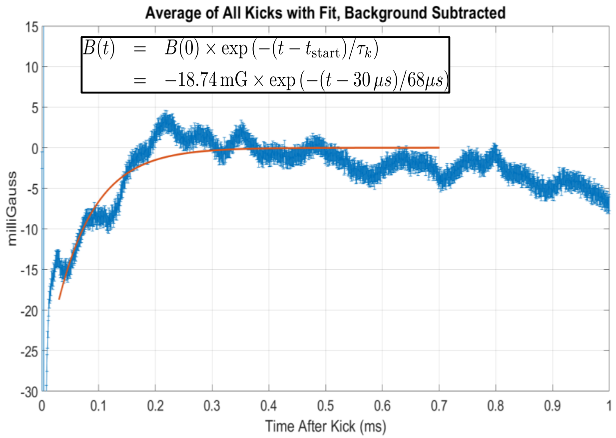
→ Net effect was small, but… complicated!

$$\boxed{B_q = -17\ \mathrm{ppb}, \delta_{B_q} = 92\ \mathrm{ppb}}$$



µ in ring

1 spot in 1 fill


Average of All Kicks with Fit, Background Subtracted

$$\begin{aligned} B(t) &= B(0) \times \exp\left(-(t - t_{\mathrm{start}})/\tau_k\right) \\ &= -18.74\,\mathrm{mG} \times \exp\left(-(t - 30\,\mu s)/68\mu s\right) \end{aligned}$$

Kickers fire on every fill

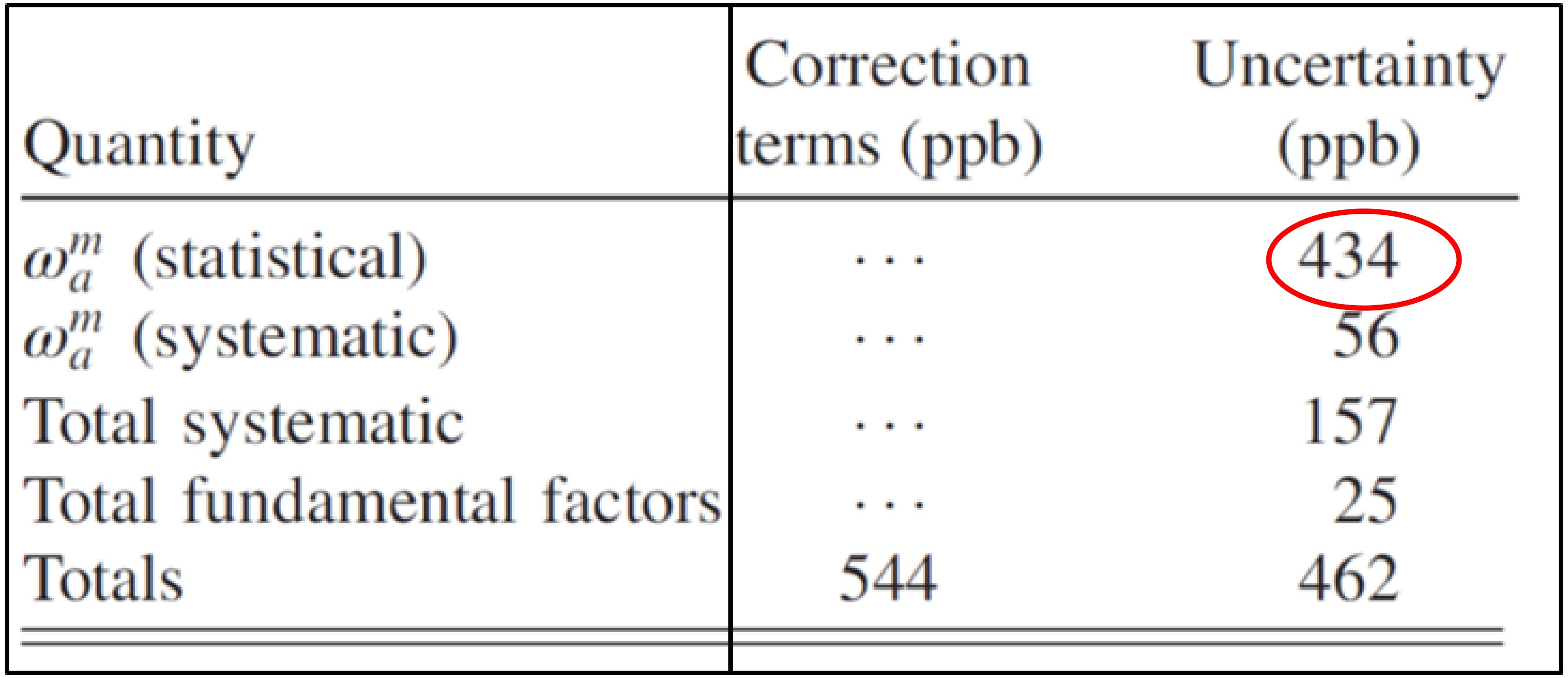
→ induces small Eddy currents

→ We measured with custom magnetometers based on the Faraday effect

$$\boxed{B_k = -27\ \mathrm{ppb}, \delta_{B_k} = 37\ \mathrm{ppb}}$$

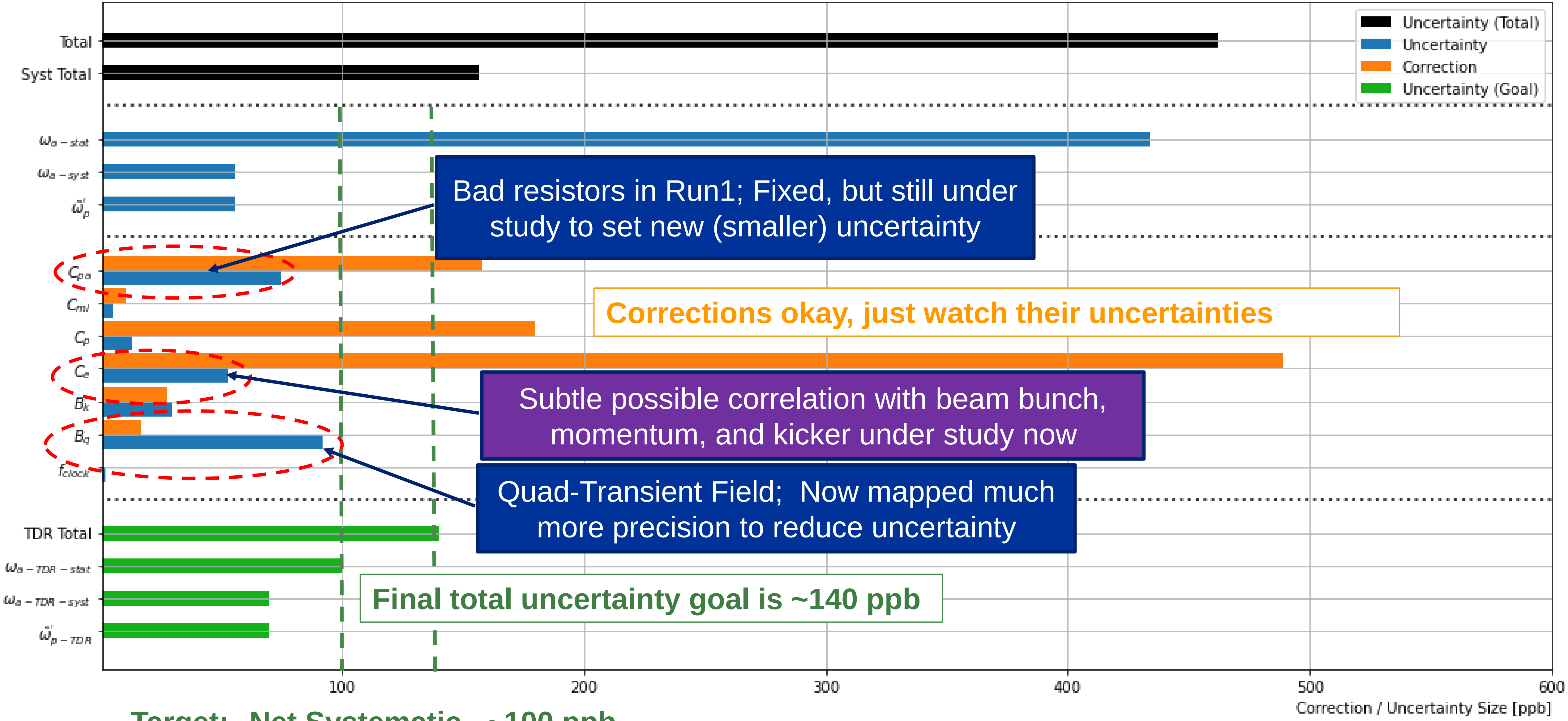
At this point, we know all the numbers in the master formula

$$a_\mu \propto \frac{f_{\rm clock} \;\; \omega_a^m \,(1 + C_e + C_p + C_{ml} + C_{pa})}{f_{\rm calib} \; \langle \omega_p'(x,y,\phi) \times M(x,y,\phi) \rangle \; (1 + B_k + B_q)}$$



Quantity	Correction terms (ppb)	Uncertainty (ppb)
ω_a^m (statistical)	$\cdots$	434
ω_a^m (systematic)	$\cdots$	56
Total systematic	$\cdots$	157
Total fundamental factors	$\cdots$	25
Totals	544	462

The Run-1 Uncertainties and Corrections and the Goals



Bad resistors in Run1; Fixed, but still under study to set new (smaller) uncertainty

