Simulating Particle Physics with Quantum Computers

Mathieu PELLEN

University of Freiburg

Colloquium - Paul Scherrer Institut, Switzerland

23rd of April 2026

universität freiburg

Outline:

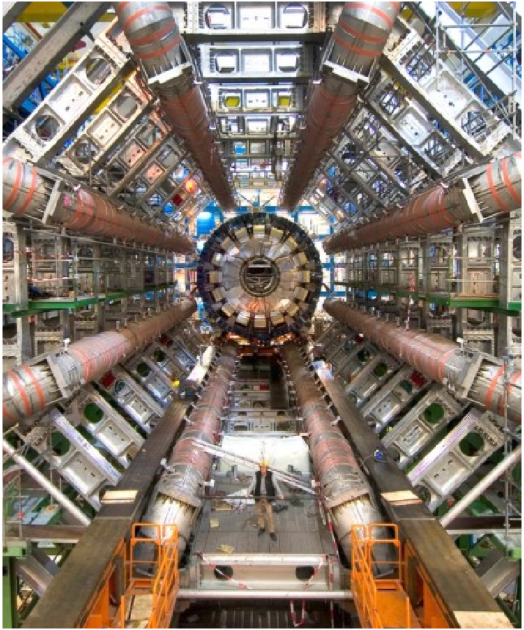
Why quantum computing and high-energy physics?

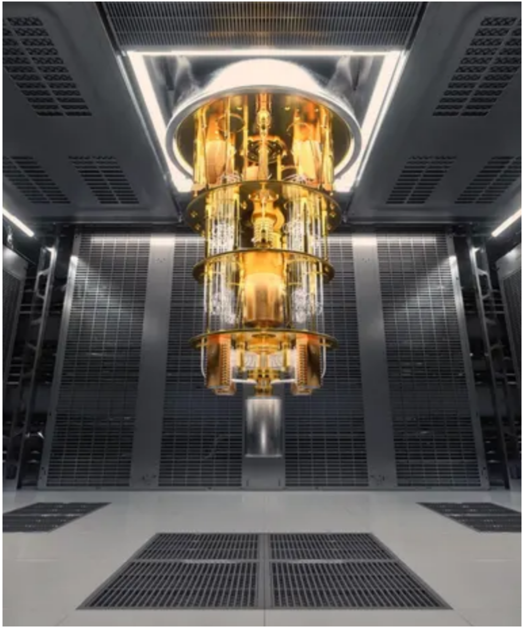
- Basics of quantum computing
  → How to build a quantum circuit

- Context and Basics of high-energy physics

How?

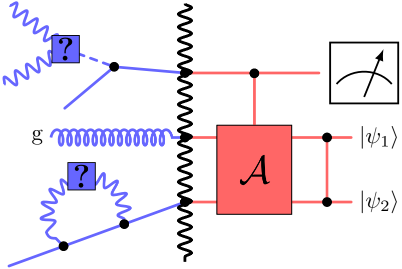
- Applications
  → Cross section → quantum integration
  → Amplitudes in QCD → dedicated quantum algorithms
  → QCD parton shower → dedicated quantum algorithms





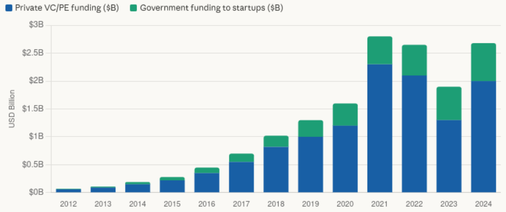
Disclaimers

- Quantum computing and high-energy physics are fast fields of research!
  → personal view on quantum computing for high-energy physics
- Beginning of quantum computing (real-world) applications
  → do not expect yet comparisons with classical state of the art now
- Not a review of all applications
  → selected applications for illustration
  → references provided to go beyond



Context in quantum computing

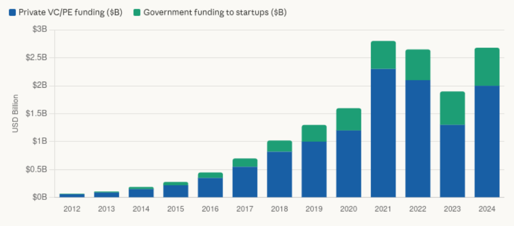
- Great commercial interest



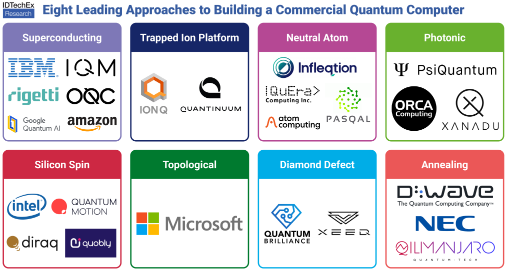
Private VC/PE = startup/venture funding only. Government = public funding to QT startups. Does not include large national lab spend (e.g. China's $10B+ in national labs). Sources: McKinsey QT Monitor 2025, ECIPE Quantum Database.

Context in quantum computing

- Great commercial interest
- Various physics approaches
  (from both big players and start-ups)

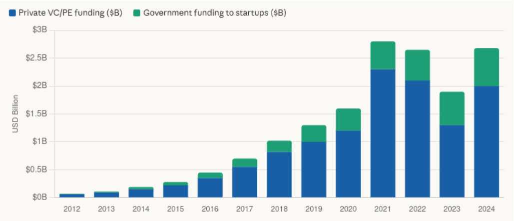


Private VC/PE = startup/venture funding only. Government = public funding to QT startups. Does not include large national lab spend (e.g. China's $10B+ in national labs). Sources: McKinsey QT Monitor 2025, ECIPE Quantum Database.

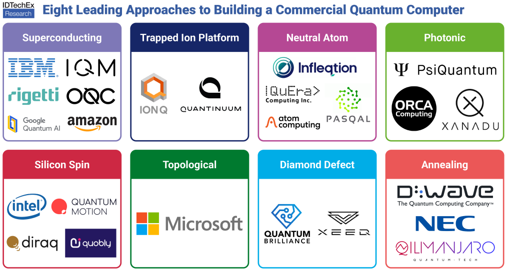


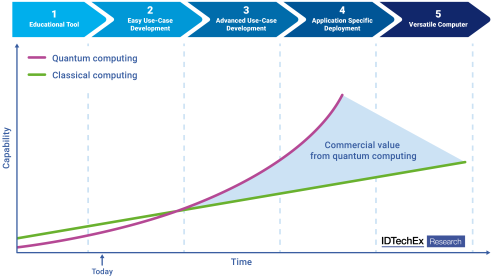
Context in quantum computing

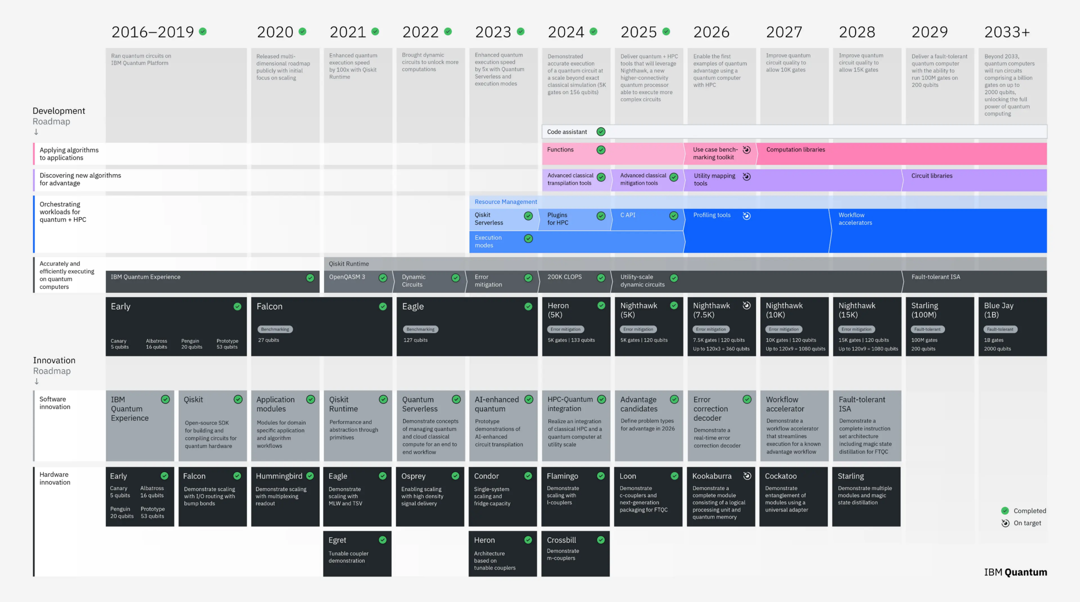
- Great commercial interest
- Various physics approaches
  (from both big players and start-ups)
- Critical moment



Private VC/PE = startup/venture funding only. Government = public funding to QT startups. Does not include large national lab spend (e.g. China's $10B+ in national labs). Sources: McKinsey QT Monitor 2025, ECIPE Quantum Database.







2016–2019 ✓ | 2020 ✓ | 2021 ✓ | 2022 ✓ | 2023 ✓ | 2024 ✓ | 2025 ✓ | 2026 | 2027 | 2028 | 2029 | 2033+

Development Roadmap ↓

- 2016–2019: Ran quantum circuits on IBM Quantum Platform
- 2020: Released multi-dimensional roadmap publicly with initial focus on scaling
- 2021: Enhanced quantum execution speed by 100x with Qiskit Runtime
- 2022: Brought dynamic circuits to unlock more computations
- 2023: Enhanced quantum execution speed by 5x with Quantum Serverless and execution modes
- 2024: Demonstrated accurate execution of a quantum circuit at a scale beyond exact classical simulation (5K gates on 156 qubits)
- 2025: Deliver quantum + HPC tools that will leverage Nighthawk, a new higher-connectivity quantum processor able to execute more complex circuits
- 2026: Enable the first examples of quantum advantage using a quantum computer with HPC
- 2027: Improve quantum circuit quality to allow 10K gates
- 2028: Improve quantum circuit quality to allow 15K gates
- 2029: Deliver a fault-tolerant quantum computer with the ability to run 100M gates on 200 qubits
- 2033+: Beyond 2033, quantum computers will run circuits comprising a billion gates on up to 2000 qubits, unlocking the full power of quantum computing

Code assistant ✓

Applying algorithms to applications: Functions ✓ | Use case benchmarking toolkit ↻ | Computation libraries

Discovering new algorithms for advantage: Advanced classical transpilation tools ✓ | Advanced classical mitigation tools ✓ | Utility mapping tools ↻ | Circuit libraries

Orchestrating workloads for quantum + HPC: Resource Management

Qiskit Serverless ✓ | Plugins for HPC ✓ | C API ✓ | Profiling tools ↻ | Workflow accelerators

Execution modes ✓

Accurately and efficiently executing on quantum computers: IBM Quantum Experience ✓

Qiskit Runtime: OpenQASM 3 ✓ | Dynamic Circuits ✓ | Error mitigation ✓ | 200K CLOPS ✓ | Utility-scale dynamic circuits ✓ | Fault-tolerant ISA

Early ✓ — Canary 5 qubits, Albatross 16 qubits, Penguin 20 qubits, Prototype 53 qubits

Falcon ✓ — Benchmarking — 27 qubits

Eagle ✓ — Benchmarking — 127 qubits

Heron (5K) ✓ — Error mitigation — 5K gates | 133 qubits

Nighthawk (5K) ✓ — Error mitigation — 5K gates | 120 qubits

Nighthawk (7.5K) ↻ — Error mitigation — 7.5K gates | 120 qubits — Up to 120x3 = 360 qubits

Nighthawk (10K) — Error mitigation — 10K gates | 120 qubits — Up to 120x9 = 1080 qubits

Nighthawk (15K) — Error mitigation — 15K gates | 120 qubits — Up to 120x9 = 1080 qubits

Starling (100M) — Fault-tolerant — 100M gates — 200 qubits

Blue Jay (1B) — Fault-tolerant — 1B gates — 2000 qubits

Innovation Roadmap ↓

Software innovation

- IBM Quantum Experience ✓
- Qiskit ✓ — Open-source SDK for building and compiling circuits for quantum hardware
- Application modules ✓ — Modules for domain specific application and algorithm workflows
- Qiskit Runtime ✓ — Performance and abstraction through primitives
- Quantum Serverless ✓ — Demonstrate concepts of managing quantum and cloud classical compute for an end to end workflow
- AI-enhanced quantum ✓ — Prototype demonstrations of AI-enhanced circuit transpilation
- HPC-Quantum integration ✓ — Realize an integration of classical HPC and a quantum computer at utility scale
- Advantage candidates ✓ — Define problem types for advantage in 2026
- Error correction decoder ✓ — Demonstrate a real-time error correction decoder
- Workflow accelerator — Demonstrate a workflow accelerator that streamlines execution for a known advantage workflow
- Fault-tolerant ISA — Demonstrate a complete instruction set architecture including magic state distillation for FTQC

Hardware innovation

- Early ✓ — Canary 5 qubits, Albatross 16 qubits, Penguin 20 qubits, Prototype 53 qubits
- Falcon ✓ — Demonstrate scaling with I/O routing with bump bonds
- Hummingbird ✓ — Demonstrate scaling with multiplexing readout
- Eagle ✓ — Demonstrate scaling with MLW and TSV
- Egret ✓ — Tunable coupler demonstration
- Osprey ✓ — Enabling scaling with high density signal delivery
- Condor ✓ — Single-system scaling and fridge capacity
- Heron ✓ — Architecture based on tunable couplers
- Flamingo ✓ — Demonstrate scaling with l-couplers
- Crossbill ✓ — Demonstrate m-couplers
- Loon ✓ — Demonstrate c-couplers and next-generation packaging for FTQC
- Kookaburra ↻ — Demonstrate a complete module consisting of a logical processing unit and quantum memory
- Cockatoo — Demonstrate entanglement of modules using a universal adapter
- Starling — Demonstrate multiple modules and magic state distillation

✓ Completed

↻ On target

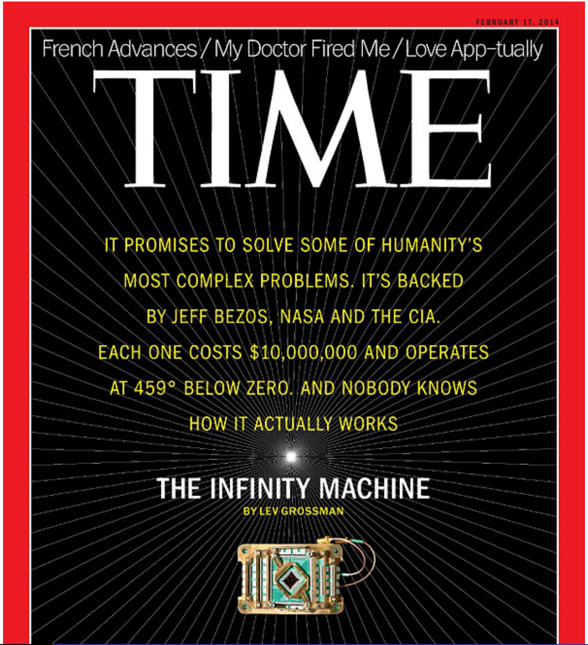
IBM **Quantum**

Why quantum computing and high-energy physics?

