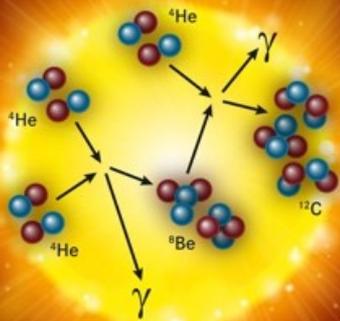


# Modern Theory of Nuclear Forces



Introduction

Chiral perturbation theory

Two-nucleon interactions and chiral symmetry

Chiral EFT as a precision tool

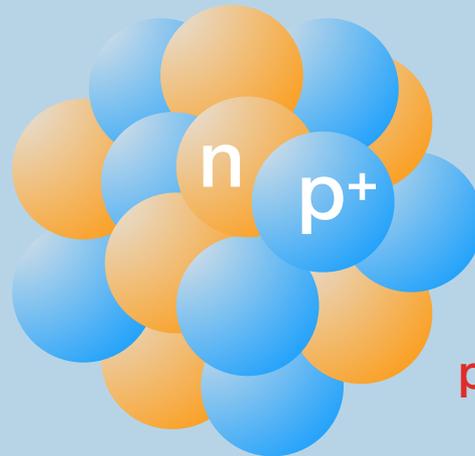
Three-nucleon forces

Summary

Classical physics + gravity:  $F = G \frac{m_1 m_2}{r^2}$

Atom: QM + Coulomb:  $F = k_C \frac{q_1 q_2}{r^2}$

Atomic nucleus: QM/QFT + ??



But what holds  
protons and neutrons  
together?