Corrections okay, just watch their uncertainties

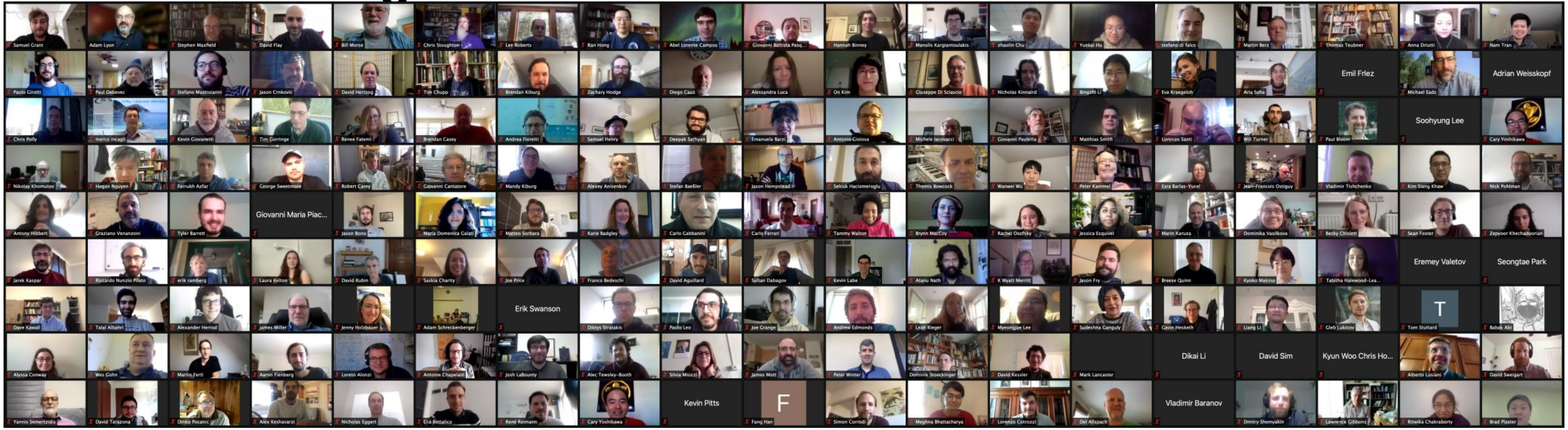
Subtle possible correlation with beam bunch, momentum, and kicker under study now

Quad-Transient Field; Now mapped much more precision to reduce uncertainty

Final total uncertainty goal is ~140 ppb

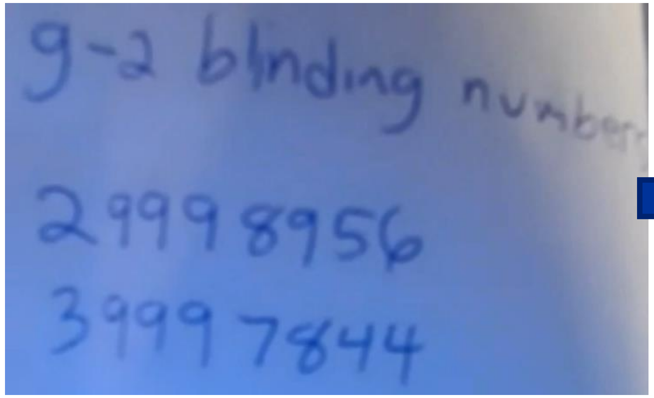
Target: Net Systematic ~100 ppb
Target: Statistical at ~100 ppb

The "Unblinding"



UW envelope

FNAL envelope

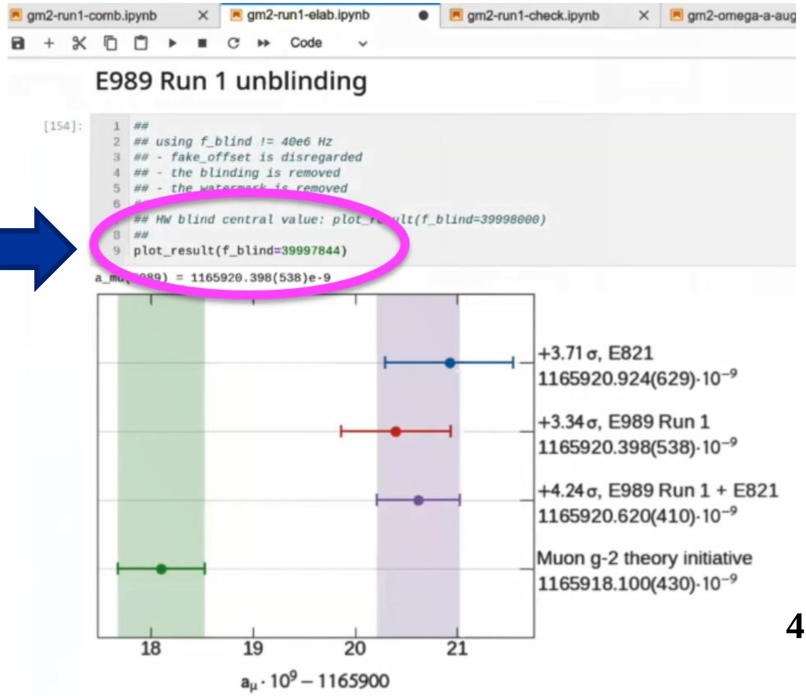


g-2 blinding number

29998956

39997844

Same numbers!

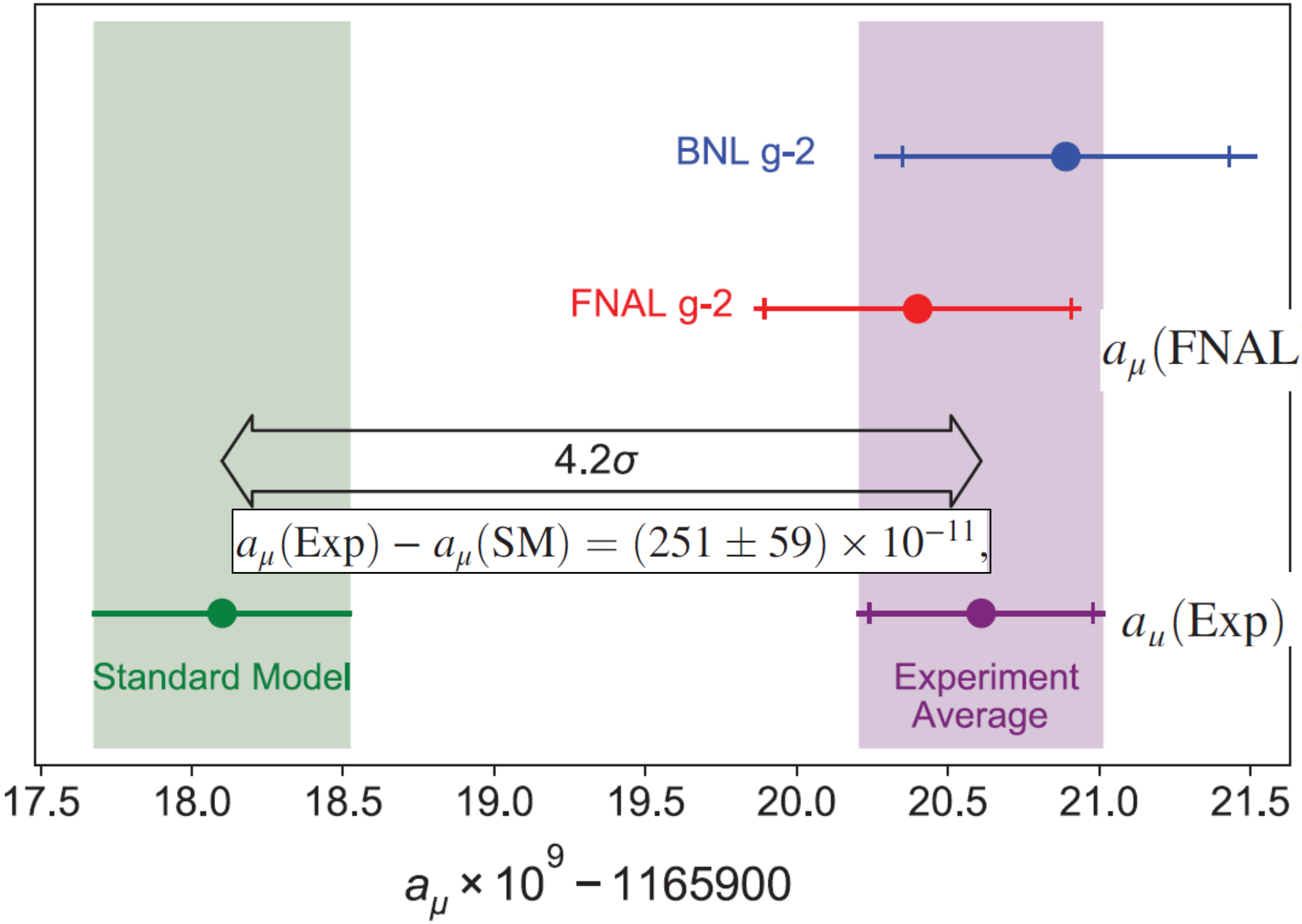


E989 Run 1 unblinding

```
##
## using f_blind != 40e6 Hz
## - fake_offset is disregarded
## - the blinding is removed
## - the watermark is removed
##
## HW blind central value: plot_result(f_blind=39998000)
##
plot_result(f_blind=39997844)
```

```
a_mu(E989) = 1165920.398(538)e-9
```