- Basics of quantum computing
  → How to build a quantum circuit

Literature:

- *Quantum Computation and Quantum Information*, Nielsen and Chuang
- *Programming Quantum Computers*, Johnston, Harrigan, and Gimeno-Segovia



FEBRUARY 17, 2014

French Advances / My Doctor Fired Me / Love App-tually

TIME

IT PROMISES TO SOLVE SOME OF HUMANITY'S MOST COMPLEX PROBLEMS. IT'S BACKED BY JEFF BEZOS, NASA AND THE CIA. EACH ONE COSTS $10,000,000 AND OPERATES AT 459° BELOW ZERO. AND NOBODY KNOWS HOW IT ACTUALLY WORKS

THE INFINITY MACHINE

BY LEV GROSSMAN

Back to quantum mechanics...
→ Any state can be written as

$$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle = \begin{pmatrix}\alpha \\ \beta\end{pmatrix},$$

with $|\alpha|^2 + |\beta|^2 = 1$

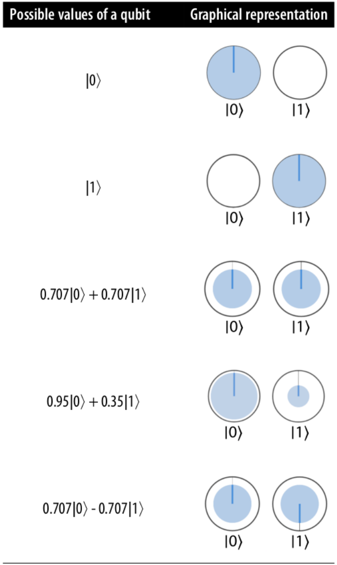
Back to quantum mechanics...

→ Any state can be written as

$$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle = \begin{pmatrix}\alpha\\ \beta\end{pmatrix},$$

with $|\alpha|^2 + |\beta|^2 = 1$

→ Representation



Possible values of a qubit	Graphical representation
$\|0\rangle$	$\|0\rangle$ $\|1\rangle$
$\|1\rangle$	$\|0\rangle$ $\|1\rangle$
$0.707\|0\rangle + 0.707\|1\rangle$	$\|0\rangle$ $\|1\rangle$
$0.95\|0\rangle + 0.35\|1\rangle$	$\|0\rangle$ $\|1\rangle$
$0.707\|0\rangle - 0.707\|1\rangle$	$\|0\rangle$ $\|1\rangle$

→ In quantum mechanics (and in quantum computing), any operation A is unitary

$$\psi \xrightarrow{A} \psi'$$

$$A\,|\psi\rangle = |\psi'\rangle \;\text{ with } A^\dagger A = A A^\dagger = \mathrm{Id}$$

$$|\psi\rangle \;—\!\!—\boxed{A}—\!\!—\; |\psi'\rangle$$

Example of gates (I)

Pauli-X (X)

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\psi = \alpha \left|0\right\rangle + \beta \left|1\right\rangle \rightarrow \ \psi' = \alpha \left|1\right\rangle + \beta \left|0\right\rangle$$

Example of gates (I)

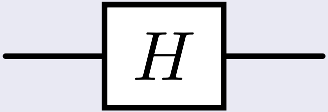
Pauli-X (X)

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\psi = \alpha |0\rangle + \beta |1\rangle \rightarrow \psi' = \alpha |1\rangle + \beta |0\rangle$$

Hadamard (H)

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$



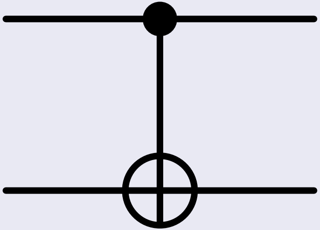
$$\psi = \alpha |0\rangle + \beta |1\rangle \rightarrow \psi' = \frac{\alpha+\beta}{\sqrt{2}} |0\rangle + \frac{\alpha-\beta}{\sqrt{2}} |1\rangle$$

Example of gates (II)

Controlled not (CNOT, CX)

$$CX = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$|00\rangle \rightarrow |00\rangle \,;\; |01\rangle \rightarrow |01\rangle \,;$$
$$|10\rangle \rightarrow |11\rangle \,;\; |11\rangle \rightarrow |10\rangle \,.$$



- If 0 → nothing happens; If 1 → Pauli-X (X)!
- *Control* qubit (top) and *target* qubit (bottom)

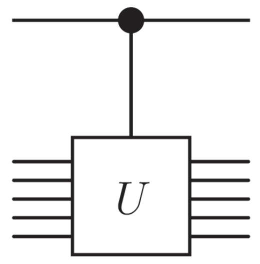
Example of gates (III)

Generalised controlled gate (CU)

$$CU = \begin{bmatrix} \mathrm{Id}_2 & 0 \\ 0 & U \end{bmatrix}$$

→ One *control* qubit and ...
... many *target* qubits

NB Also possible:
Many *control* qubits and ...
... one/many *target* qubit



Measurement process

→ State: $|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$

→ Probabilities: $\begin{cases} \text{for} & |0\rangle : \alpha^2 \\ \text{for} & |1\rangle : \beta^2 \end{cases} \quad \text{with } |\alpha|^2 + |\beta|^2 = 1!$

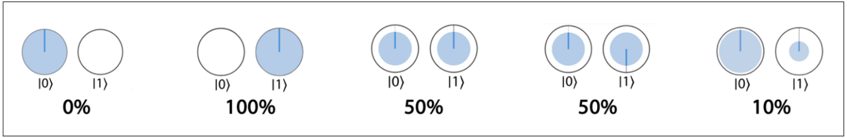
→ Measurement = squaring/collapse the amplitude!

Measurement process

→ State: $|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$

→ Probabilities: $\begin{cases} \text{for} \quad |0\rangle : \alpha^2 \\ \text{for} \quad |1\rangle : \beta^2 \end{cases}$ with $|\alpha|^2 + |\beta|^2 = 1$!

→ Measurement = squaring/collapse the amplitude!



NB: Probability obtained after many measurements/shots! $\langle\psi|\psi\rangle$!

→ α and β encode information that can be measured/computed!

- Live example with QISKIT [open-source software development kit from IBM]

- Live example with QISKIT [open-source software development kit from IBM]

Quantum-computing applications

- Key parameters of quantum computers:
  - Number of qubits
    → amount of operations in parallel
  - Depth/number of gate operations
    → amount of operations in sequence (before decoherence/noise)
  - Error rates
    → Fault-tolerant era $\sim$ 2030 (depending on the technology)
- Expressibility is a combination of number of qubits and depth
- Implementation of a problem in quantum computing
  = implementation with low-level language with restrictions (unitary operations)

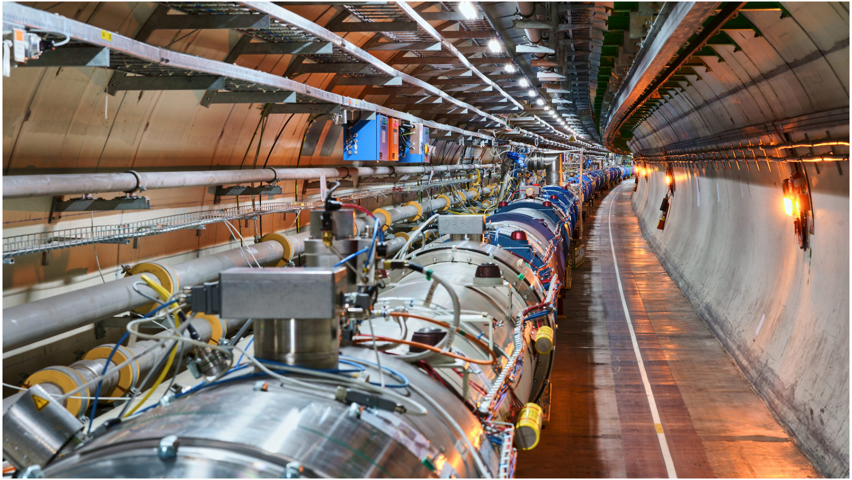
Why quantum computing and high-energy physics?

- Basics of high-energy physics and context

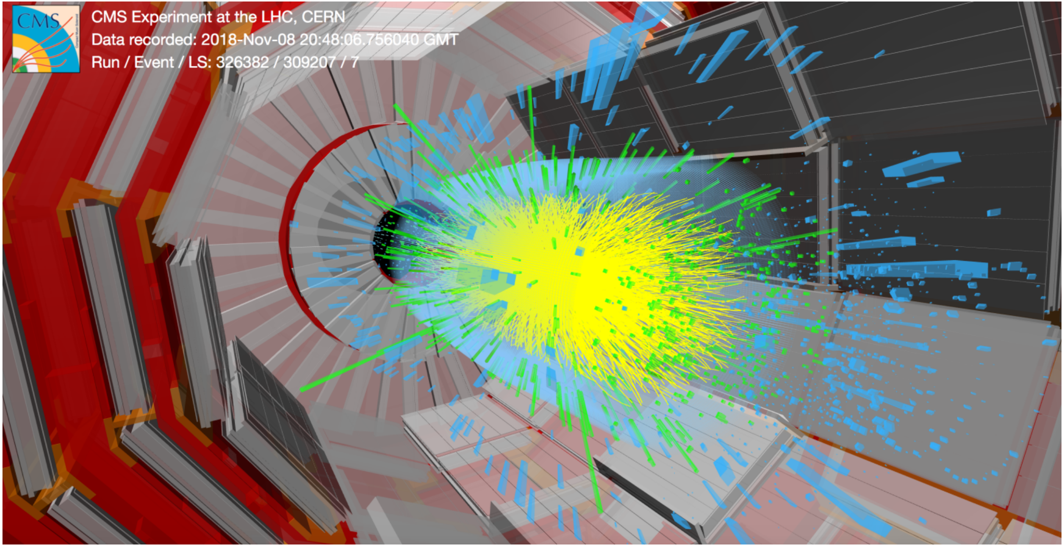
High-energy physics = LHC @ CERN, Geneva (Switzerland)

→ 27km-long tunnel where protons are collided at high-energy (13.6 TeV currently)

→ Allow to probe fundamental interactions

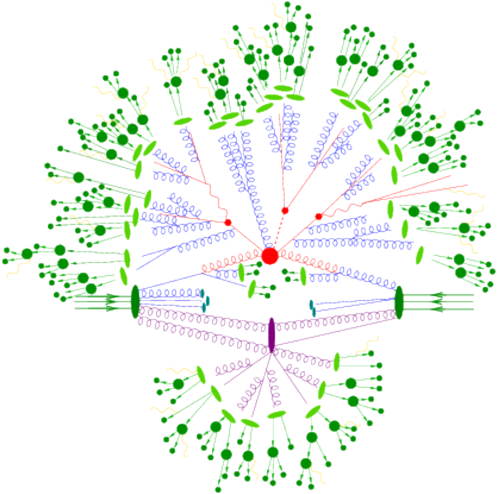


Life at the LHC (in reality)



Life at the LHC (on the theory side)

- Factorisation principle:
  - **high-energy** (perturbative)
    → Hard-scattering, parton-shower
  - **low-energy** (non-perturbative)
    → parton distribution function,
    hadronisation, underlying events, ...
- **Focus of the presentation: high energy!**
  → Calculated from first principles

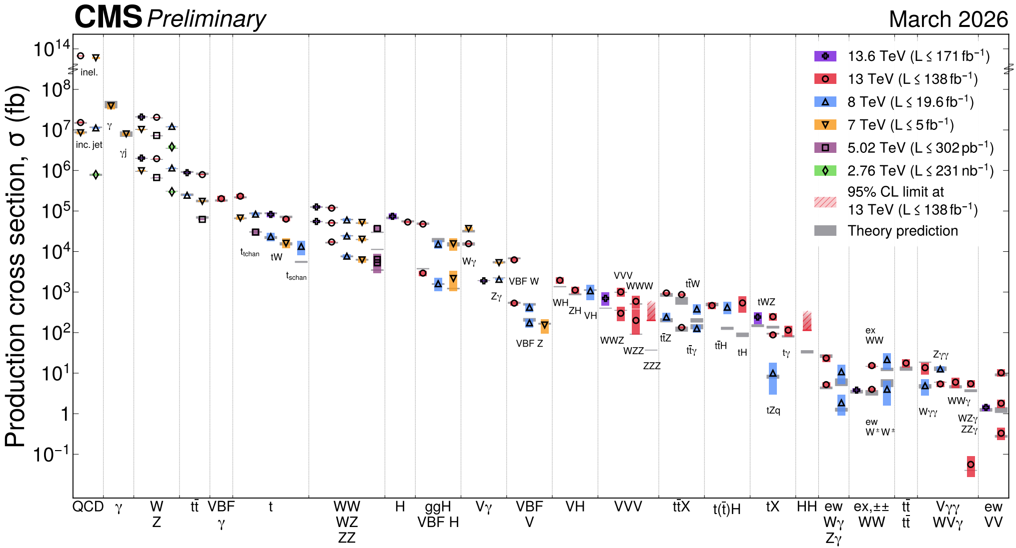


[source: Sherpa]

- Comparison between theory and experiment to learn about particle physics!

[source: bing image creator, Engel, Pellen]

- Comparison between theory and experiment to learn about particle physics!

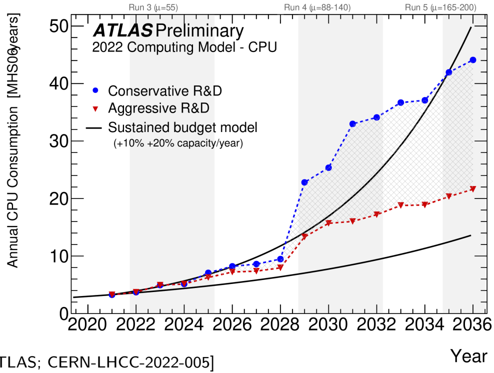


Computing problem in high-energy physics

→ Event generation:
$\sim 15\%$ of ~ 3 billion cpuh.y^{-1} for ATLAS

[Buckley; 1908.00167], [Valassi et al.; 2004.13687]

Computing problem in high-energy physics



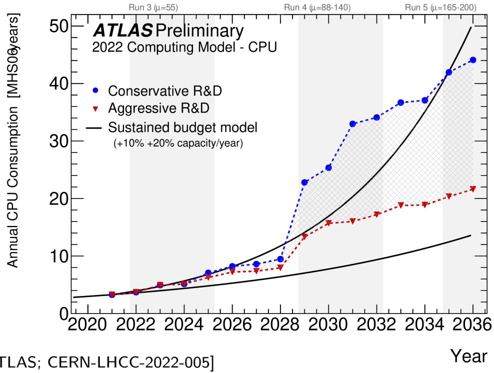
[ATLAS; CERN-LHCC-2022-005]

→ Event generation:

$\sim$ 15% of $\sim$ 3 billion cpuh.y^{-1} for ATLAS

[Buckley; 1908.00167], [Valassi et al.; 2004.13687]

Computing problem in high-energy physics



[ATLAS; CERN-LHCC-2022-005]

→ Event generation:

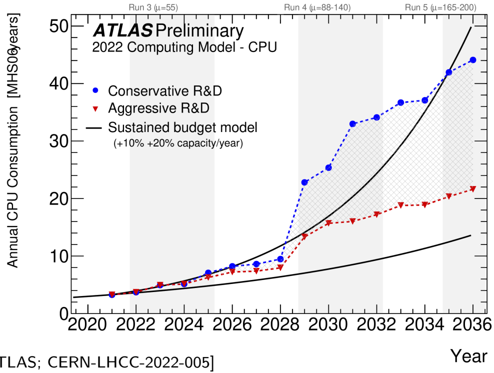
$\sim 15\%$ of ~ 3 billion cpuh.y^{-1} for ATLAS

[Buckley; 1908.00167], [Valassi et al.; 2004.13687]

- Solutions: GPU & AI

[Borowka et al.; 1811.11720], [Carrazza et al.; 2002.12921, 2009.06635, 2106.10279], [Bothmann et al.; 2106.06507], [Cruz-Martinez, De Laurentis, MP; 2502.07060], [Seymour, Sule; 2403.08692, 2511.19633], [Valassi; 2510.05392], [Bothmann et al.; 2604.03511]

Computing problem in high-energy physics



[ATLAS; CERN-LHCC-2022-005]

→ Event generation:

$\sim$ 15% of $\sim$ 3 billion cpuh.y^{-1} for ATLAS

[Buckley; 1908.00167], [Valassi et al.; 2004.13687]

- Solutions: GPU & AI

[Borowka et al.; 1811.11720], [Carrazza et al.; 2002.12921, 2009.06635, 2106.10279], [Bothmann et al.; 2106.06507], [Cruz-Martinez, De Laurentis, MP; 2502.07060], [Seymour, Sule; 2403.08692, 2511.19633], [Valassi; 2510.05392], [Bothmann et al.; 2604.03511]

- Can quantum computing be of any use in HEP?
  → to compute things faster/more efficiently?
  → to compute new things?
- Quantum computing provides a natural framework for QFT calculations?

Reviews

- [Gray, Terashi; Gray:2022fou] (selected topics), [Delgado et al.; 2203.08805] (Snowmass), [Klco et al.; 2107.04769] (lattice), [Di Meglio et al.; 2307.03236] (CERN Quantum Initiative)

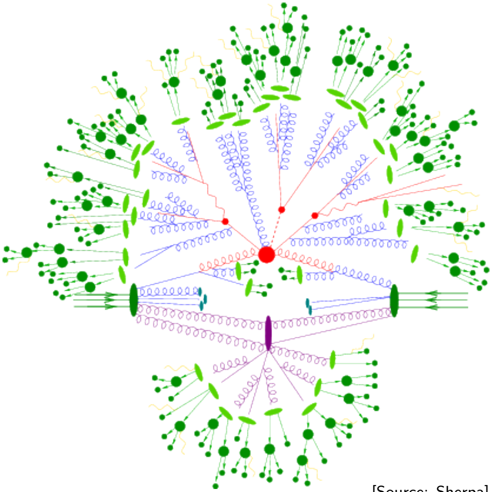
Selected references

- Amplitude/loop integrals: [Ramirez-Uribe et al.; 2105.08703], [Bepari, Malik, Spannowsky, Williams; 2010.00046], **[Chawdhry, MP; 2303.04818]**, **[Chawdhry, MP, Williams; 2507.07194]**, [Bashore, Moretti, Vitos; 2507.14252], [Ochoa-Oregon et al.; 2508.04019]
- Monte Carlo (integration): [Bravo-Prieto et al; 2110.06933], **[Agliardi, Grossi, MP, Prati; 2201.01547]**, [Martínez de Lejarza et al.; 2401.03023], [Martínez de Lejarza, Cieri, Rodrigo; 2204.06496], [Cruz-Martinez, Robbiati, Carrazza; 2308.05657], **[Williams, MP; 2502.14647]**, [Martínez de Lejarza et al.; 2409.12236], [Pyretzidis, Martínez de Lejarza, Rodrigo; 2506.19965]
- Parton shower: **[Bauer, de Jong, Nachman, Provasoli; 1904.03196, 1901.08148]**, [Bepari, Malik, Spannowsky, Williams; 2010.00046], [Williams, Malik, Spannowsky, Bepari; 2109.13975], [Deliyannis et al.; 2203.10018], [Chigusa, Yamazaki; 2204.12500], [Gustafson, Prestel, Spannowsky, Williams; 2207.10694], **[Bauer, Chigusa, Yamazaki; 2310.19881]**
- Others: [Perez-Salinas, Cruz-Martinez, Alhajri, Carrazza: 2011.13934], [Bauer, Freytsis, Nachman; 2102.05044], [Martínez de Lejarza et al.; 2503.16073]

Quantum applications in high-energy physics

→ Quantum applications still in their infancy!

- Is it possible?

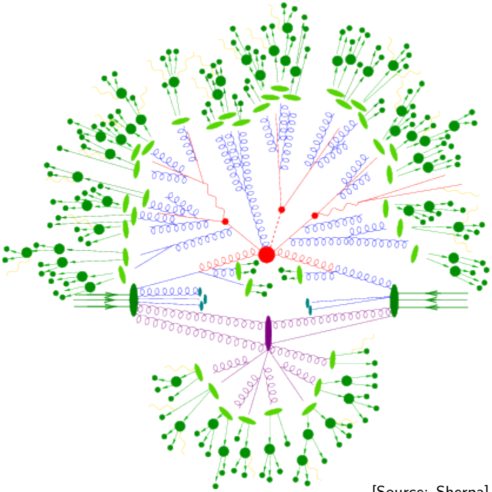


[Source: Sherpa]

Quantum applications in high-energy physics

→ Quantum applications still in their infancy!

- Is it possible?
- Is there a (theoretical) quantum advantage?

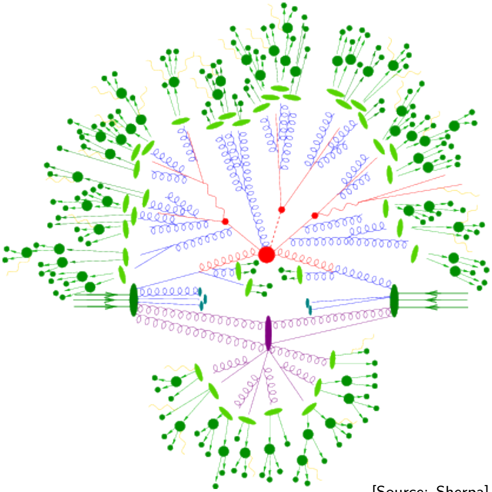


[Source: Sherpa]

Quantum applications in high-energy physics

→ Quantum applications still in their infancy!

- Is it possible?
- Is there a (theoretical) quantum advantage?
- Is it more resource efficient/faster than CPU/GPU/ML approaches?



[Source: Sherpa]

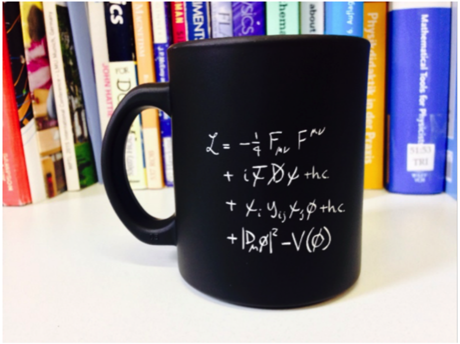
How?

- Applications

  → Cross section → quantum integration

  → Amplitudes in QCD → dedicated quantum algorithms

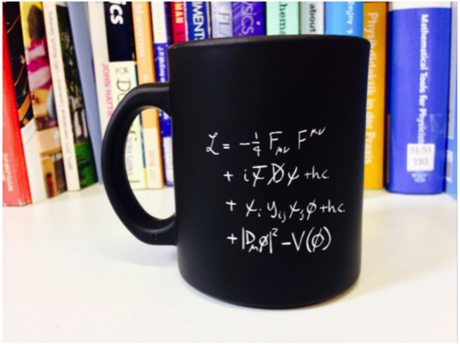
  → QCD parton shower → dedicated quantum algorithms



[source: CERN]

The Standard Model of particle physics

- Particles are represented by fields
- Lagrangian displays interactions of fields
- Based on gauge symmetry
  → $SU_{\mathrm{QCD}}(3) \times SU_{\mathrm{L}}(2) \times U_{\mathrm{Y}}(1)$



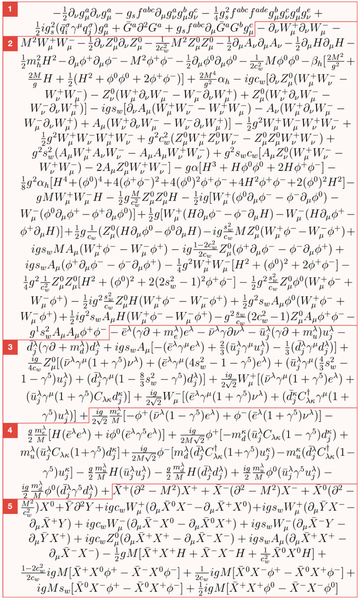
[source: CERN]

The Standard Model of particle physics

- Particles are represented by fields
- Lagrangian displays interactions of fields
- Based on gauge symmetry
  → $SU_{\mathrm{QCD}}(3) \times SU_{\mathrm{L}}(2) \times U_{\mathrm{Y}}(1)$

- Strong force
  → Quantum chromodynamics (QCD) mediated by gluon
- Electromagnetic force
  → Quantum electrodynamics (QED) mediated by photon
- Weak force
  → Weak interaction mediated by W and Z