We confirm the BNL result
The combined discrepancy with SM increases

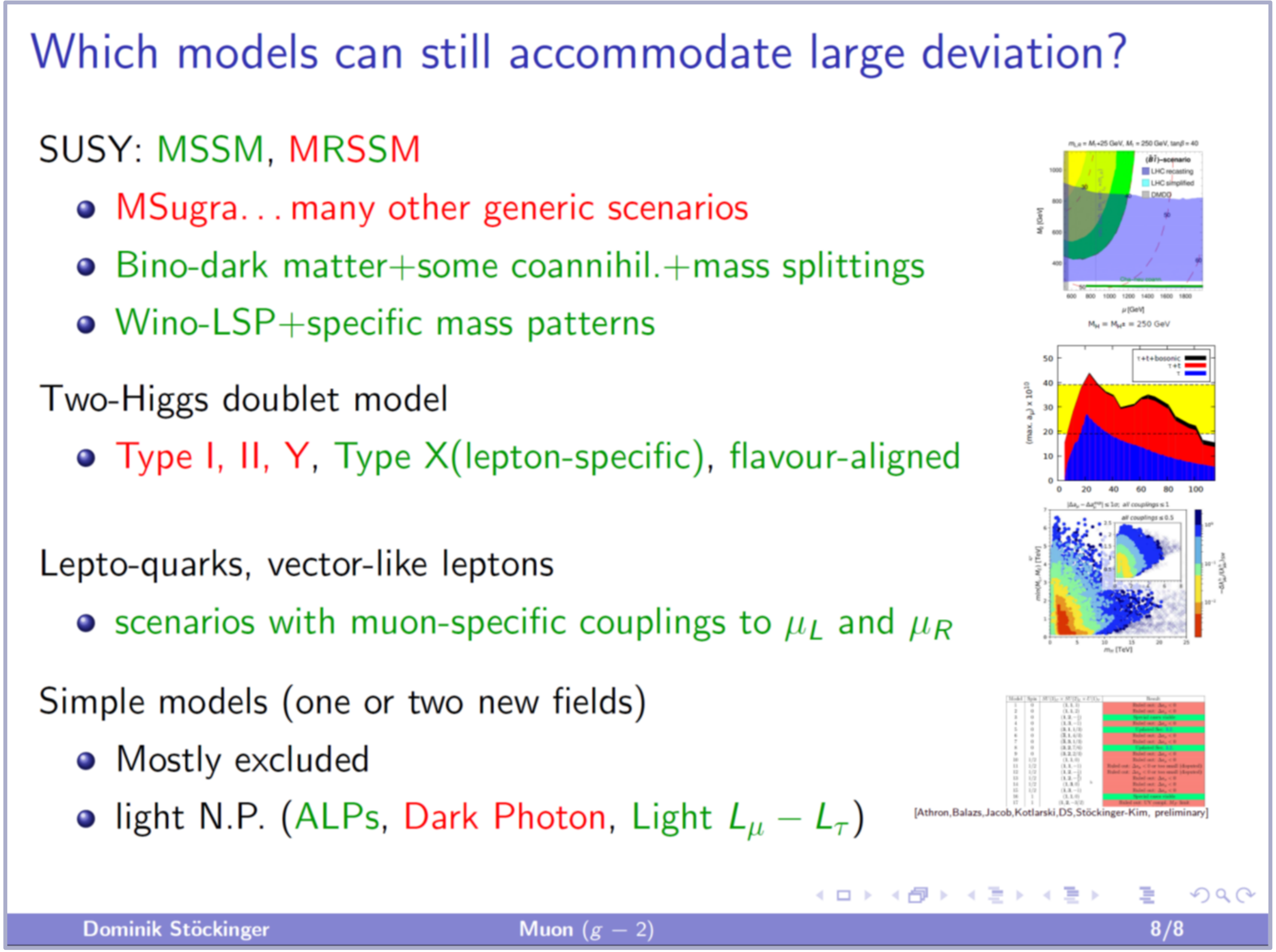


$$a_\mu(\mathrm{FNAL}) = 116\,592\,040(54) \times 10^{-11} \quad (0.46\ \mathrm{ppm})$$

$$a_\mu(\mathrm{Exp}) - a_\mu(\mathrm{SM}) = (251 \pm 59) \times 10^{-11},$$

$$a_\mu(\mathrm{Exp}) = 116\,592\,061(41) \times 10^{-11} \quad (0.35\ \mathrm{ppm})$$

What could it mean? … a literature summary by Dominik Stockinger



Which models can still accommodate large deviation?

SUSY: MSSM, MRSSM

- MSugra. . . many other generic scenarios
- Bino-dark matter+some coannihil.+mass splittings
- Wino-LSP+specific mass patterns

Two-Higgs doublet model

- Type I, II, Y, Type X(lepton-specific), flavour-aligned

Lepto-quarks, vector-like leptons

- scenarios with muon-specific couplings to μ_L and μ_R

Simple models (one or two new fields)

- Mostly excluded
- light N.P. (ALPs, Dark Photon, Light $L_\mu - L_\tau$)

[Athron,Balazs,Jacob,Kotlarski,DS,Stöckinger-Kim, preliminary]

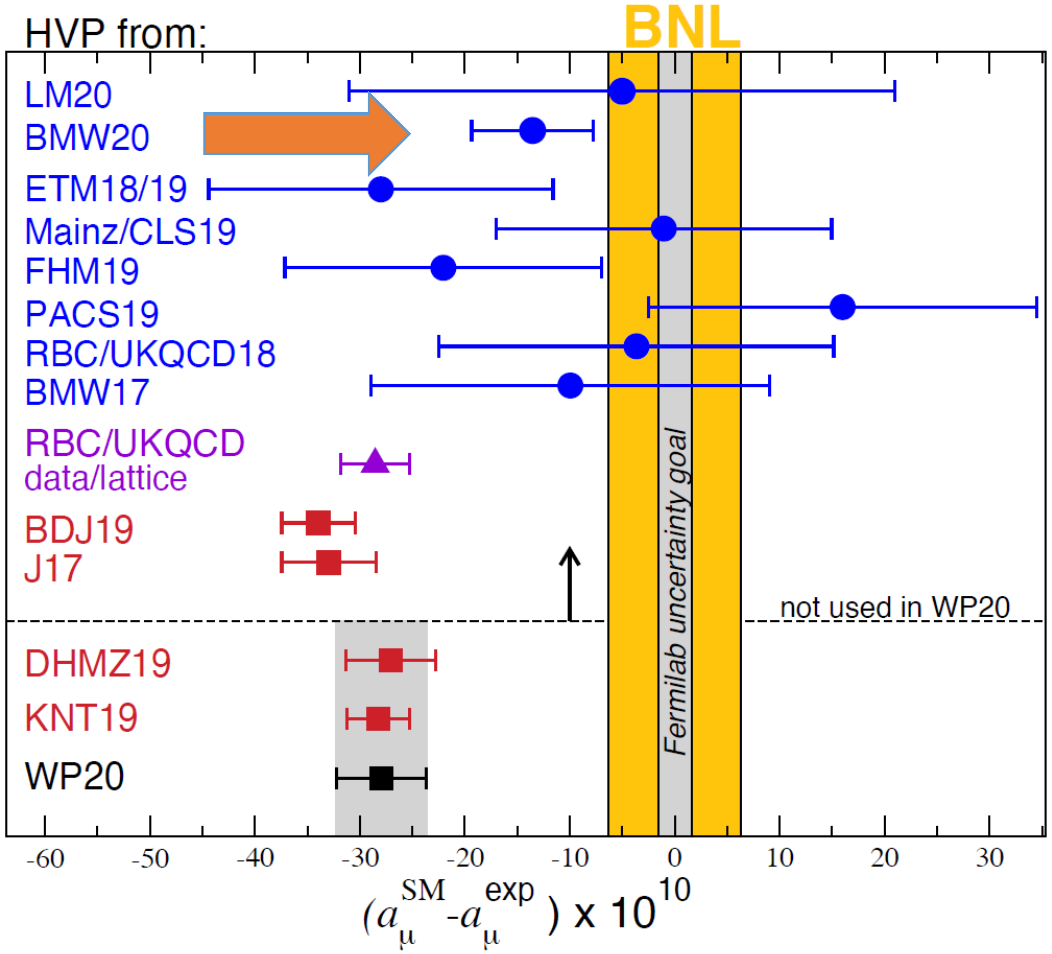
Dominik Stöckinger Muon $(g-2)$ 8/8

RED = “no”
GREEN = “maybe”

The April 7 result release PRL has 300 citations as of this morning … so many ideas have emerged

Or maybe the SM will shift per new Lattice result?

- *The BMW collaboration's result is the first of its kind at sub-percent precision; it is compared to decades of expt. results*
- *We look forward to continued efforts by all lattice groups as we require the SM precision to increase over time*



Ab-initio lattice QCD(+QED) calculations are maturing

Difficult problem: scales from $2m_\pi$ to several GeV enter; cross-checks needed at high precision

Hybrid window method restricts scales that enter from lattice/dispersive data

Dispersive, $e^+e^- \to$ hadrons (20+ years of experiments)

Article

Leading hadronic contribution to the muon magnetic moment from lattice QCD

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The standard model of particle physics describes the vast majority of experiments and observations involving elementary particles. Any deviation from its predictions would be a sign of new, fundamental physics. One long-standing discrepancy concerns the anomalous magnetic moment of the muon, a measure of the magnetic field surrounding that particle. Standard-model predictions[1] exhibit disagreement with measurements[2] that is tightly scattered around 3.7 standard deviations. Today, theoretical and measurement errors are comparable; however, ongoing and planned experiments aim to reduce the measurement error by a factor of four. Theoretically, the dominant source of error is the leading-order hadronic vacuum polarization (LO-HVP) contribution. For the upcoming measurements, it is essential to evaluate the prediction for this contribution with independent methods and to reduce its uncertainties. The most precise, model-independent determinations so far rely on dispersive techniques, combined with measurements of the cross-section of electron–positron annihilation into hadrons[3–6]. To eliminate our reliance on these experiments, here we use ab initio quantum chromodynamics (QCD) and quantum electrodynamics simulations to compute the LO-HVP contribution. We reach sufficient precision to discriminate between the measurement of the anomalous magnetic moment of the muon and the predictions of dispersive methods. Our result favours the experimentally measured value over those obtained using the dispersion relation. Moreover, the methods used and developed in this work will enable further increased precision as more powerful computers become available.

The muon is an ephemeral sibling of the electron. It is 207 times more massive, but has the same electric charge and spin. Similarly to the electron, it behaves like a tiny magnet, characterized by a magnetic moment. This quantity is proportional to the spin and charge of the muon and inversely proportional to twice its mass. Dirac's relativistic quantum mechanics predicts that the constant of proportionality, g_μ, should be equal to 2. However, in a relativistic quantum field theory such as the standard model, this prediction receives small corrections due to quantum vacuum fluctuations. These corrections are called the anomalous magnetic moment and are quantified by $(g_\mu - 2)/2$. They were measured to a precision of 0.54 ppm at the Brookhaven National Laboratory in the early 2000s[2], and have been calculated with a comparable precision (see ref.[7] for a recent review).

At this level of precision, all of the interactions of the standard model contribute. The leading contributions are electromagnetic and described by quantum electrodynamics (QED), but the one that dominates the theoretical error is induced by the strong interaction and requires solving the highly nonlinear equations of QCD at low energies. This contribution is determined by the LO-HVP, which describes how the propagation of a virtual photon is modified by the presence of quark and gluon fluctuations in the vacuum. Here we compute this LO-HVP contribution to $(g_\mu - 2)/2$, denoted by $a_\mu^{\rm LO\text{-}HVP}$, using ab initio simulations in QCD and QED.

QCD is a generalized version of QED. The Euclidean Lagrangian for this theory is $\mathcal{L} = [1/(4e^2)]F_{\mu\nu}F_{\mu\nu} + [1/(2g^2)]\mathrm{tr}(G_{\mu\nu}G_{\mu\nu}) + \Sigma_f \bar\psi_f[\gamma_\mu(\partial_\mu + iq_fA_\mu + iB_\mu) + m_f]\psi_f$, where γ_μ are the Dirac matrices, f runs over the 'flavours' of quarks, m_f are the masses and the q_f are the charges of quarks in units of the electron charge, e. Moreover, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and $G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu + [B_\mu, B_\nu]$ and g is the QCD coupling constant. In electrodynamics the gauge potential A_μ is a real-valued field, whereas in QCD B_μ is a 3 × 3 Hermitian matrix field. The different flavours of quarks are represented by independent fermionic fields, ψ_f. These fields have an additional 'colour' index in QCD, which runs from 1 to 3. In this work, we include both QED and QCD, as well as four non-degenerate quark flavours

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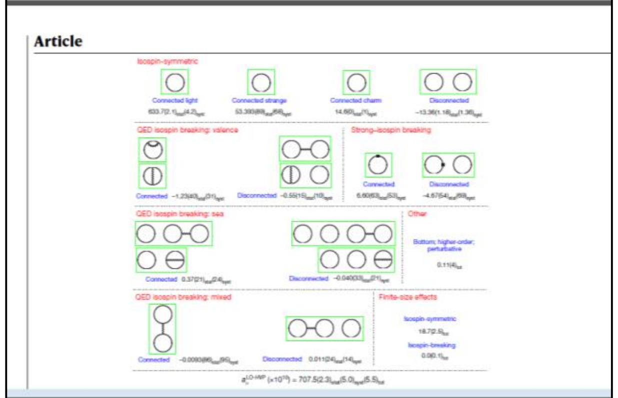


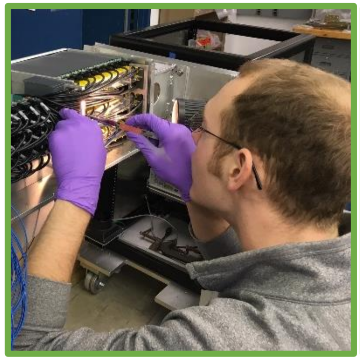
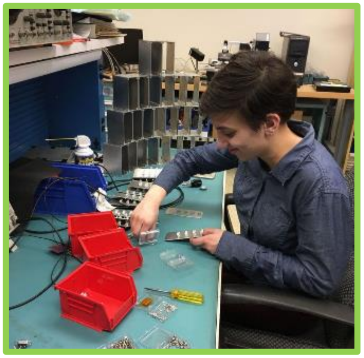
Article

Now first published lattice result with sub-percent precision available (BMW20), cross-checks are crucial to establish or refute high-precision lattice methodology (same situation as for HLbL) $\Rightarrow$ Theory Initiative as a platform to do this

The University of Washington g-2 Team: Grad Students / Postdocs / Faculty/Scientists

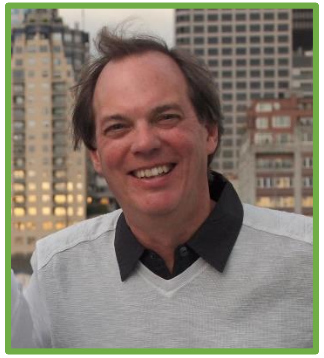


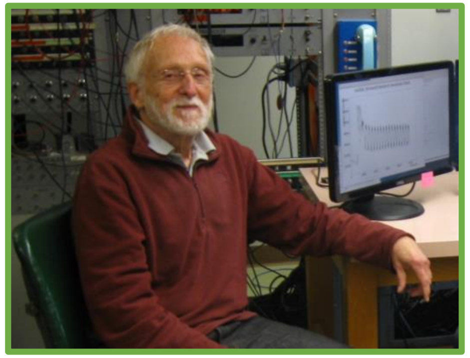




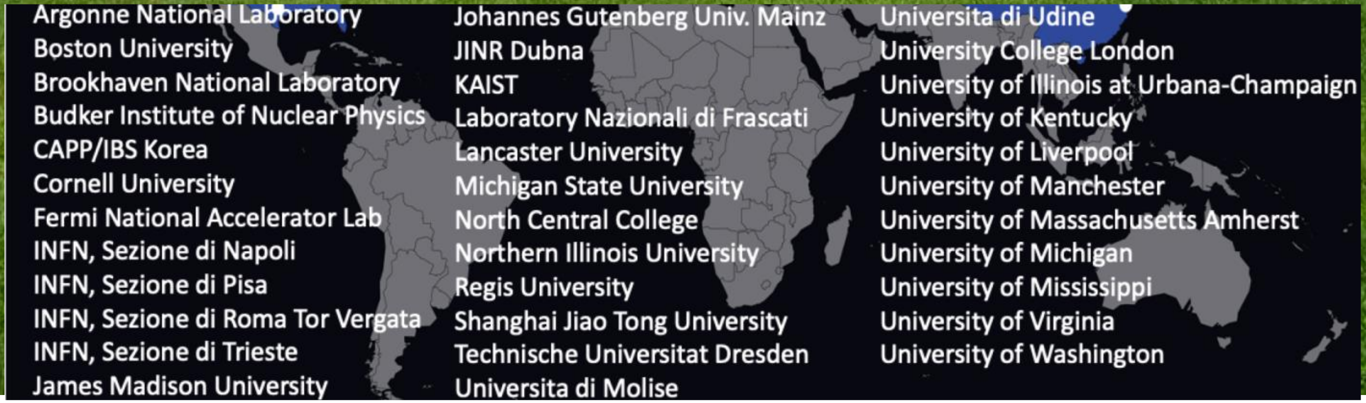


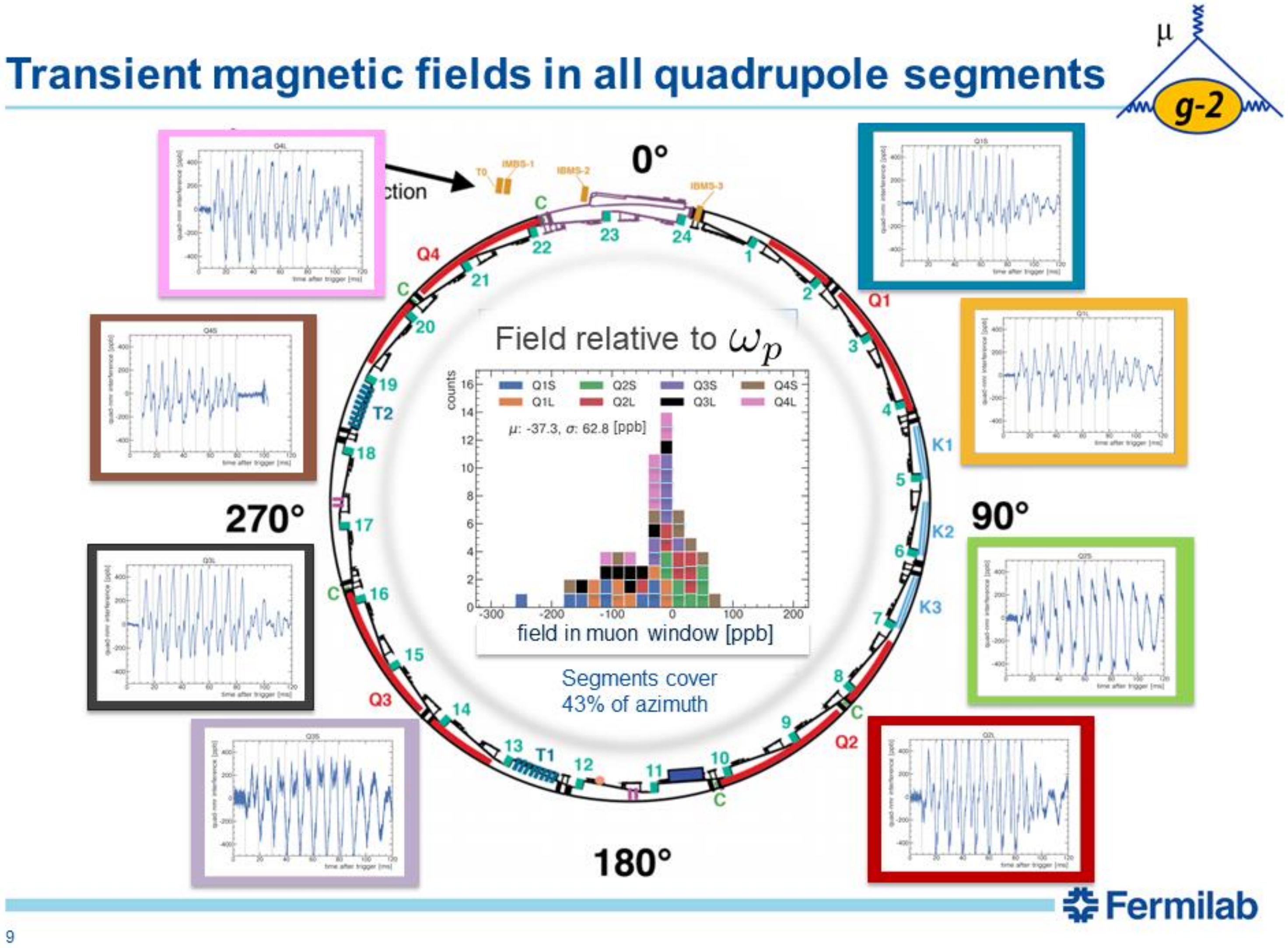
It’s just too early to say…
Our error will go down (by a lot) and the SM will improve further
The fun is just beginning

The g-2 Collaboration in Elba, Spring 2019
(when we had hoped to open the box)



Argonne National Laboratory
Boston University
Brookhaven National Laboratory
Budker Institute of Nuclear Physics
CAPP/IBS Korea
Cornell University
Fermi National Accelerator Lab
INFN, Sezione di Napoli
INFN, Sezione di Pisa
INFN, Sezione di Roma Tor Vergata
INFN, Sezione di Trieste
James Madison University
Johannes Gutenberg Univ. Mainz
JINR Dubna
KAIST
Laboratory Nazionali di Frascati
Lancaster University
Michigan State University
North Central College
Northern Illinois University
Regis University
Shanghai Jiao Tong University
Technische Universitat Dresden
Universita di Molise
Universita di Udine
University College London
University of Illinois at Urbana-Champaign
University of Kentucky
University of Liverpool
University of Manchester
University of Massachusetts Amherst
University of Michigan
University of Mississippi
University of Virginia
University of Washington

Overcoming the Quad-Transient by mapping in great detail around the ring will reduce the systematic considerably



Transient magnetic fields in all quadrupole segments

Segments cover 43% of azimuth

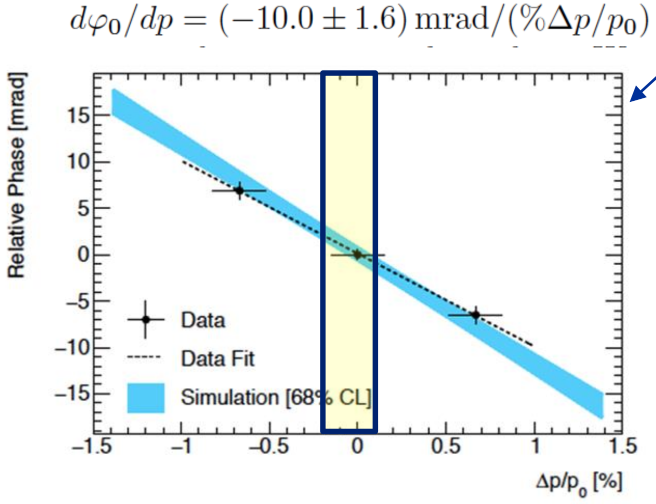
UW built PEEK trolley and probe for inside quad measurements during pulsing