→ W and Z get their mass from Higgs mechanism



$$
\begin{gathered}
-\frac{1}{2}\partial_\nu g^a_\mu \partial_\nu g^a_\mu - g_s f^{abc}\partial_\mu g^a_\nu g^b_\mu g^c_\nu - \frac{1}{4} g_s^2 f^{abc} f^{ade} g^b_\mu g^c_\nu g^d_\mu g^e_\nu + \\
\frac{1}{2} i g_s^2 (\bar{q}^\sigma_i \gamma^\mu q^\sigma_j) g^a_\mu + \bar{G}^a \partial^2 G^a + g_s f^{abc}\partial_\mu \bar{G}^a G^b g^c_\mu - \partial_\nu W^+_\mu \partial_\nu W^-_\mu - \\
M^2 W^+_\mu W^-_\mu - \frac{1}{2}\partial_\nu Z^0_\mu \partial_\nu Z^0_\mu - \frac{1}{2c_w^2} M^2 Z^0_\mu Z^0_\mu - \frac{1}{2}\partial_\mu A_\nu \partial_\mu A_\nu - \frac{1}{2}\partial_\mu H \partial_\mu H - \\
\frac{1}{2} m_h^2 H^2 - \partial_\mu\phi^+\partial_\mu\phi^- - M^2\phi^+\phi^- - \frac{1}{2}\partial_\mu\phi^0\partial_\mu\phi^0 - \frac{1}{2c_w^2} M\phi^0\phi^0 - \beta_h[\frac{2M^2}{g^2} + \\
\frac{2M}{g}H + \frac{1}{2}(H^2+\phi^0\phi^0+2\phi^+\phi^-)] + \frac{2M^4}{g^2}\alpha_h - igc_w[\partial_\nu Z^0_\mu(W^+_\mu W^-_\nu - \\
W^+_\nu W^-_\mu) - Z^0_\nu(W^+_\mu\partial_\nu W^-_\mu - W^-_\mu\partial_\nu W^+_\mu) + Z^0_\mu(W^+_\nu\partial_\nu W^-_\mu - \\
W^-_\nu\partial_\nu W^+_\mu)] - igs_w[\partial_\nu A_\mu(W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - A_\nu(W^+_\mu\partial_\nu W^-_\mu - \\
W^-_\mu\partial_\nu W^+_\mu) + A_\mu(W^+_\nu\partial_\nu W^-_\mu - W^-_\nu\partial_\nu W^+_\mu)] - \frac{1}{2}g^2 W^+_\mu W^-_\mu W^+_\nu W^-_\nu + \\
\frac{1}{2}g^2 W^+_\mu W^-_\nu W^+_\mu W^-_\nu + g^2c_w^2(Z^0_\mu W^+_\mu Z^0_\nu W^-_\nu - Z^0_\mu Z^0_\mu W^+_\nu W^-_\nu) + \\
g^2 s_w^2(A_\mu W^+_\mu A_\nu W^-_\nu - A_\mu A_\mu W^+_\nu W^-_\nu) + g^2 s_w c_w[A_\mu Z^0_\nu(W^+_\mu W^-_\nu - \\
W^+_\nu W^-_\mu) - 2A_\mu Z^0_\mu W^+_\nu W^-_\nu] - g\alpha[H^3 + H\phi^0\phi^0 + 2H\phi^+\phi^-] - \\
\frac{1}{8}g^2\alpha_h[H^4+(\phi^0)^4+4(\phi^+\phi^-)^2+4(\phi^0)^2\phi^+\phi^-+4H^2\phi^+\phi^-+2(\phi^0)^2H^2] - \\
gMW^+_\mu W^-_\mu H - \frac{1}{2}g\frac{M}{c_w^2}Z^0_\mu Z^0_\mu H - \frac{1}{2}ig[W^+_\mu(\phi^0\partial_\mu\phi^- - \phi^-\partial_\mu\phi^0) - \\
W^-_\mu(\phi^0\partial_\mu\phi^+ - \phi^+\partial_\mu\phi^0)] + \frac{1}{2}g[W^+_\mu(H\partial_\mu\phi^- - \phi^-\partial_\mu H) - W^-_\mu(H\partial_\mu\phi^+ - \\
\phi^+\partial_\mu H)] + \frac{1}{2}g\frac{1}{c_w}(Z^0_\mu(H\partial_\mu\phi^0 - \phi^0\partial_\mu H) - ig\frac{s_w^2}{c_w}MZ^0_\mu(W^+_\mu\phi^- - W^-_\mu\phi^+) + \\
igs_wMA_\mu(W^+_\mu\phi^- - W^-_\mu\phi^+) - ig\frac{1-2c_w^2}{2c_w}Z^0_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) + \\
igs_wA_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) - \frac{1}{4}g^2W^+_\mu W^-_\mu[H^2+(\phi^0)^2+2\phi^+\phi^-] - \\
\frac{1}{4}g^2\frac{1}{c_w^2}Z^0_\mu Z^0_\mu[H^2+(\phi^0)^2+2(2s_w^2-1)^2\phi^+\phi^-] - \frac{1}{2}g^2\frac{s_w^2}{c_w}Z^0_\mu\phi^0(W^+_\mu\phi^- + \\
W^-_\mu\phi^+) - \frac{1}{2}ig^2\frac{s_w^2}{c_w}Z^0_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) + \frac{1}{2}g^2s_wA_\mu\phi^0(W^+_\mu\phi^- + \\
W^-_\mu\phi^+) + \frac{1}{2}ig^2s_wA_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) - g^2\frac{s_w}{c_w}(2c_w^2-1)Z^0_\mu A_\mu\phi^+\phi^- - \\
g^1 s_w^2 A_\mu A_\mu\phi^+\phi^- - \bar{e}^\lambda(\gamma\partial+m_e^\lambda)e^\lambda - \bar{\nu}^\lambda\gamma\partial\nu^\lambda - \bar{u}^\lambda_j(\gamma\partial+m_u^\lambda)u^\lambda_j - \\
\bar{d}^\lambda_j(\gamma\partial+m_d^\lambda)d^\lambda_j + igs_wA_\mu[-(\bar{e}^\lambda\gamma^\mu e^\lambda) + \frac{2}{3}(\bar{u}^\lambda_j\gamma^\mu u^\lambda_j) - \frac{1}{3}(\bar{d}^\lambda_j\gamma^\mu d^\lambda_j)] + \\
\frac{ig}{4c_w}Z^0_\mu[(\bar{\nu}^\lambda\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar{e}^\lambda\gamma^\mu(4s_w^2-1-\gamma^5)e^\lambda) + (\bar{u}^\lambda_j\gamma^\mu(\frac{4}{3}s_w^2 - \\
1-\gamma^5)u^\lambda_j) + (\bar{d}^\lambda_j\gamma^\mu(1-\frac{8}{3}s_w^2-\gamma^5)d^\lambda_j)] + \frac{ig}{2\sqrt{2}}W^+_\mu[(\bar{\nu}^\lambda\gamma^\mu(1+\gamma^5)e^\lambda) + \\
(\bar{u}^\lambda_j\gamma^\mu(1+\gamma^5)C_{\lambda\kappa}d^\kappa_j)] + \frac{ig}{2\sqrt{2}}W^-_\mu[(\bar{e}^\lambda\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar{d}^\kappa_j C^\dagger_{\lambda\kappa}\gamma^\mu(1 + \\
\gamma^5)u^\lambda_j)] + \frac{ig}{2\sqrt{2}}\frac{m_e^\lambda}{M}[-\phi^+(\bar{\nu}^\lambda(1-\gamma^5)e^\lambda) + \phi^-(\bar{e}^\lambda(1+\gamma^5)\nu^\lambda)] - \\
\frac{g}{2}\frac{m_e^\lambda}{M}[H(\bar{e}^\lambda e^\lambda) + i\phi^0(\bar{e}^\lambda\gamma^5e^\lambda)] + \frac{ig}{2M\sqrt{2}}\phi^+[-m_d^\kappa(\bar{u}^\lambda_jC_{\lambda\kappa}(1-\gamma^5)d^\kappa_j) + \\
m_u^\lambda(\bar{u}^\lambda_jC_{\lambda\kappa}(1+\gamma^5)d^\kappa_j] + \frac{ig}{2M\sqrt{2}}\phi^-[m_d^\lambda(\bar{d}^\lambda_jC^\dagger_{\lambda\kappa}(1+\gamma^5)u^\kappa_j) - m_u^\kappa(\bar{d}^\lambda_jC^\dagger_{\lambda\kappa}(1 - \\
\gamma^5)u^\kappa_j] - \frac{g}{2}\frac{m_u^\lambda}{M}H(\bar{u}^\lambda_ju^\lambda_j) - \frac{g}{2}\frac{m_d^\lambda}{M}H(\bar{d}^\lambda_jd^\lambda_j) + \frac{ig}{2}\frac{m_u^\lambda}{M}\phi^0(\bar{u}^\lambda_j\gamma^5u^\lambda_j) - \\
\frac{ig}{2}\frac{m_d^\lambda}{M}\phi^0(\bar{d}^\lambda_j\gamma^5d^\lambda_j) + \bar{X}^+(\partial^2-M^2)X^+ + \bar{X}^-(\partial^2-M^2)X^- + \bar{X}^0(\partial^2 - \\
\frac{M^2}{c_w^2})X^0 + \bar{Y}\partial^2Y + igc_wW^+_\mu(\partial_\mu\bar{X}^0X^- - \partial_\mu\bar{X}^+X^0) + igs_wW^+_\mu(\partial_\mu\bar{Y}X^- - \\
\partial_\mu\bar{X}^+Y) + igc_wW^-_\mu(\partial_\mu\bar{X}^-X^0 - \partial_\mu\bar{X}^0X^+) + igs_wW^-_\mu(\partial_\mu\bar{X}^-Y - \\
\partial_\mu\bar{Y}X^+) + igc_wZ^0_\mu(\partial_\mu\bar{X}^+X^+ - \partial_\mu\bar{X}^-X^-) + igs_wA_\mu(\partial_\mu\bar{X}^+X^+ - \\
\partial_\mu\bar{X}^-X^-) - \frac{1}{2}gM[\bar{X}^+X^+H + \bar{X}^-X^-H + \frac{1}{c_w^2}\bar{X}^0X^0H] + \\
\frac{1-2c_w^2}{2c_w}igM[\bar{X}^+X^0\phi^+ - \bar{X}^-X^0\phi^-] + \frac{1}{2c_w}igM[\bar{X}^0X^-\phi^+ - \bar{X}^0X^+\phi^-] + \\
igMs_w[\bar{X}^0X^-\phi^+ - \bar{X}^0X^+\phi^-] + \frac{1}{2}igM[\bar{X}^+X^+\phi^0 - \bar{X}^-X^-\phi^0]
\end{gathered}
$$

[source: symmetrymagazine, Thomas Gutierrez]

The Standard Model of particle physics

- Particles are represented by fields
- Lagrangian displays interactions of fields
- Based on gauge symmetry
  → $SU_{\rm QCD}(3) \times SU_{\rm L}(2) \times U_{\rm Y}(1)$

- Strong force
  → Quantum chromodynamics (QCD) mediated by gluon
- Electromagnetic force
  → Quantum electrodynamics (QED) mediated by photon
- Weak force
  → Weak interaction mediated by W and Z

→ W and Z get their mass from Higgs mechanism

Perturbation theory and Feynman diagrams

Quark and gluon interaction

$$\mathcal{L}_\psi = \sum_q \overline{\psi}_q \left(\mathrm{i}\not{\partial} - g_\mathrm{s} \frac{\lambda_a}{2} \not{G}^a - m_q \right) \psi_q$$

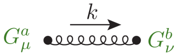
After quantisation:
(e.g. with path integral formalism)
→ *Feynman rules* in momentum space

Perturbation theory and Feynman diagrams

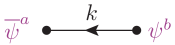
Quark and gluon interaction

$$\mathcal{L}_\psi = \sum_q \overline{\psi}_q \left(\mathrm{i}\not{\partial} - g_\mathrm{s} \frac{\lambda_a}{2} \not{G}^a - m_q \right) \psi_q$$

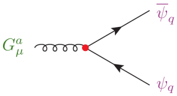
After quantisation:
(e.g. with path integral formalism)
→ *Feynman rules* in momentum space



$$\frac{-\mathrm{i}\delta^{ab} g_{\mu\nu}}{k^2 + \mathrm{i}\epsilon}$$



$$\frac{\mathrm{i}(\not{k} + m_q)}{k^2 - m_q^2 + \mathrm{i}\epsilon}$$



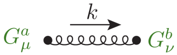
$$-\,\mathrm{i} g_\mathrm{s} \frac{\lambda^a}{2} \gamma_\mu$$

Perturbation theory and Feynman diagrams

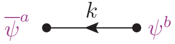
Quark and gluon interaction

$$\mathcal{L}_\psi = \sum_q \overline{\psi}_q \left(\mathrm{i}\not{\partial} - g_\mathrm{s} \frac{\lambda_a}{2} \not{G}^a - m_q \right) \psi_q$$

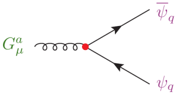
After quantisation:
(e.g. with path integral formalism)
→ *Feynman rules* in momentum space



$$\frac{-\mathrm{i}\delta^{ab} g_{\mu\nu}}{k^2 + \mathrm{i}\epsilon}$$



$$\frac{\mathrm{i}(\not{k} + m_q)}{k^2 - m_q^2 + \mathrm{i}\epsilon}$$



$$-\mathrm{i} g_\mathrm{s} \frac{\lambda^a}{2} \gamma_\mu$$

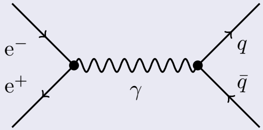
→ Building blocks for transition **amplitudes**:

$$\langle f | \, S \, | i \rangle = (2\pi)^4 \delta \left(\sum_j p_j - \sum_j k_j \right) \mathrm{i}\mathcal{M}_{\mathbf{fi}}, \quad | i \rangle \neq | f \rangle$$

→ **Cross section**: Probability to measure a scattering process

$$\sigma \propto \int \mathrm{d}\Phi |\mathcal{M}|^2$$

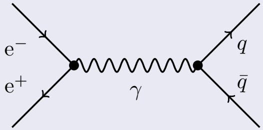
$\mathrm{e}^+(p_1)\,\mathrm{e}^-(p_2) \to q(k_1)\,\bar{q}(k_2)$ [QED]



→ **Cross section**: Probability to measure a scattering process

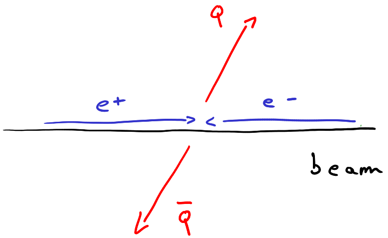
$$\sigma \propto \int \mathrm{d}\Phi |\mathcal{M}|^2$$

$$\mathrm{e}^+(p_1)\,\mathrm{e}^-(p_2) \to q(k_1)\,\bar{q}(k_2) \quad \text{[QED]}$$



- $\mathrm{d}\Phi \propto \mathrm{d}\phi\, \mathrm{d}\cos\theta$
- $|\mathcal{M}|^2 \propto 1 + \cos^2\theta$

$$\Rightarrow \quad \sigma_{\text{incl}} \propto \int_0^{2\pi} \mathrm{d}\phi \int_{-1}^{+1} \mathrm{d}\cos\theta\,(1+\cos^2\theta)$$

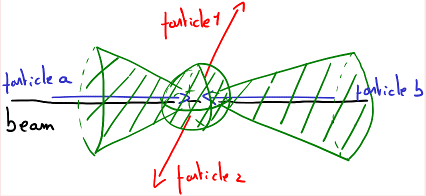


- Experimental measurements performed in parts of the phase space
  $\rightarrow$ where detectors are placed
- Experiments also measure observables
  $\rightarrow$ transverse mom. $\left[p_{\mathrm{T}}(p) = \sqrt{p_x^2 + p_y^2}\right]$, rapidity $\left[y(p) = \frac{1}{2}\log\left(\frac{|\vec{p}|+p_z}{|\vec{p}|-p_z}\right)\right]$ ...
- Analytical integration not an option!

- Experimental measurements performed in parts of the phase space
  → where detectors are placed
- Experiments also measure observables
  → transverse mom. $\left[p_{\mathrm{T}}(p) = \sqrt{p_x^2 + p_y^2}\right]$, rapidity $\left[y(p) = \frac{1}{2}\log\left(\frac{|\vec{p}|+p_z}{|\vec{p}|-p_z}\right)\right]$ ...
- Analytical integration not an option!

$\Rightarrow$ Solution: **Monte Carlo integration!**

→ Trade integration variables (angles, energies) vs. random numbers (x_i)



Example: $\phi = 2\pi \cdot x_1$, $\cos\theta = 2x_2 - 1$

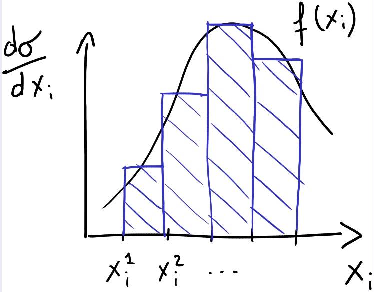
$$\Rightarrow\ \sigma \propto \int_0^1 \mathrm{d}x_1 \cdots \int_0^1 \mathrm{d}x_n\, f(x_1,\cdots,x_n)\,\Theta(g(x_1,\cdots,x_n))$$

- $\Theta(t)$ encodes experimental selection
- g highly complex function with singularities

$$\sigma \propto \int_0^1 \mathrm{d}x_1 \cdots \int_0^1 \mathrm{d}x_n \, f(x_1, \cdots, x_n) \, \Theta(g(x_1, \cdots, x_n))$$

- Cross section:

$$\sigma = \sum_{i,l} c_{il} \frac{\mathrm{d}\sigma}{\mathrm{d}x_l^i}$$



- **Integrating = guessing the values of a function at specific points (Riemann sum)**
- More complex calculations ...