The muons that escape (lost) during a fill have a slightly different phase compared to those that remain stored

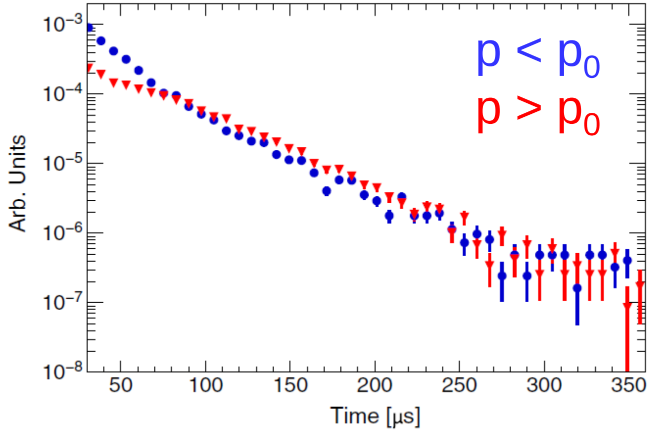
$$a_\mu \propto \frac{f_{\mathrm{clock}}\;\omega_a^m\,(1+C_e+C_p+C_{ml}+C_{pa})}{f_{\mathrm{calib}}\;\langle \omega_p'(x,y,\phi)\times M(x,y,\phi)\rangle\;(1+B_k+B_q)}$$

Because of a double correlation. We measured both and determined this effect to be TINY

$$\Delta\omega_a = \frac{d\phi}{dt} = \boxed{\frac{d\phi}{dp}}\cdot\boxed{\frac{dp}{dt}} \neq 0$$



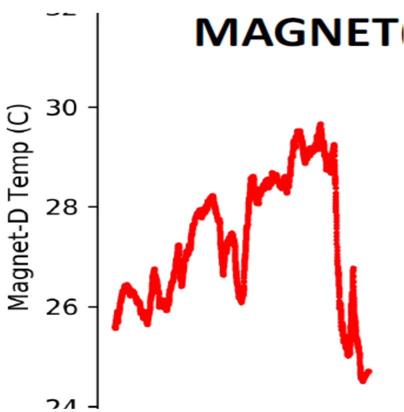
Phase depends on momentum



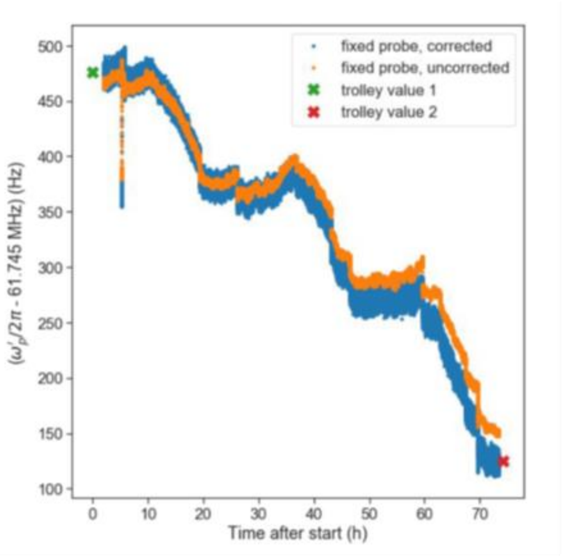
Loss rate depends on momentum

$$\boxed{C_{ml} = -11\ \mathrm{ppb},\ \delta_{C_{ml}} = 5\ \mathrm{ppb}}$$

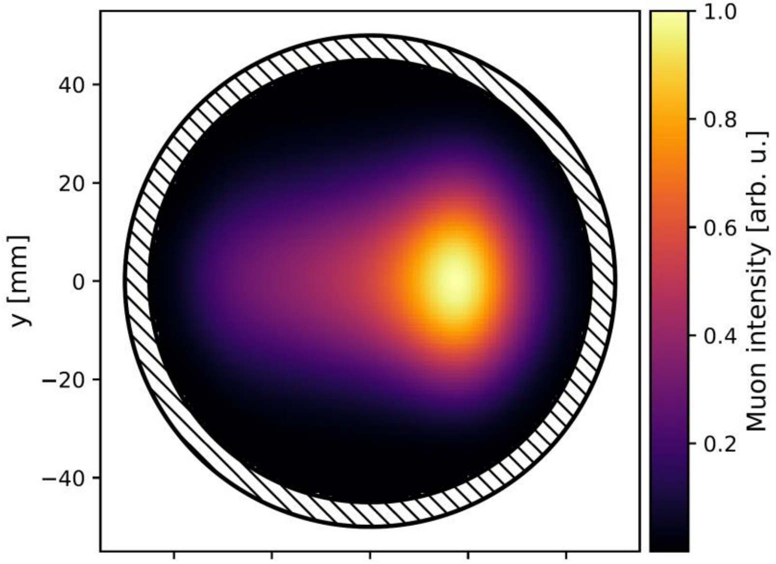
Run-1 commissioning Challenges … (resolved by now)

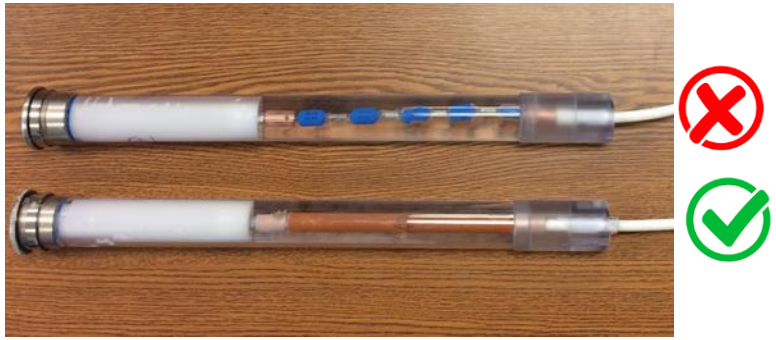


Hall T unstable
→ B changing
→ Gains changing

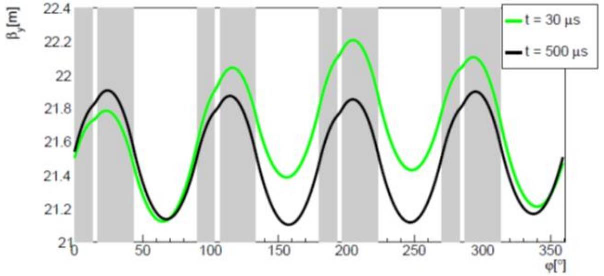


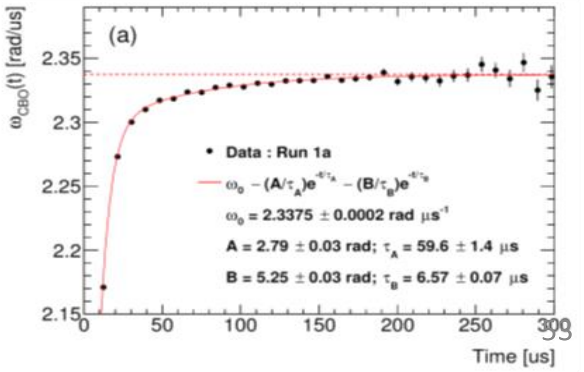
Kicker sparks limited range to below optimum





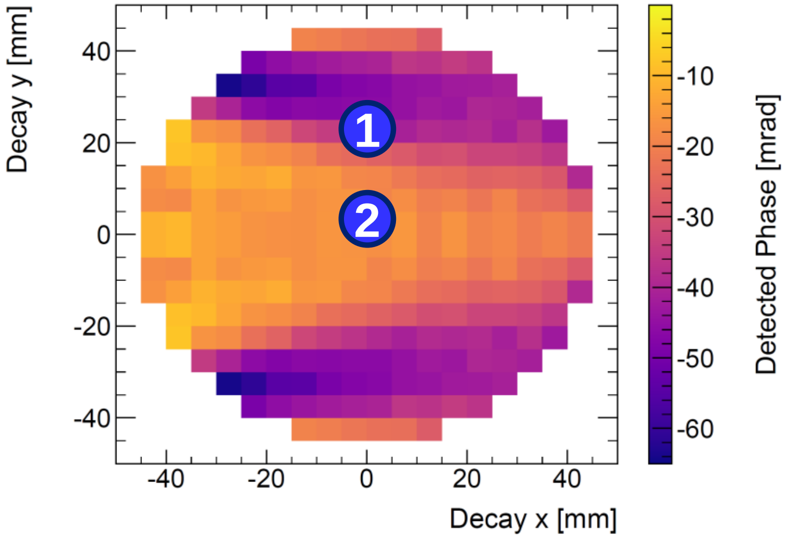
2/32 ESQ resistors "damaged"





The damaged resistors allowed the optical lattice to evolve during a fill… and that turned out to be a very tough problem to evaluate

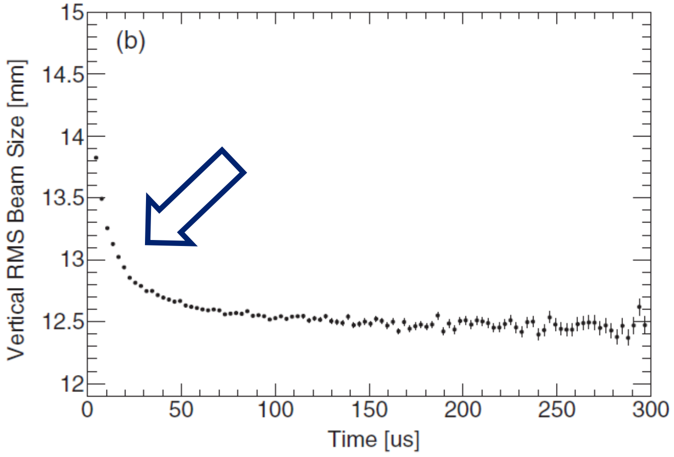
$$a_\mu \propto \frac{f_{\text{clock}}\ \omega_a^m \left(1 + C_e + C_p + C_{ml} + C_{pa}\right)}{f_{\text{calib}}\ \langle \omega_p'(x,y,\phi) \times M(x,y,\phi)\rangle \left(1 + B_k + B_q\right)}$$



Detector acceptance couples phase to decay X-Y coordinate inside storage volume

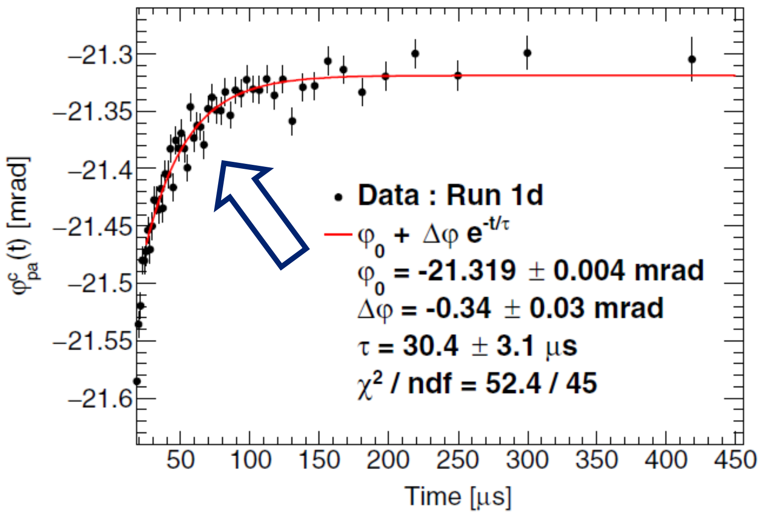
(Peak of "wiggle" plot different for 1 and 2 slightly)

This will always be true



Bad resistors squeezed the vertical width during the fill !