  ... more integration variables ...

  ... more computing resources!

Quantum Amplitude Estimate (QAE)

[Brassard, Hoyer, Mosca, Tapp; Quantum Amplitude Amplification and Estimation; quant-ph/0005055]

$$\mathcal{A}|0\rangle = \sqrt{1-a}|\Psi_0\rangle + \sqrt{a}|\Psi_1\rangle$$

QAE estimates a with high probability such that the estimation error scales as $\mathcal{O}(1/M)$ [as opposed to $\mathcal{O}(1/\sqrt{M})$], M: number of applications of $\mathcal{A}$

Quantum Amplitude Estimate (QAE)

[Brassard, Hoyer, Mosca, Tapp; Quantum Amplitude Amplification and Estimation; quant-ph/0005055]

$$\mathcal{A}|0\rangle = \sqrt{1-a}|\Psi_0\rangle + \sqrt{a}|\Psi_1\rangle$$

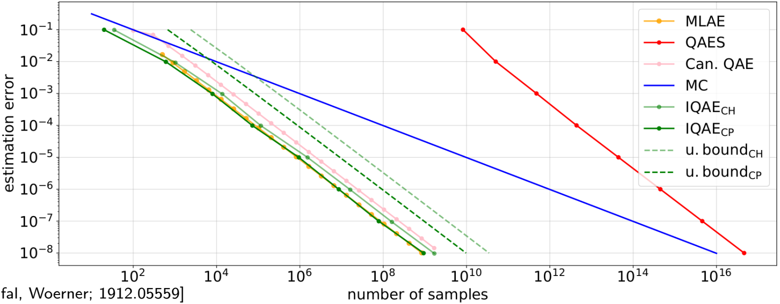
QAE estimates a with high probability such that the estimation error scales as $\mathcal{O}(1/M)$ [as opposed to $\mathcal{O}(1/\sqrt{M})$], M: number of applications of $\mathcal{A}$

Quantum Amplitude Estimate (QAE)

[Brassard, Hoyer, Mosca, Tapp; Quantum Amplitude Amplification and Estimation; quant-ph/0005055]

$$\mathcal{A}|0\rangle = \sqrt{1-a}|\Psi_0\rangle + \sqrt{a}|\Psi_1\rangle$$

QAE estimates a with high probability such that the estimation error scales as $\mathcal{O}(1/M)$ [as opposed to $\mathcal{O}(1/\sqrt{M})$], M: number of applications of $\mathcal{A}$



[Grinko, Gacon, Zoufal, Woerner; 1912.05559]

Resulting estimation error for $a = 1/2$ and 95% confidence level with respect to the required total number of oracle queries.

Quantum integration

Extension to

$$\mathcal{A}|0\rangle = \sum_i a_i |\Psi_i\rangle$$

→ Definition of a piece-wise function with $f(x_i) = a_i$.

Quantum integration

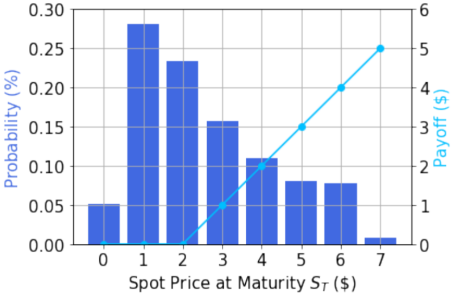
Extension to

$$\mathcal{A}|0\rangle = \sum_i a_i |\Psi_i\rangle$$

→ Definition of a piece-wise function with $f(x_i) = a_i$.

Initially used in finance for simple functions in 1D [Zoufal, Lucchi, Woerner; 1904.00043]

[Woerner and Egger; 1806.06893], [Stamatopoulos et al.; 1905.02666, 2111.12509], [Rebentrost, Gupt, Bromley; Phys.Rev.A 98 (2018) 022321]



$$I = \int \mathrm{d}x\, f(x) g(x)$$

[Zoufal, Lucchi, Woerner; 1904.00043]

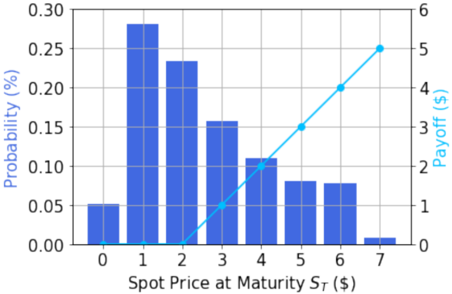
Quantum integration

Extension to

$$\mathcal{A}|0\rangle = \sum_i a_i |\Psi_i\rangle$$

→ Definition of a piece-wise function with $f(x_i) = a_i$.

Initially used in finance for simple functions in 1D [Zoufal, Lucchi, Woerner; 1904.00043]
[Woerner and Egger; 1806.06893], [Stamatopoulos et al.; 1905.02666, 2111.12509], [Rebentrost, Gupt, Bromley; Phys.Rev.A 98 (2018) 022321]



[Zoufal, Lucchi, Woerner; 1904.00043]

$$I = \int \mathrm{d}x\, f(x) g(x)$$

- In finance:
  - f: probability
  - g: payoff
- In high-energy physics:
  - f: $|\mathcal{M}|^2$
  - g: $\Theta(\Phi - \Phi_c)$

Idea to apply QAE to high-energy physics [Agliardi, Grossi, MP, Prati; 2201.01547]
→ Followed by [Martínez de Lejarza et al.; 2204.06496, 2401.03023, 2409.12236,2506.19965], [Williams, MP; 2502.14647]

Idea to apply QAE to high-energy physics [Agliardi, Grossi, MP, Prati; 2201.01547]
→ Followed by [Martínez de Lejarza et al.; 2204.06496, 2401.03023, 2409.12236,2506.19965], [Williams, MP; 2502.14647]

Two-step procedure

1. Encode the $a_i = f(x_i)$ of the integrand f into the state $\mathcal{A}|0\rangle = \sum_i a_i |\Psi_i\rangle$
2. Guess (=integrate) the a_i using QAE

Idea to apply QAE to high-energy physics [Agliardi, Grossi, MP, Prati; 2201.01547]
→ Followed by [Martínez de Lejarza et al.; 2204.06496, 2401.03023, 2409.12236,2506.19965], [Williams, MP; 2502.14647]

Two-step procedure

1. Encode the $a_i = f(x_i)$ of the integrand f into the state $\mathcal{A}|0\rangle = \sum_i a_i |\Psi_i\rangle$
2. Guess (=integrate) the a_i using QAE

Encode the a_i

- Analytical expression of the integrand (quantum arithmetics)
- Numerical implementation of the integrand (following application)

→ Both approached used for classical Monte Carlo

[Williams, MP; 2502.14647]

Quantum arithmetics + use of QUANTINUUM Quantum Monte Carlo Integrator [Akhalwaya et al.; 2308.06081], based on [Herbert; 2105.09100]

[Williams, MP; 2502.14647]

Quantum arithmetics + use of QUANTINUUM Quantum Monte Carlo Integrator

[Akhalwaya et al.; 2308.06081], based on [Herbert; 2105.09100]

- 2D integral

$$\int \prod_{i=1}^{N_I} \mathrm{d}x_i \frac{\sum_{S_k \in I} \alpha_k \prod_{j \in S_k} x_j^{n_j}}{\prod_{p=1}^{N_P} (x_p - M_{op}^2)^2 + M_{op}^2 \Gamma_{op}^2} \quad \rightarrow \quad \int_0^{s} \int_0^{s-s_2} \mathrm{d}s_1 \mathrm{d}s_2 \frac{s_1^2 s - s_1 M_\tau^2 s + s_1 M_\tau^2 s_2}{(s_2 - M_\mathrm{W}^2)^2 + (M_\mathrm{W} \Gamma_\mathrm{W})^2}$$

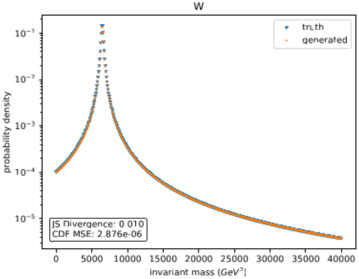
[Williams, MP; 2502.14647]

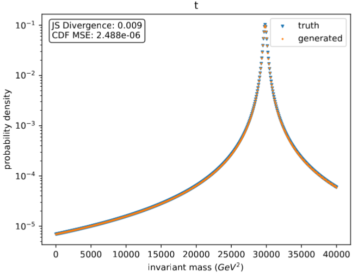
Quantum arithmetics + use of QUANTINUUM Quantum Monte Carlo Integrator [Akhalwaya et al.; 2308.06081], based on [Herbert; 2105.09100]

- 2D integral

$$\int \prod_{i=1}^{N_I} \mathrm{d}x_i \frac{\sum_{S_k \in I} \alpha_k \prod_{j \in S_k} x_j^{n_j}}{\prod_{p=1}^{N_P} (x_p - M_{op}^2)^2 + M_{op}^2 \Gamma_{op}^2} \quad \to \quad \int_0^s \int_0^{s-s_2} \mathrm{d}s_1 \mathrm{d}s_2 \frac{s_1^2 s - s_1 M_T^2 s + s_1 M_T^2 s_2}{(s_2 - M_W^2)^2 + (M_W \Gamma_W)^2}$$

- Building blocks → Propagator (using Fourier expansion) [Rosenkranz et al.; 2405.21058]





[Williams, MP; 2502.14647]

Quantum arithmetics + use of QUANTINUUM Quantum Monte Carlo Integrator [Akhalwaya et al.; 2308.06081], based on [Herbert; 2105.09100]

- 2D integral

$$\int \prod_{i=1}^{N_I} \mathrm{d}x_i \frac{\sum_{S_k \in I} \alpha_k \prod_{j \in S_k} x_j^{n_j}}{\prod_{p=1}^{N_P} (x_p - M_{op}^2)^2 + M_{op}^2 \Gamma_{op}^2} \quad \rightarrow \quad \int_0^s \int_0^{s-s_2} \mathrm{d}s_1 \mathrm{d}s_2 \frac{s_1^2 s - s_1 M_T^2 s + s_1 M_T^2 s_2}{(s_2 - M_W^2)^2 + (M_W \Gamma_W)^2}$$

- Building blocks → Propagator (using Fourier expansion) [Rosenkranz et al.; 2405.21058]
- Resource estimate → Fault-tolerant era

Compilation	**Resource**	**Metric**	**Precision**		
			10%	**1%**	**0.1%**
NISQ	Number of qubits	Largest across circuits	28	28	28
	CX gates	Total number across circuits	7.39×10^7	6.15×10^8	5.09×10^9
		Total depth across circuits	4.34×10^7	3.61×10^8	2.99×10^9
		Number in largest circuit	3.39×10^7	2.71×10^8	1.20×10^9
		Depth of largest circuit	1.99×10^7	1.59×10^8	7.03×10^8
	All gates	Total number across circuits	1.50×10^8	1.24×10^9	1.03×10^{10}
		Total depth across circuits	8.02×10^7	6.67×10^8	5.52×10^9
		Number in largest circuit	6.86×10^7	5.49×10^8	2.42×10^9
		Depth of largest circuit	3.68×10^7	2.94×10^8	1.30×10^9
Fault tolerant	Number of qubits	Largest across circuits	41	41	41
	T gates	Total number across circuits	3.39×10^9	4.08×10^{10}	2.72×10^{11}
		Total depth across circuits	3.29×10^9	3.95×10^{10}	2.63×10^{11}
		Number in largest circuit	1.65×10^9	2.14×10^{10}	6.97×10^{10}
		Depth of largest circuit	1.60×10^9	2.07×10^{10}	6.75×10^{10}

How?

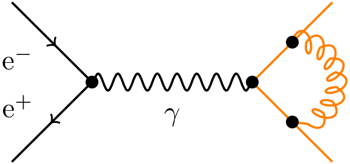
- Applications
  - → Cross section → quantum integration
  - → Amplitudes in QCD → dedicated quantum algorithms
  - → QCD parton shower → dedicated quantum algorithms

Quantum implementation of QCD amplitude

→ Numerical implementation of integrand

$$\mathcal{M} \sim \sum \mathrm{Tr}\left(T^{a_1} \ldots T^{a_n}\right) \mathcal{K}(1, \ldots, n)$$

- Kinematic part: made of spinors and tensors (and kinematic invariants)
- Colour part: made of SU(3) generators of QCD

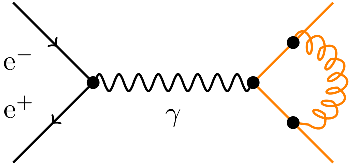


Quantum implementation of QCD amplitude

→ Numerical implementation of integrand

$$\mathcal{M} \sim \sum \mathrm{Tr}\left(T^{a_1} \dots T^{a_n}\right) \mathcal{K}(1, \dots, n)$$

- Kinematic part: made of spinors and tensors (and kinematic invariants)
- Colour part: made of SU(3) generators of QCD



- First step towards a full quantum amplitude/Monte Carlo
- Useful for a quantum parton shower (see later)

Colour algebra in QCD - crash course

- $[T^a, T^b] = \mathrm{i} f_{abc} T^c$.
- $T^a, T^c, \ldots$: SU(3) generators
- Gell-Mann matrices

$$T^1 = \frac{1}{2}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad T^2 = \frac{1}{2}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad T^3 = \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad T^4 = \frac{1}{2}\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix},$$

$$T^5 = \frac{1}{2}\begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \quad T^6 = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad T^7 = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad T^8 = \frac{1}{2\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}.$$

Attention! T^a are not unitary!