This should never happen



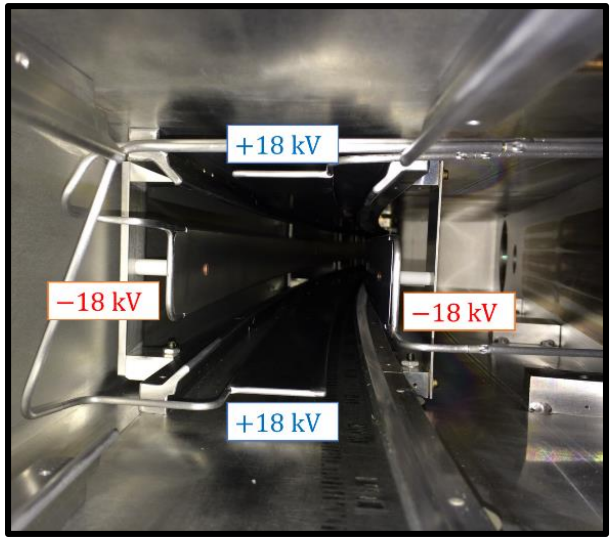
Creates a measured $\phi(t)$ that had to be removed from the fit

An unfortunate reality → C_{pa}

$$\boxed{C_{pa} = -158\ \text{ppb},\ \delta_{C_{pa}} = 75\ \text{ppb}}$$

Two transients effects perturbed B within the kicker and quadrupole plates at injection

$$a_\mu \propto \frac{f_{\text{clock}}\ \ \omega_a^m\,(1+C_e+C_p+C_{ml}+C_{pa})}{f_{\text{calib}}\ \langle \omega_p'(x,y,\phi)\times M(x,y,\phi)\rangle\ (1+B_k+B_q)}$$



Quads pulsed every fill

→ induces mechanical vibrations (43% of ring)

→ oscillating conductor perturbs B field

8 bunch sequence; 10 ms spacing

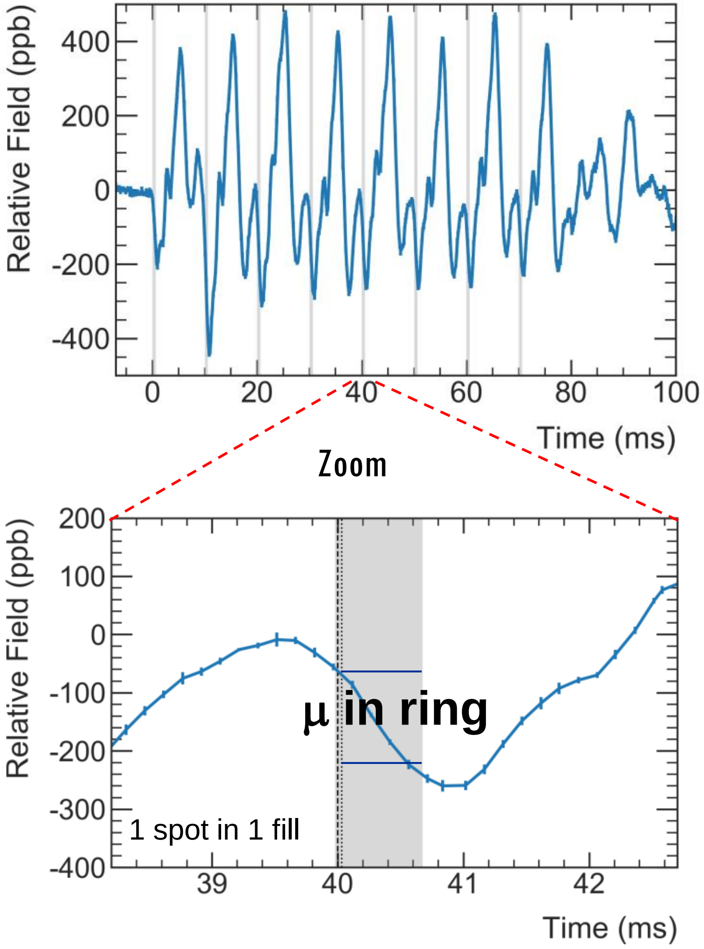
→ close to 100 Hz natural resonance!!!

Special NMR probes used to map the effect →

Lucky, small when muons are present, averaged over bunches and reduced by quad coverage of azimuth

Uncertainty large now because we have not yet mapped all quads; takes time

Expect δ_{B_q} reduction x2-3 in future



$$\boxed{B_q = -17\ \text{ppb}, \delta_{B_q} = 92\ \text{ppb}}$$

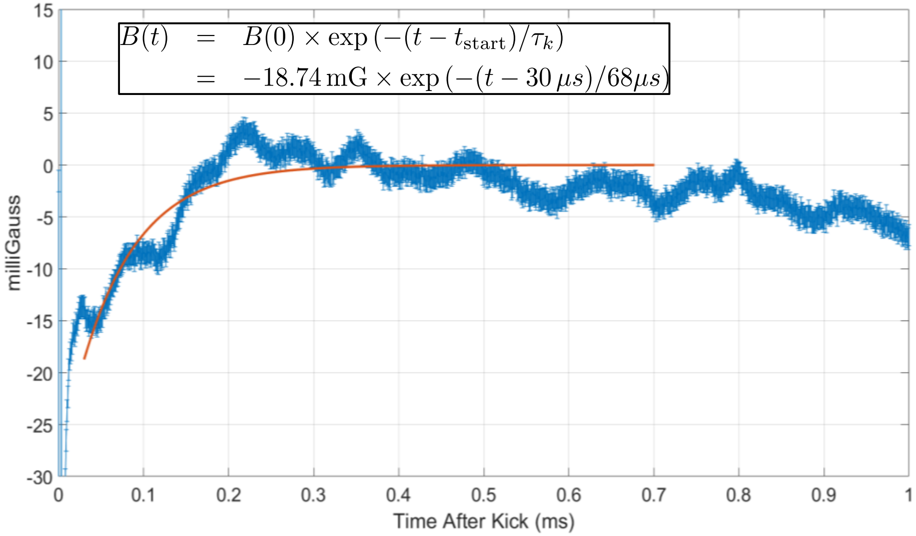
Eddy currents are produced when Kicker fires → leaves a small decaying magnetic field

$$a_\mu \propto \frac{f_{\text{clock}} \;\; \omega_a^m \left(1 + C_e + C_p + C_{ml} + C_{pa}\right)}{f_{\text{calib}} \; \langle \omega_p'(x,y,\phi) \times M(x,y,\phi)\rangle \; \left(1 + B_k + B_q\right)}$$

- The ~ 220 G kicker pulse produces a transient magnetic field for 150 ns in the storage volume → eddy currents
- **2 Faraday magnetometers** installed between the kicker plates measured the rotation of polarized light in a crystal due to the transient field
- Consistent results for both magnetometers
- Signal was fitted with an exponential function

$$\Delta B(t) = \Delta B(0) \exp\left(-t/\tau_k\right)$$

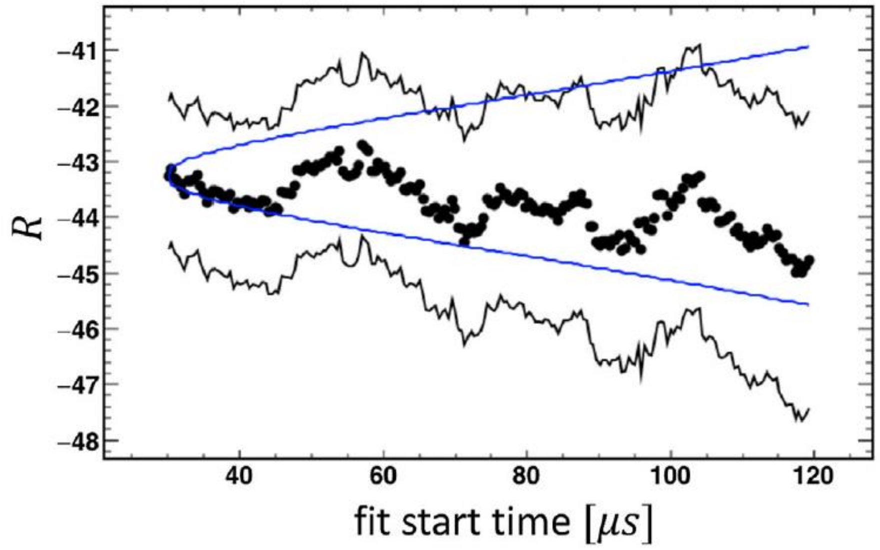
Magnetometer between kicker plates

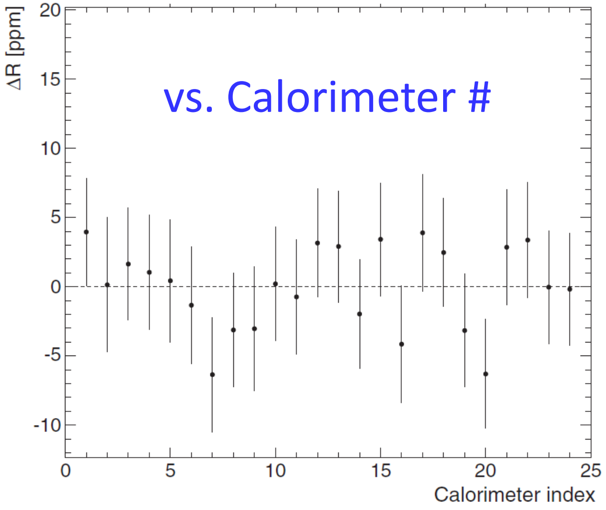


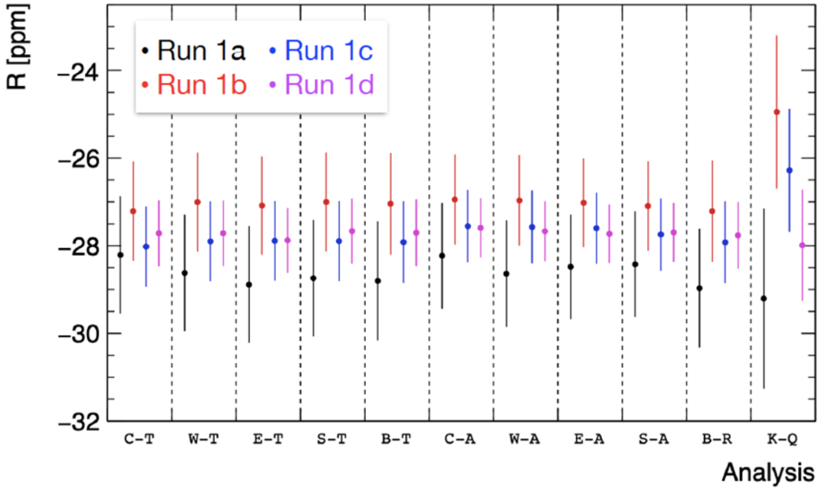
$$\boxed{B_k = -27\ \text{ppb},\ \delta_{B_k} = 37\ \text{ppb}}$$

Consistency checks. *R* = (blinded) ω_a^m in PPM

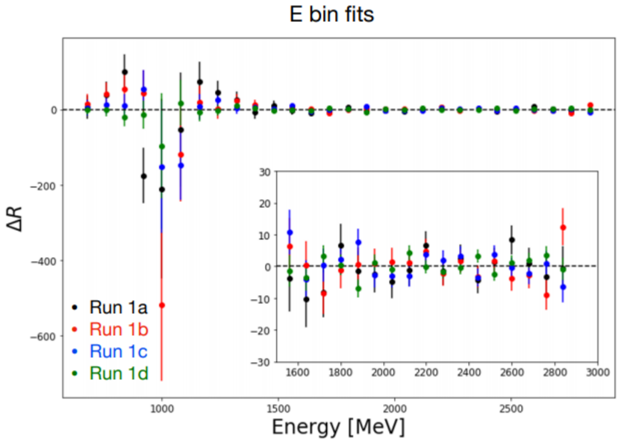
$$a_\mu \propto \frac{f_{\mathrm{clock}}\ \omega_a^m\,(1+C_e+C_p+C_{ml}+C_{pa})}{f_{\mathrm{calib}}\ \langle\omega'_p(x,y,\phi)\times M(x,y,\phi)\rangle\,(1+B_k+B_q)}$$







Comparison of analysis methods by 6 teams over 4 subgroups of Run-1



Takeaway: Statistical Uncertainty = 434 ppb
Systematic uncertainty = 56 ppb

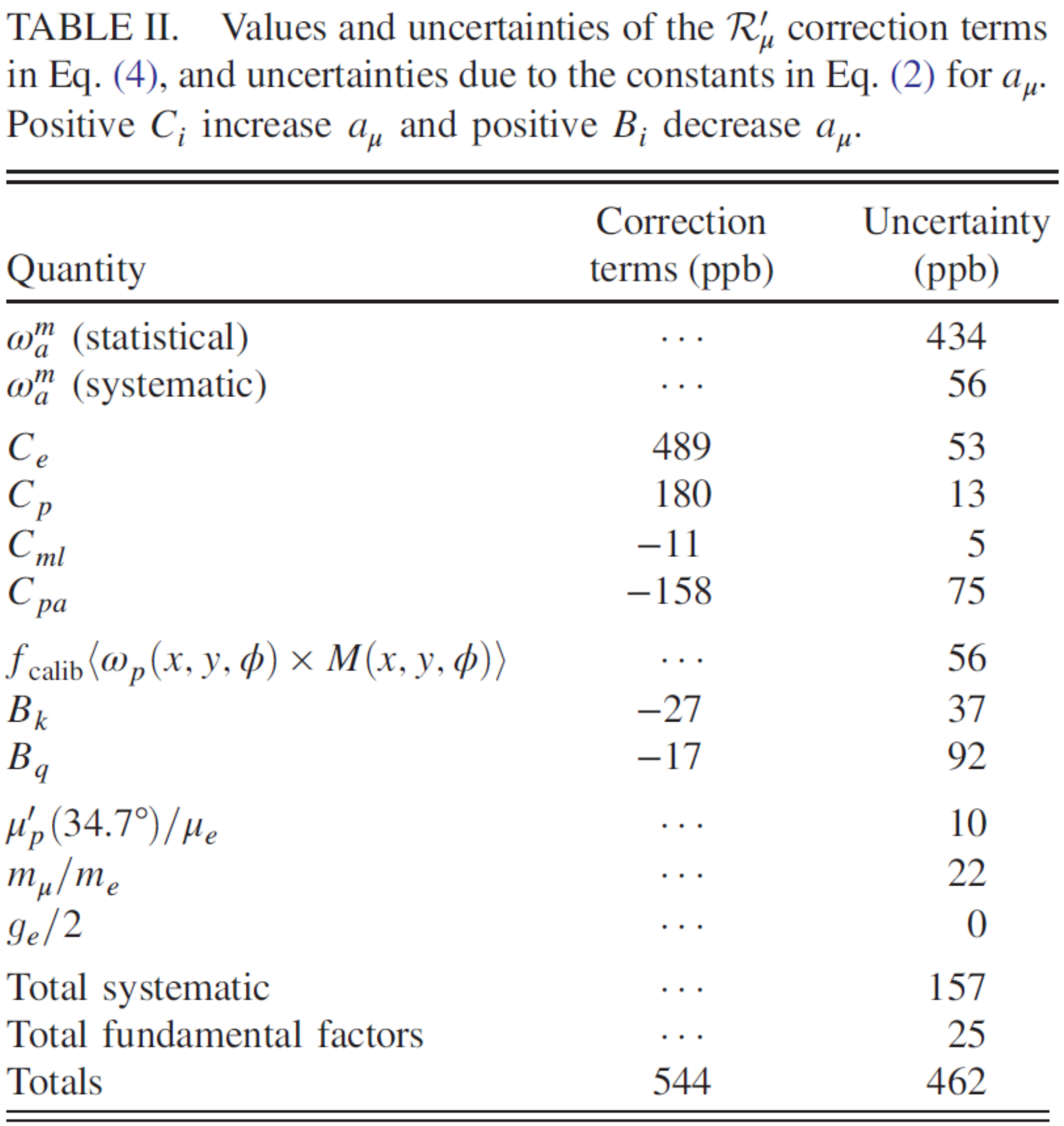
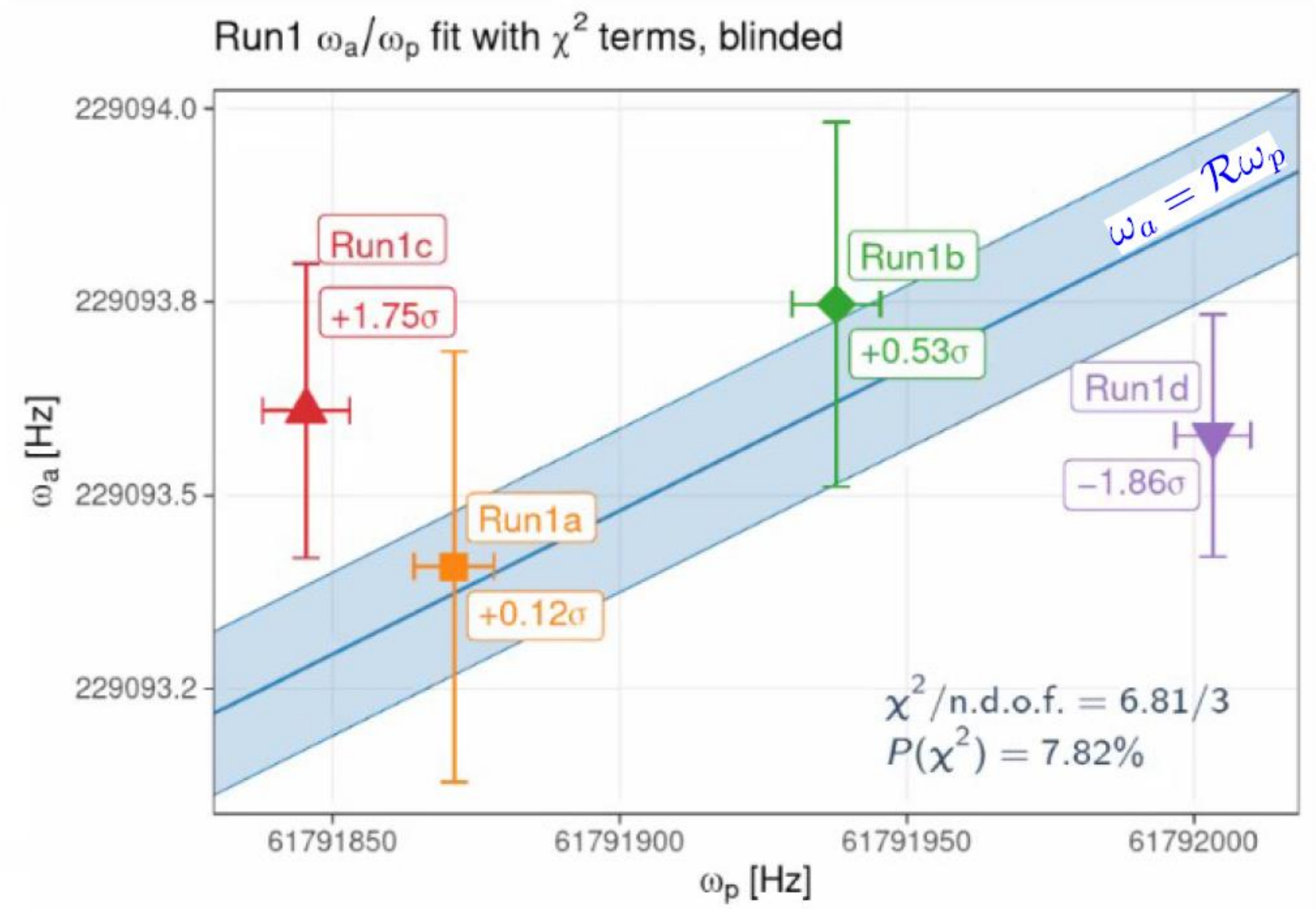


TABLE II. Values and uncertainties of the $\mathcal{R}'_\mu$ correction terms in Eq. (4), and uncertainties due to the constants in Eq. (2) for a_μ. Positive C_i increase a_μ and positive B_i decrease a_μ.