→ In our example, colour factor: $T^a_{ij} T^a_{jk} = C_F \delta_{ik}$

Quantum implementation of colour

→ Implementation of the (discrete) colour into qubits:

- Gluon: 8 colours → 3 qubits ($2^3 = 8$)
- Quark: 3 colours → 2 qubits ($2^2 = 4$)

Quantum implementation of colour

→ Implementation of the (discrete) colour into qubits:

- Gluon: 8 colours → 3 qubits ($2^3 = 8$)
- Quark: 3 colours → 2 qubits ($2^2 = 4$)

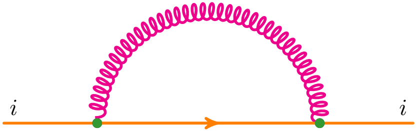
Make non-unitary matrices unitary again!

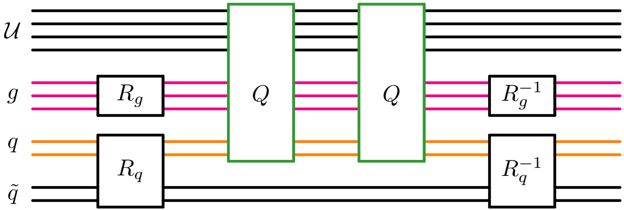
→ Extend dimension and modify them

$$\overline{T^1} = \frac{1}{2}\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad \overline{T^2} = \frac{1}{2}\begin{pmatrix} 0 & -\mathrm{i} & 0 & 0 \\ \mathrm{i} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad \overline{T^3} = \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad \overline{T^4} = \frac{1}{2}\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

$$\overline{T^5} = \frac{1}{2}\begin{pmatrix} 0 & 0 & -\mathrm{i} & 0 \\ 0 & 1 & 0 & 0 \\ \mathrm{i} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad \overline{T^6} = \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad \overline{T^7} = \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -\mathrm{i} & 0 \\ 0 & \mathrm{i} & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad \overline{T^8} = \frac{1}{2\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

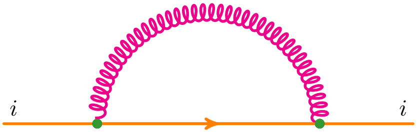
Example

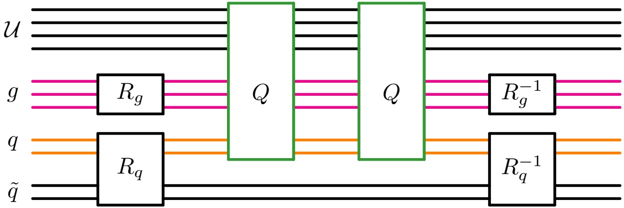




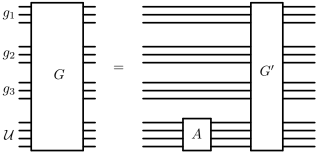
- One-to-one correspondence between Feynman diagram and circuit
- Gates for *qqg* (Q) and ggg (G) vertices to simulate QCD (colour) interaction

Example

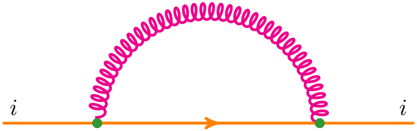


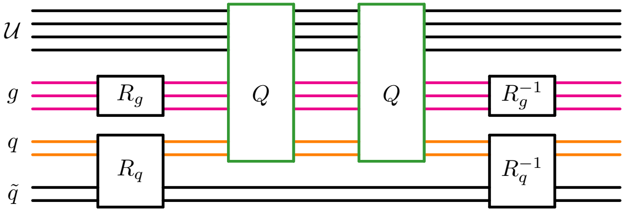


- One-to-one correspondence between Feynman diagram and circuit
- Gates for qqg (Q) and ggg (G) vertices to simulate QCD (colour) interaction

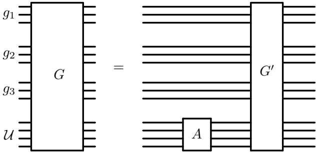


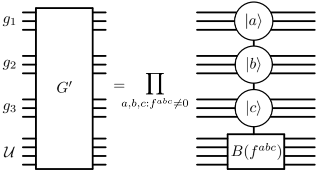
Example



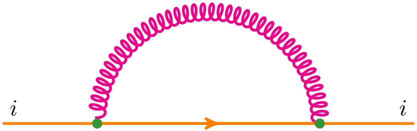


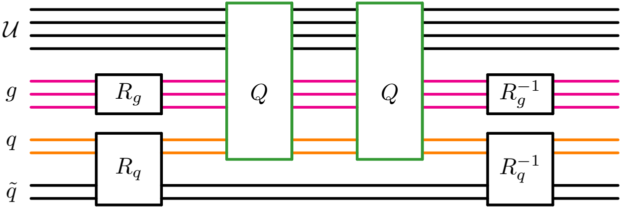
- One-to-one correspondence between Feynman diagram and circuit
- Gates for qqg (Q) and ggg (G) vertices to simulate QCD (colour) interaction



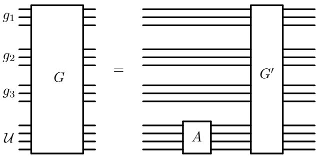


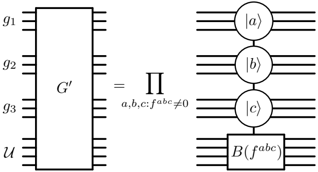
Example

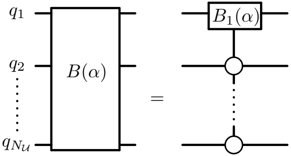




$$\langle\Omega|_{\mathcal{U}} \prod_{i=1}^{N_{ops}} \left\{B(\alpha_i)A\right\} |\Omega\rangle_{\mathcal{U}} = \prod_{i=1}^{N_{ops}} \alpha_i$$

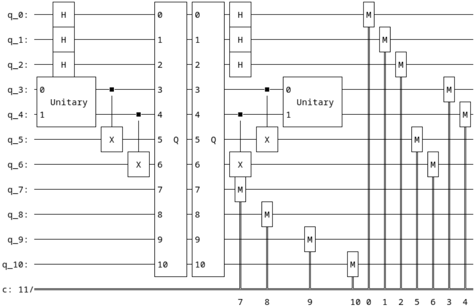






→ Trace defined in $|00000000000\rangle = |0_{11}\rangle$ state: $|\psi\rangle = \frac{\mathcal{C}}{\mathcal{N}}|0_{11}\rangle + \ldots$

$$\Rightarrow \frac{27415}{N_{\text{shots}}=1000000} \sim \left(\frac{\mathcal{C}}{\mathcal{N}} = \frac{(N_c{=}3)C_F}{N_c^{n_q=1}(N_c^2-1)^{n_g=1}}\right)^2 \qquad \sim \alpha^2 \text{ in intro}$$

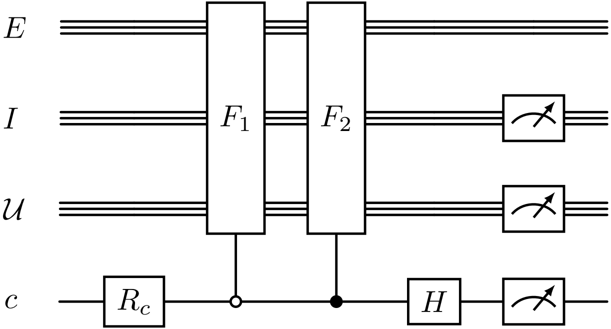


```
Total counts are: {'00000010101': 226, '00010010011': 342, '00000010110': 225, '00000001101': 696, '0001
0010010': 362, '00000010001': 872, '00010001101': 2006, '00000001100': 643, '00010001100': 2006, '000100
00110': 1051, '00000001111': 638, '00000010010': 904, '00010000010': 1057, '00100010101': 52353, '000100
10100': 3342, '00100000101': 6942, '00010000111': 1046, '00000010111': 210, '00100001101': 36223, '00100
010111': 51877, '00000001110': 643, '00010010000': 4838, '00000010100': 5421, '00010000100': 1035, '0010
0000000': 107280, '00010000011': 1075, '00100000111': 6795, '00100001011': 145043, '00100010010': 10275,
 '00010000000': 65548, '00000000000': 27415, '00010001111': 2031, '00100000110': 7004, '00000001011': 25
51, '00100001111': 36471, '00010010111': 8866, '00010000101': 1077, '00100010110': 52220, '00010010110':
 9080, '00100010100': 185856, '00100001100': 36173, '00010010101': 8860, '00100010000': 14129, '00100001
110': 36925, '00100000100': 6858, '00100010011': 10182, '00100000001': 6950, '00010001110': 2018, '00100
000011': 6983, '00000010011': 815, '00010001011': 7957, '00010010001': 340, '00100010001': 10163, '00010
000001': 1092, '00100000010': 7010}
```

→ Colour factors encoded in one single state $|0_{11}\rangle$ (as needed for QAE)
→ Any colour factor can be computed

[Chawdhry, MP; 2303.04818]

→ Interference between two amplitudes (F_1 and F_2)



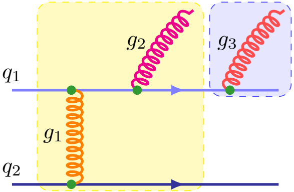
→ c is an equal superposition of F_1 and F_2

→ Making the measurement (∼ square/collapse the amplitude):

$$|\mathcal{M}|^2 = |F_1 + F_2|^2 = |F_1|^2 + |F_2|^2 + 2\,\mathrm{Re}\,\{F_1.F_2^*\}$$

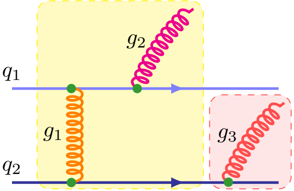
→ Common building block factored out

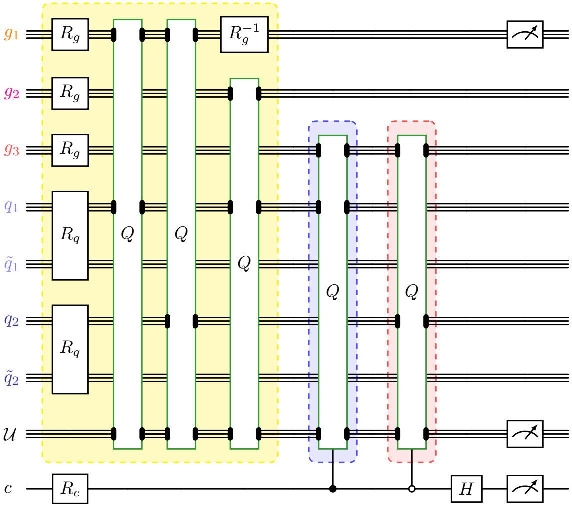
F_1:



and

F_2:

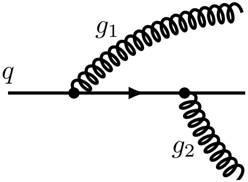




[Chawdhry, MP, Williams; 2507.07194]

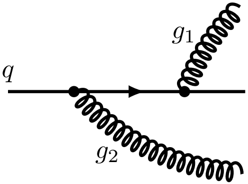
Common building block factored out + simple operation to obtained second amplitude
→ Hint for quadratic improvement with respect to classical approach

F_1:

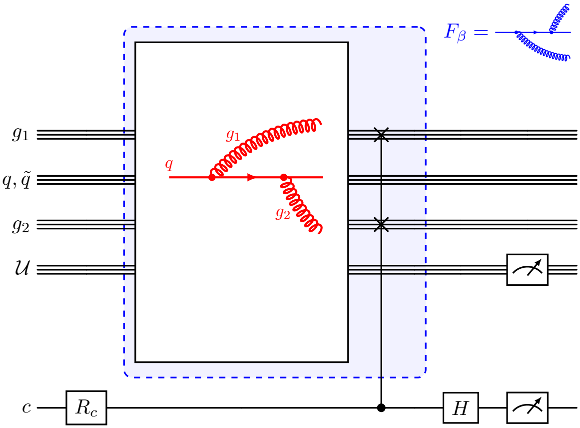


and

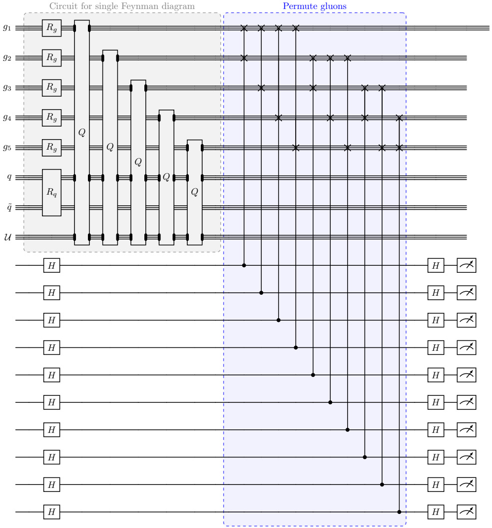
F_2:

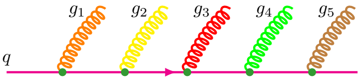


[Chawdhry, MP, Williams; 2507.07194]

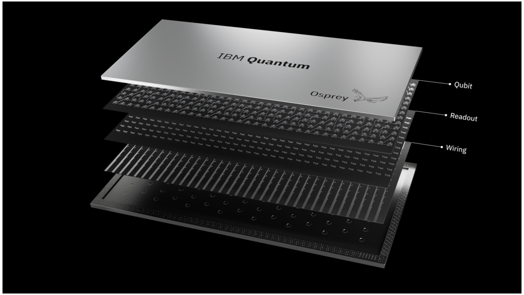


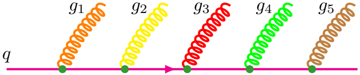
→ Application to IBM 120 qubits superconducting with squared lattice (Nightawk generation) [Chawdhry, Pascuzzi, MP; in preparation]



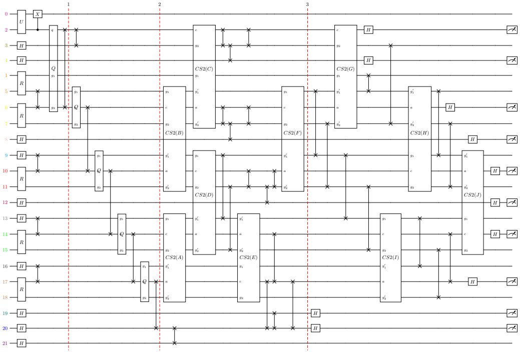


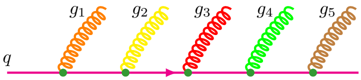
→ Application to IBM 120 qubits superconducting with squared lattice (Nightawk generation) [Chawdhry, Pascuzzi, MP; in preparation]