Quantity	Correction terms (ppb)	Uncertainty (ppb)
ω_a^m (statistical)	⋯	434
ω_a^m (systematic)	⋯	56
C_e	489	53
C_p	180	13
C_{ml}	−11	5
C_{pa}	−158	75
$f_{\text{calib}}\langle\omega_p(x,y,\phi)\times M(x,y,\phi)\rangle$	⋯	56
B_k	−27	37
B_q	−17	92
$\mu'_p(34.7°)/\mu_e$	⋯	10
m_μ/m_e	⋯	22
$g_e/2$	⋯	0
Total systematic	⋯	157
Total fundamental factors	⋯	25
Totals	544	462


Run1 ω_a/ω_p fit with χ^2 terms, blinded
229094.0
229093.8
229093.5
229093.2
ω_a [Hz]
61791850
61791900
61791950
61792000
ω_p [Hz]
Run1c
+1.75σ
Run1a
+0.12σ
Run1b
+0.53σ
Run1d
−1.86σ
$\omega_a = \mathcal{R}\omega_p$
χ^2/n.d.o.f. = 6.81/3
$P(\chi^2)$ = 7.82%

Two corrections involve a time dependence to the average ensemble phase constant if measured vs. time-in-fill

$$a_\mu \propto \frac{f_{\text{clock}} \;\; \omega_a^m \left(1 + C_e + C_p + C_{ml} + C_{pa}\right)}{f_{\text{calib}} \; \langle \omega_p'(x,y,\phi) \times M(x,y,\phi) \rangle \; (1 + B_k + B_q)}$$

$$N(t) = N_0(t) e^{-t/\gamma\tau_\mu} \left[1 + A\cos\left(\omega_a t + \varphi_0\right)\right]$$

What if phase is not a constant? →

$$\cos\left(\omega_a t + \varphi_0(t)\right) \rightarrow \cos\left(\omega_a t + \varphi_0 + \varphi' t + \ldots\right)$$

$$\cos\left(\left(\underbrace{\omega_a + \varphi'}_{\omega_a' \neq \omega_a}\right) t + \varphi_0 + \ldots\right)\right)$$

Phase constant φ_0 is the orientation of the muon ensemble average spin at time *t* = 0 “injection”

It has no important “physical” meaning, but it is assumed to be constant

Fit Equation

$$N_0\, e^{-\frac{t}{\gamma\tau}}\,(1 + A \cdot A_{BO}(t)\cos(\omega_a\, t + \phi \cdot \phi_{BO}(t)\,)\,) \cdot N_{\mathrm{CBO}}(t) \cdot N_{\mathrm{VW}}(t) \cdot N_y(t) \cdot N_{2\mathrm{CBO}}(t) \cdot J(t)$$

$$A_{\mathrm{BO}}(t) = 1 + A_A \cos(\omega_{\mathrm{CBO}}(t) + \phi_A)e^{-\frac{t}{\tau_{\mathrm{CBO}}}}$$

$$\phi_{\mathrm{BO}}(t) = 1 + A_\phi \cos(\omega_{\mathrm{CBO}}(t) + \phi_\phi)e^{-\frac{t}{\tau_{\mathrm{CBO}}}}$$

Red = free parameters

Blue= fixed parameters

$$N_{\mathrm{CBO}}(t) = 1 + A_{\mathrm{CBO}} \cos(\omega_{\mathrm{CBO}}(t) + \phi_{\mathrm{CBO}})e^{-\frac{t}{\tau_{\mathrm{CBO}}}}$$

$$N_{2\mathrm{CBO}}(t) = 1 + A_{2\mathrm{CBO}} \cos(2\omega_{\mathrm{CBO}}(t) + \phi_{2\mathrm{CBO}})e^{-\frac{t}{2\tau_{\mathrm{CBO}}}}$$

$$N_{\mathrm{VW}}(t) = 1 + A_{\mathrm{VW}} \cos(\omega_{\mathrm{VW}}(t)t + \phi_{\mathrm{VW}})e^{-\frac{t}{\tau_{\mathrm{VW}}}}$$

$$N_y(t) = 1 + A_y \cos(\omega_y(t)t + \phi_y)e^{-\frac{t}{\tau_y}}$$

$$J(t) = 1 - k_{LM}\int_{t_0}^{t} \Lambda(t)dt$$

Muon Loss term

ω_y, ω_{vw} vertical oscillations

ω_{CBO}, $\omega_{2\mathrm{CBO}}$, radial oscillation

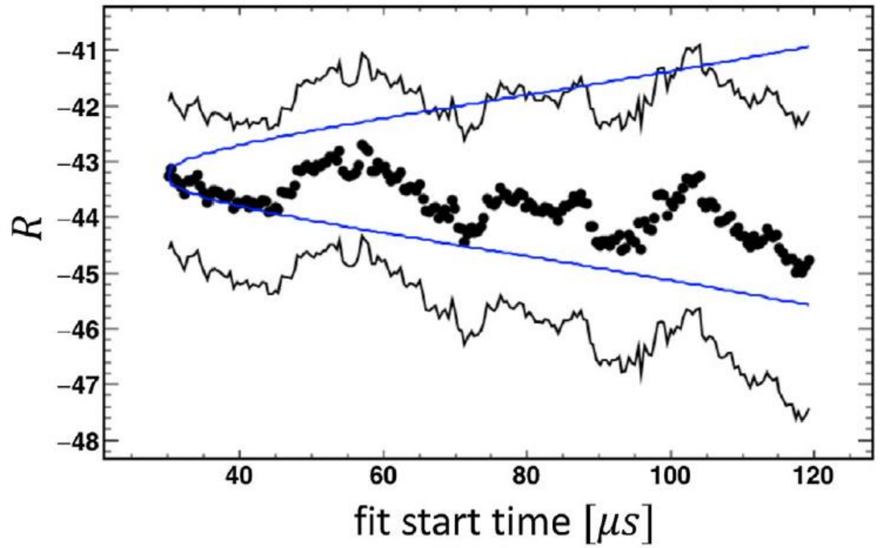
$$\omega_{\mathrm{CBO}}(t) = \omega_0 t + Ae^{-\frac{t}{\tau_A}} + Be^{-\frac{t}{\tau_B}}$$

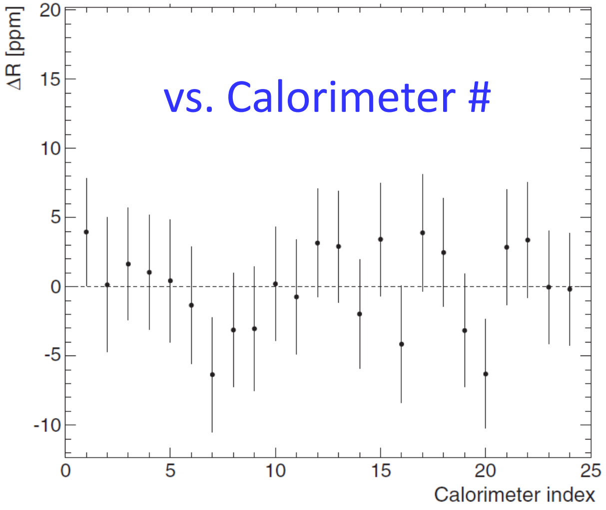
$$\omega_y(t) = F\omega_{\mathrm{CBO}(t)}\sqrt{2\omega_c/F\omega_{\mathrm{CBO}}(t) - 1}$$

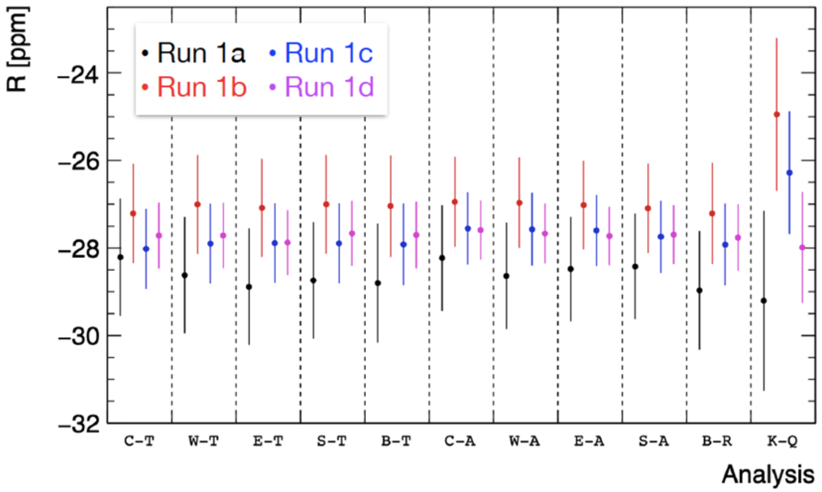
$$\omega_{\mathrm{VW}}(t) = \omega_c - 2\omega_y(t)$$

Consistency checks. *R* = (blinded) ω_a^m in PPM

$$a_\mu \propto \frac{f_{\text{clock}}\ \omega_a^m\,(1+C_e+C_p+C_{ml}+C_{pa})}{f_{\text{calib}}\ \langle \omega'_p(x,y,\phi)\times M(x,y,\phi)\rangle\,(1+B_k+B_q)}$$







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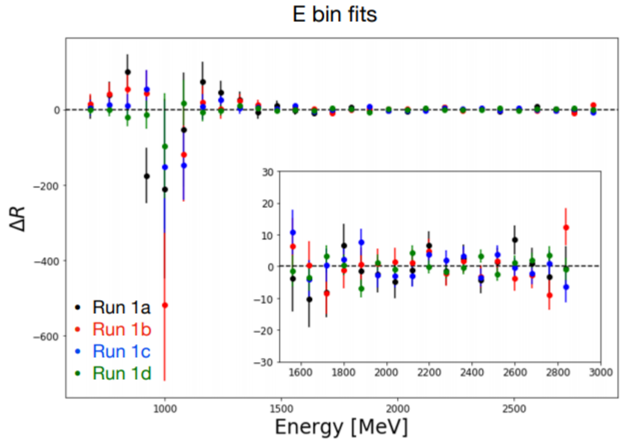

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