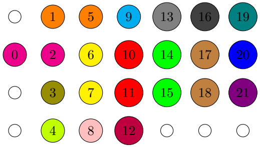


→ Application to IBM 120 qubits superconducting with squared lattice (Nightawk generation) [Chawdhry, Pascuzzi, MP; in preparation]



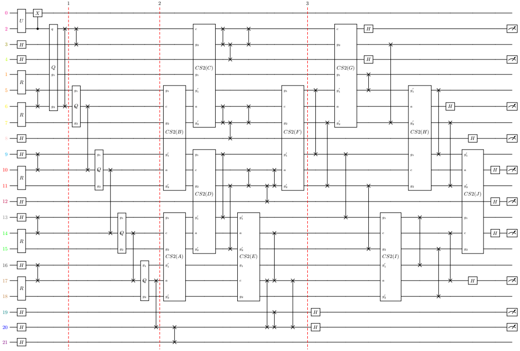


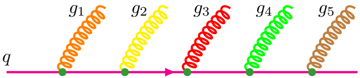
Initial:



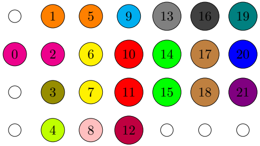
○	1	5	9	13	16	19
0	2	6	10	14	17	20
○	3	7	11	15	18	21
○	4	8	12	○	○	○

→ Application to IBM 120 qubits superconducting with squared lattice (Nightawk generation) [Chawdhry, Pascuzzi, MP; in preparation]

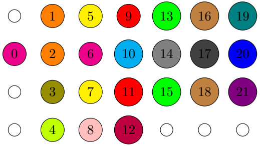




Initial:



Step 1:



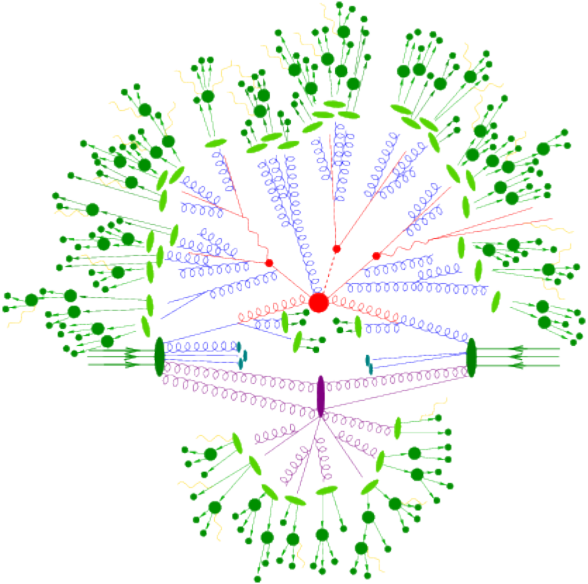
How?

- Applications

  → Cross section → quantum integration

  → Amplitudes in QCD → dedicated quantum algorithms

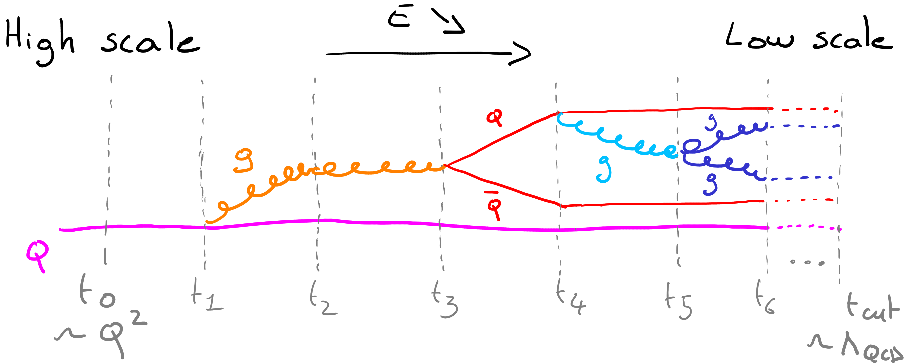
  → QCD parton shower → dedicated quantum algorithms



[source: Sherpa]

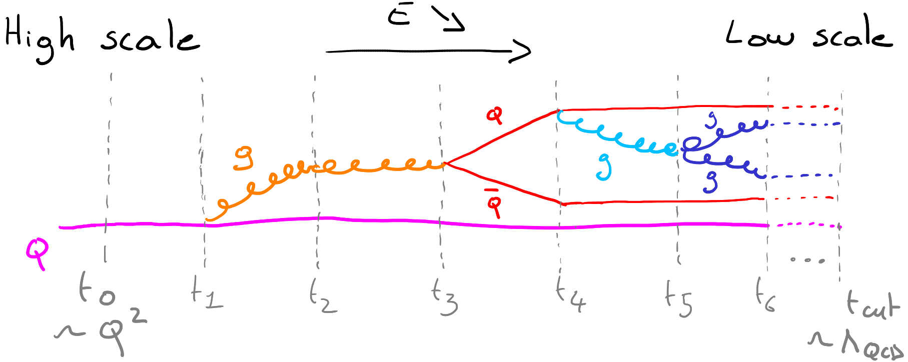
Parton shower

Algorithmic and approximate way of emitting an arbitrary number of particles



Parton shower

Algorithmic and approximate way of emitting an arbitrary number of particles



Advantages

- Universal

Drawbacks

- Approximate

→ First proposal for Quantum Parton Shower [Bauer, de Jong, Nachman, Provasoli; 1904.03196]

Toy model:
$$\begin{aligned}\mathcal{L} =& \bar{f}_1(i\partial\!\!\!/ + m_1)f_1 + \bar{f}_2(i\partial\!\!\!/ + m_2)f_2 + (\partial_\mu\phi)^2 \\ &+ g_1\bar{f}_1 f_1\phi + g_2\bar{f}_2 f_2\phi + g_{12}\left[\bar{f}_1 f_2 + \bar{f}_2 f_1\right]\phi\end{aligned}$$

- Splittings: $f_i \to f_j\phi$ and $\phi \to f_i\bar{f}_j$
- Mixing between f_1 and f_2

→ First proposal for Quantum Parton Shower [Bauer, de Jong, Nachman, Provasoli; 1904.03196]

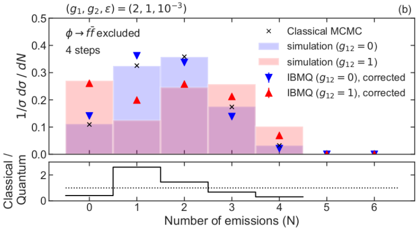
Toy model:
$$\begin{aligned}\mathcal{L} =& \bar{f}_1(i\partial\!\!\!/ + m_1)f_1 + \bar{f}_2(i\partial\!\!\!/ + m_2)f_2 + (\partial_\mu \phi)^2 \\ &+ g_1\bar{f}_1 f_1\phi + g_2\bar{f}_2 f_2\phi + g_{12}\left[\bar{f}_1 f_2 + \bar{f}_2 f_1\right]\phi\end{aligned}$$

- Splittings: $f_i \to f_j\phi$ and $\phi \to f_i\bar{f}_j$
- Mixing between f_1 and f_2
- **Idea**: Evolution with (quantum) superposition of f_1 and f_2
  → allows to take into account interference between different histories
- Classically requires a reweighting strategy at each step (costly)
  → claim of exponential speed-up

→ First proposal for Quantum Parton Shower [Bauer, de Jong, Nachman, Provasoli; 1904.03196]

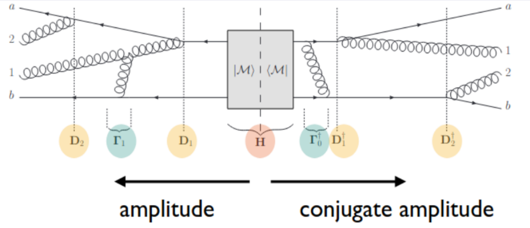
Toy model:
$$\mathcal{L} = \bar{f}_1(i\partial\!\!\!/ + m_1)f_1 + \bar{f}_2(i\partial\!\!\!/ + m_2)f_2 + (\partial_\mu\phi)^2 \\ + g_1\bar{f}_1 f_1\phi + g_2\bar{f}_2 f_2\phi + g_{12}\left[\bar{f}_1 f_2 + \bar{f}_2 f_1\right]\phi$$

- Splittings: $f_i \to f_j\phi$ and $\phi \to f_i\bar{f}_j$
- Mixing between f_1 and f_2
- **Idea**: Evolution with (quantum) superposition of f_1 and f_2
  → allows to take into account interference between different histories
- Classically requires a reweighting strategy at each step (costly)
  → claim of exponential speed-up



Outlook

- In SM, mixing for γ/Z → EW PS
- Account for different PS history (and interference) → Amplitude PS
  [Nagy, Soper; 0706.0017,1902.02105], [Plätzer et al.; 1201.0260,1802.08531,1808.00332], [Isaacson, Prestel; 1806.10102]



- Perfect framework for implementation of previous quantum interaction gates

Quantum simulation in high-energy physics

- Is it possible?
  → **Yes.**

Quantum simulation in high-energy physics

- Is it possible?
  → **Yes.**

- Is there a quantum advantage?
  → **In principle, yes. In practice, not yet.**

Quantum simulation in high-energy physics

- Is it possible?
  → **Yes.**

- Is there a quantum advantage?
  → **In principle, yes. In practice, not yet.**

- Is it more resource efficient/faster than CPU/GPU/AI approchaes?
  → **At the moment, not known.**

- Are we witnessing a quantum revolution?

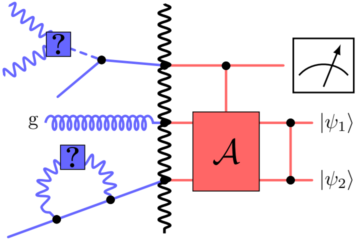
- When can we do *state-of-the-art* quantum computations?

Conclusion

Quantum simulation of High-Energy physics

→ High-energy physics: computationally-intensive field

- **GPU & AI**: Technical, general-purpose approaches
- **Quantum computing**: Tailored to quantum systems (at very high energies)



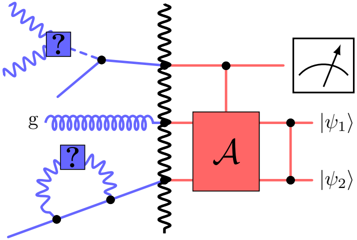
Conclusion

Quantum simulation of High-Energy physics

→ High-energy physics: computationally-intensive field

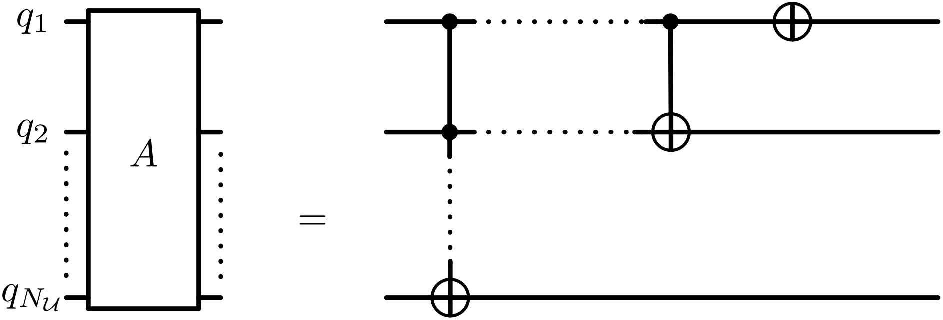
- **GPU & AI**: Technical, general-purpose approaches
- **Quantum computing**: Tailored to quantum systems (at very high energies)

- Several interesting applications already
- Powerful quantum computers becoming available
  → entering era with non-trivial applications
- Exciting emerging field with many new ideas!



Back-up slides

BACK-UP


q_1
q_2
$q_{N_\mathcal{U}}$
A
$=$

$$B_1(\alpha) = \begin{pmatrix} \sqrt{1-|\alpha|^2} & \alpha \\ -\alpha & \sqrt{1-|\alpha|^2} \end{pmatrix} \tag{1}$$

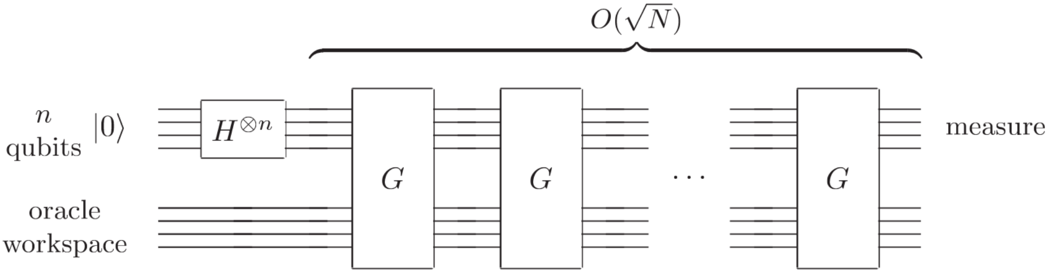
$$B(\alpha)A\,|k\rangle = \begin{cases} \alpha\,|0\rangle + \sqrt{1-|\alpha|^2}\,|1\rangle & \text{if } k=0 \\ |k+1\rangle & \text{if } 0<k<2^{N_{\mathcal{U}}}-1 \\ \sqrt{1-|\alpha|^2}\,|0\rangle - \alpha\,|1\rangle & \text{if } k=2^{N_{\mathcal{U}}}-1 \end{cases} \tag{2}$$

$$\langle\Omega|_{\mathcal{U}} B(\alpha)A|\Omega\rangle_{\mathcal{U}} = \alpha \tag{3}$$

$$\langle\Omega|_{\mathcal{U}} \prod_{i=1}^{N_{ops}} \{B(\alpha_i)A\}\,|\Omega\rangle_{\mathcal{U}} = \prod_{i=1}^{N_{ops}} \alpha_i \tag{4}$$

Grover algorithm/iteration

- Very general quantum algorithm
- Quadratic speed up
  → $\mathcal{O}(\sqrt{N})$ operations instead of $\mathcal{O}(N)$
- Most famous example: unstructured database search



Quantum Amplitude Estimate (QAE)

[Brassard, Hoyer, Mosca, Tapp; Quantum Amplitude Amplification and Estimation; quant-ph/0005055]

$$\mathcal{A}|0\rangle = \sqrt{1-a}|\Psi_0\rangle + \sqrt{a}|\Psi_1\rangle$$

QAE estimates a with high probability such that the estimation error scales as $\mathcal{O}(1/M)$ [as opposed to $\mathcal{O}(1/\sqrt{M})$]

M: number of applications of $\mathcal{A}$

Quantum Amplitude Estimate (QAE)

[Brassard, Hoyer, Mosca, Tapp; Quantum Amplitude Amplification and Estimation; quant-ph/0005055]

$$\mathcal{A}|0\rangle = \sqrt{1-a}|\Psi_0\rangle + \sqrt{a}|\Psi_1\rangle$$

QAE estimates a with high probability such that the estimation error scales as $\mathcal{O}(1/M)$ [as opposed to $\mathcal{O}(1/\sqrt{M})$]

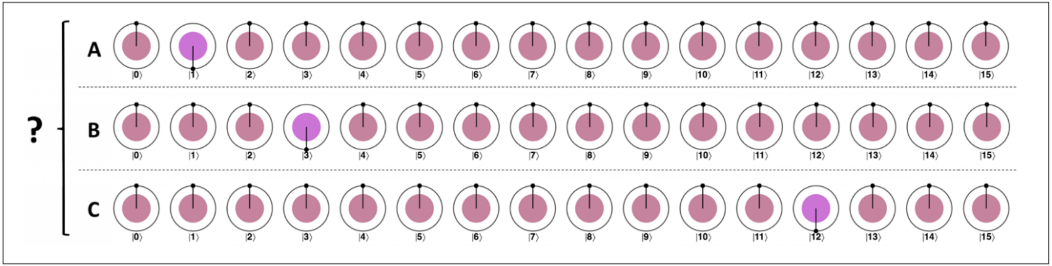
M: number of applications of $\mathcal{A}$

→ What the (orignal) algorithm provides:

- An estimate: $\tilde{a} = \sin^2(\tilde{\theta}_a)$
  with $\tilde{\theta}_a = y\pi/M$, $y \in \{0, ..., M-1\}$, and $M = 2^n$
- A success probability (that can be increased by repeating the algorithm)
- A bound: $|a - \tilde{a}| \leq \mathcal{O}\left(1/M\right)$

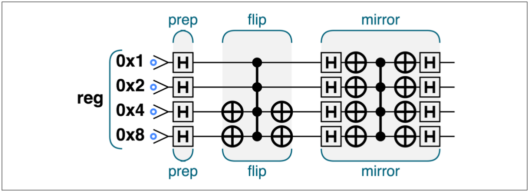
Grover algorithm/iteration

- Example (from [Johnston, Harrigan, Gimeno-Segovia; Programming Quantum Computers])

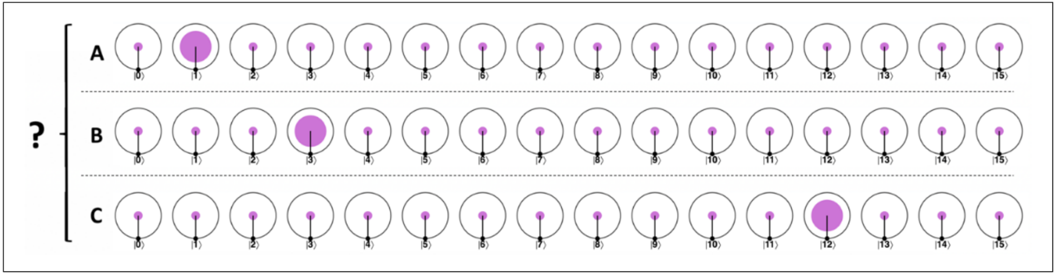


→ What solution is contained in our quantum register?

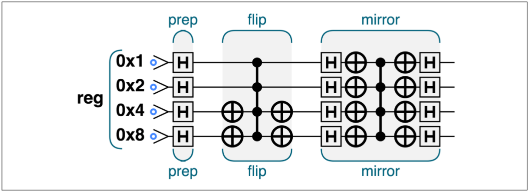
Grover algorithm/iteration



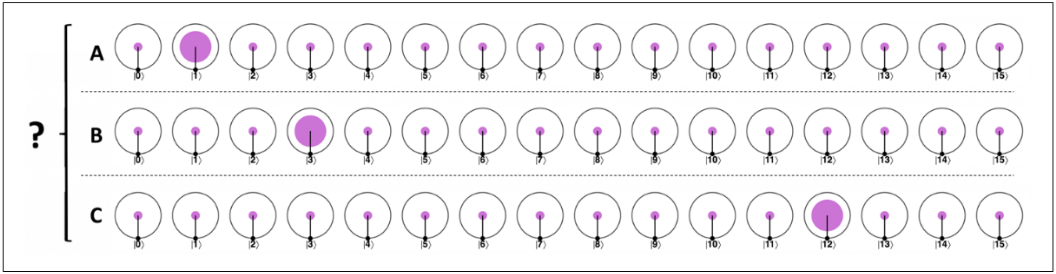
→ Applying a Grover iteration



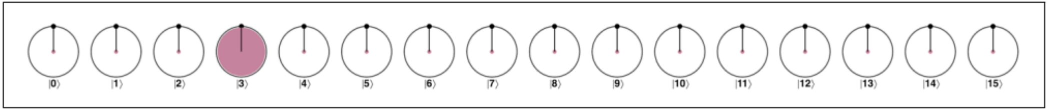
Grover algorithm/iteration



→ Applying a Grover iteration



→ Applying it twice (*e.g.* solution B)



Applications

- **$\mathrm{e}^+\mathrm{e}^- \to \mathrm{q\bar{q}}$** (in QED) → 1D problem

$$\sigma \sim \int_{-1}^{1}\int_{0}^{2\pi} \mathrm{d}\cos\theta \mathrm{d}\phi \left(1+\cos^2\theta\right)$$

- **$\mathrm{e}^+\mathrm{e}^- \to \mathrm{q\bar{q}'W}$** → 2D problem

$$\begin{aligned}\sigma &\sim \int_{M_\mathrm{W}^2}^{s}\int_{0}^{s_1^{\mathrm{Max}}}\int_{-1}^{1}\int_{0}^{2\pi}\int_{0}^{2\pi} \mathrm{d}\Phi_3 \left|\mathcal{M}_{\mathrm{e}^+\mathrm{e}^-\to q\bar{q}'\mathrm{W}}\right|^2 \\ &\sim \int_{M_\mathrm{W}^2}^{s}\int_{0}^{s_1^{\mathrm{Max}}} \mathrm{d}\tilde{\Phi}_3 \left|\mathcal{M}'\right|^2\end{aligned}$$

with $\mathcal{M}' = \mathcal{M}_{\mathrm{e}^+\mathrm{e}^-\to q\bar{q}'\mathrm{W}}\left(\cos\theta_1 = 0,\ \phi_1 = \pi/2,\ \phi_2 = \pi/2\right)$.

Loading of distribution / encoding into qubits

Encoding the distribution to be integrated into qubits (coefficients of states)

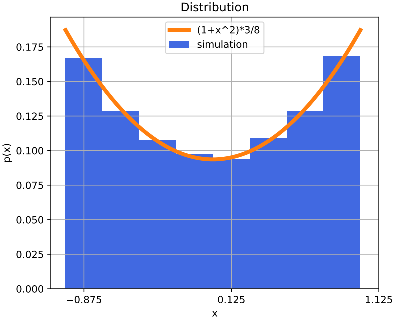
- Exact loading [Shende, Bullock, Markov, quant-ph/0406176] (resource intensive)
- Using quantum machine learning (qGAN) [Zoufal, Lucchi, Woerner; 1904.00043] (not exact)

Loading of distribution / encoding into qubits

Encoding the distribution to be integrated into qubits (coefficients of states)

- Exact loading [Shende, Bullock, Markov, quant-ph/0406176] (resource intensive)
- Using quantum machine learning (qGAN) [Zoufal, Lucchi, Woerner; 1904.00043] (not exact)

- Example: Exact loading - $1 + x^2$



→ 3 qubits: $2^3 = 8$ bins

Integration - $1 + x^2$

- Matching boundary of integration (3 qubits $\Rightarrow 2^3$ bins)

Domain	low stat.		high stat.		very high stat.		exact	
	σ	$\delta[\%]$	σ	$\delta[\%]$	σ	$\delta[\%]$	σ	$\delta[\%]$
$[-0.75; 0]$	0.345	-3.31	0.332	0.706	0.334	0.0331	0.334	-8.31×10^{-3}
$[-0.5; 0]$	0.215	-5.86	0.201	1.15	0.203	0.0986	0.203	-0.0161
$[-0.25; 0]$	0.112	-17.1	0.0939	1.87	0.0960	-0.284	0.0957	-0.0389

Integration - $1 + x^2$

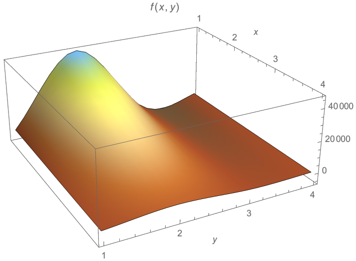
- Matching boundary of integration (3 qubits $\Rightarrow 2^3$ bins)

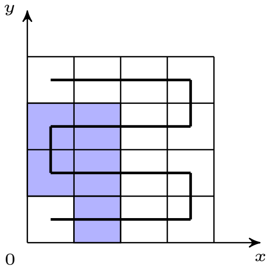
Domain	low stat.		high stat.		very high stat.		exact	
	σ	$\delta[\%]$	σ	$\delta[\%]$	σ	$\delta[\%]$	σ	$\delta[\%]$
$[-0.75; 0]$	0.345	-3.31	0.332	0.706	0.334	0.0331	0.334	-8.31×10^{-3}
$[-0.5; 0]$	0.215	-5.86	0.201	1.15	0.203	0.0986	0.203	-0.0161
$[-0.25; 0]$	0.112	-17.1	0.0939	1.87	0.0960	-0.284	0.0957	-0.0389

- Non-matching boundary of integration

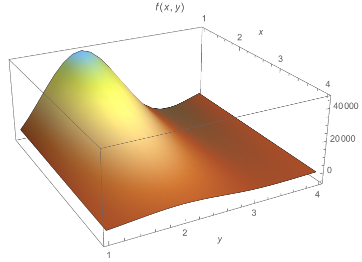
Qubits number	$[-0.7; 0.6]$				$[-0.625; 0.375]$			
	high stat.		exact		high stat.		exact	
	σ	$\delta[\%]$	σ	$\delta[\%]$	σ	$\delta[\%]$	σ	$\delta[\%]$
3	0.402	-28.0	0.406	-27.1	0.296	-28.1	0.299	-27.5
4	0.463	-17.0	0.468	-16.0	0.408	-1.07	0.412	5.96×10^{-3}
5	0.527	-5.46	0.532	-4.62	0.408	-1.07	0.412	5.96×10^{-3}
6	0.542	-2.76	0.547	-1.81	0.408	-1.07	0.412	5.96×10^{-3}

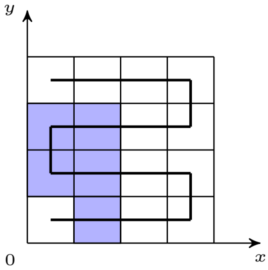
Integration - 2D $[e^+e^- \to \mathbf{q\bar{q}}'W$ with angles fixed$]$





Integration - 2D [$e^+e^- \to \mathbf{q\bar{q}}'W$ with angles fixed]





Qubits number	Grid dim.	$\mathcal{S}_1$ σ	$\mathcal{S}_1$ $\delta[\%]$	$\mathcal{S}_2$ σ	$\mathcal{S}_2$ $\delta[\%]$
4	4×4	0.55	0	0.70	-4.1
5	5×5	0.52	-4.92	0.53	-26.6
6	6×6	0.47	-14.1	0.79	9
6	7×7	0.62	-14.4	0.70	-3.0
6	8×8	0.55	0	0.78	7.6

$\mathcal{S}_1$: matching boundary of integration
$\mathcal{S}_2$: no matching boundary of integration

[Agliardi, Grossi, MP, Prati; 2201.01547]

Working but control of uncertainty crucial!

Remarks

- Use Qiskit (IBM python software) subroutines and noiseless quantum simulation (perfect quantum computer)
- For present application, too many qubits for test on real hardware
  $\rightarrow$ 4 qubits for representation $\rightarrow$ 9 total qubits
  $\rightarrow$ 6 qubits for representation $\rightarrow$ 13 total qubits
- Largest quantum computer on IBM quantum experience:
  7 qubits previously (127 qubits now)
  $\rightarrow$ Simulators can go up to 5000 qubits

Remarks

- Use Qiskit (IBM python software) subroutines and noiseless quantum simulation (perfect quantum computer)
- For present application, too many qubits for test on real hardware
  → 4 qubits for representation → 9 total qubits
  → 6 qubits for representation → 13 total qubits
- Largest quantum computer on IBM quantum experience:
  7 qubits previously (127 qubits now)
  → Simulators can go up to 5000 qubits

Summary

- First application of quantum integration in HEP
- Theoretical quadratic speed-up
- Main challenge: error estimate

[Williams, MP; 2502.14647]

Development roadmap

QUANTINUUM

	2020	2023	2025	2027	2029
SYSTEMS:	SYSTEM MODEL H1	SYSTEM MODEL H2	QUANTINUUM HELIOS	QUANTINUUM SOL	QUANTINUUM APOLLO
PHYSICAL QUBITS:	20	56	96	192	1000's
PHYSICAL 2-QUBIT GATE ERROR:	1×10^{-3}	1×10^{-3}	$< 5 \times 10^{-4}$	$< 2 \times 10^{-4}$	1×10^{-4}
LOGICAL QUBITS:		> 12	~ 50	~ 100	100's
LOGICAL ERROR RATES:		1×10^{-3}	$< 10^{-4}$	$\sim 10^{-5}$	1×10^{-5} to $1 \times 10^{-10*}$

*analysis based on recent literature in new, novel error correcting codes predict that error could be as low as 1E-10 in Apollo (ref: arXiv:2403.16054, arXiv:2308.07915)