$\sim 10^{-15}$  m

# The Standard Model: $SU(3)_c \times SU(2)_L \times U(1)$

## Quarks

$u$ up	$c$ charm	$t$ top
$d$ down	$s$ strange	$b$ bottom

## Forces

$Z$ Z boson	$\gamma$ photon
$W$ W boson	$g$ gluon

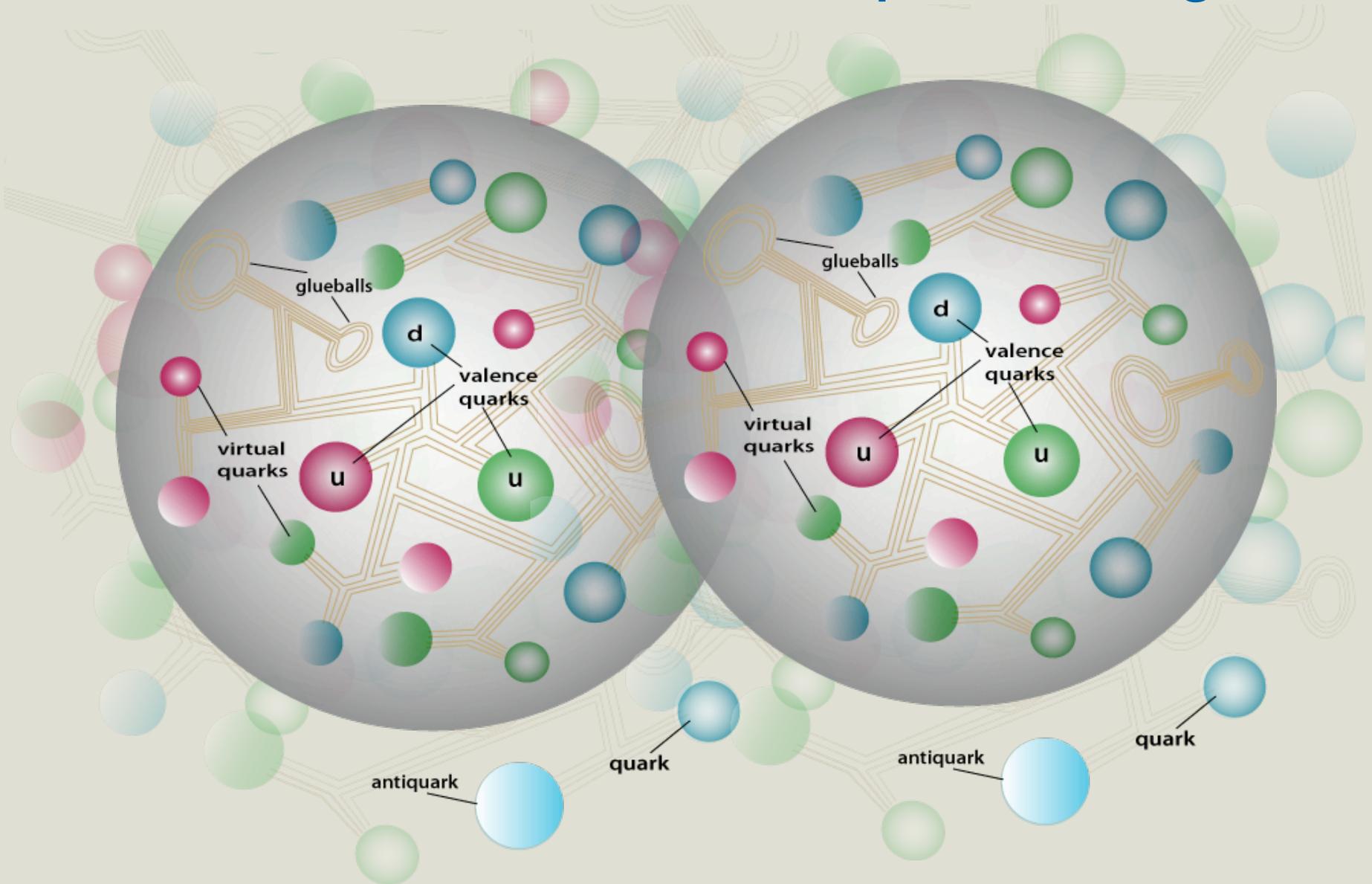
$H$   
Higgs boson

$e$ electron	$\mu$ muon	$\tau$ tau
$\nu_e$ electron neutrino	$\nu_\mu$ muon neutrino	$\nu_\tau$ tau neutrino

## Leptons

# Strong Interaction

# Deuteron as a bound state of quarks and gluons



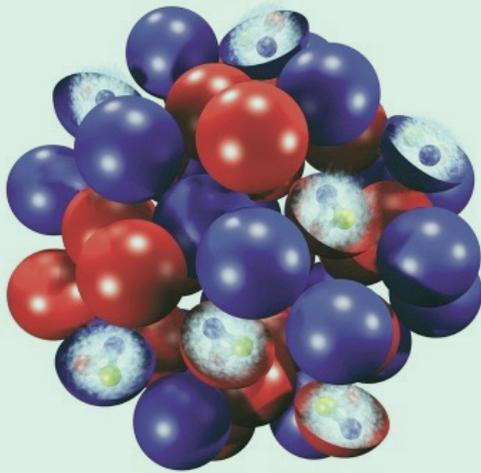
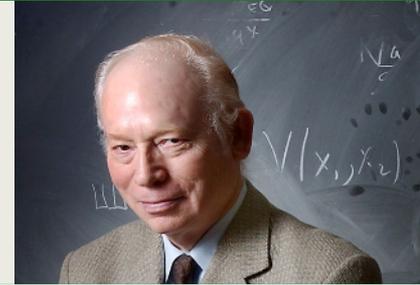
Is there a way to simplify the picture (without losing connection to QCD)?

# Degrees of freedom

Weinberg's 3rd law of progress in theoretical physics:

*You may use any degrees of freedom you like to describe a physical system, but if you use the wrong ones, you will be sorry...*

*in Asymptotic Realms of Physics, MIT Press, Cambridge, 1983*



Typical momenta of nucleons in nuclei:

$$\langle \Psi | \hat{p} | \Psi \rangle \sim 50 - 300 \text{ MeV}$$

Fermi-momentum at the saturation density:

$$p_F = (3/2\pi^2\rho)^{1/3} \sim 270 \text{ MeV}$$

⇒ non-relativistic description in the framework of the **A-body Schrödinger equation**:

$$\left[ \left( \sum_{i=1}^N \frac{-\vec{\nabla}_i^2}{2m} + \mathcal{O}(m^{-3}) \right) + \underbrace{V_{2N} + V_{3N} + V_{4N} + \dots}_{\text{derived in ChPT}} \right] |\Psi\rangle = E |\Psi\rangle$$

# Degrees of freedom

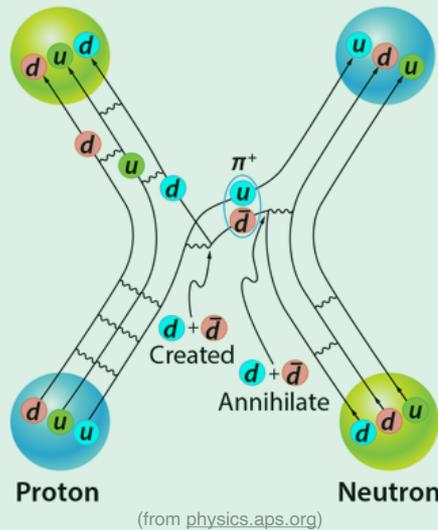
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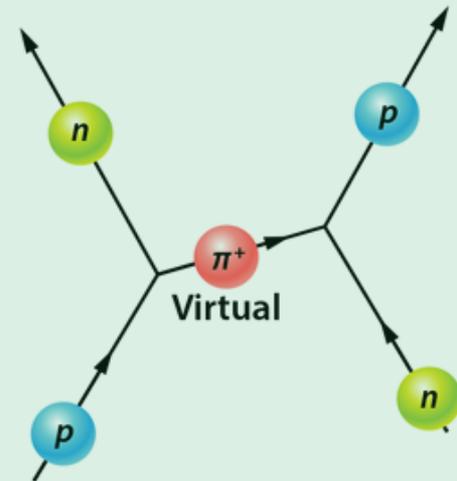
*in Asymptotic Realms of Physics, MIT Press, Cambridge, 1983*



Fundamental degrees of freedom



Effective degrees of freedom



low resolution



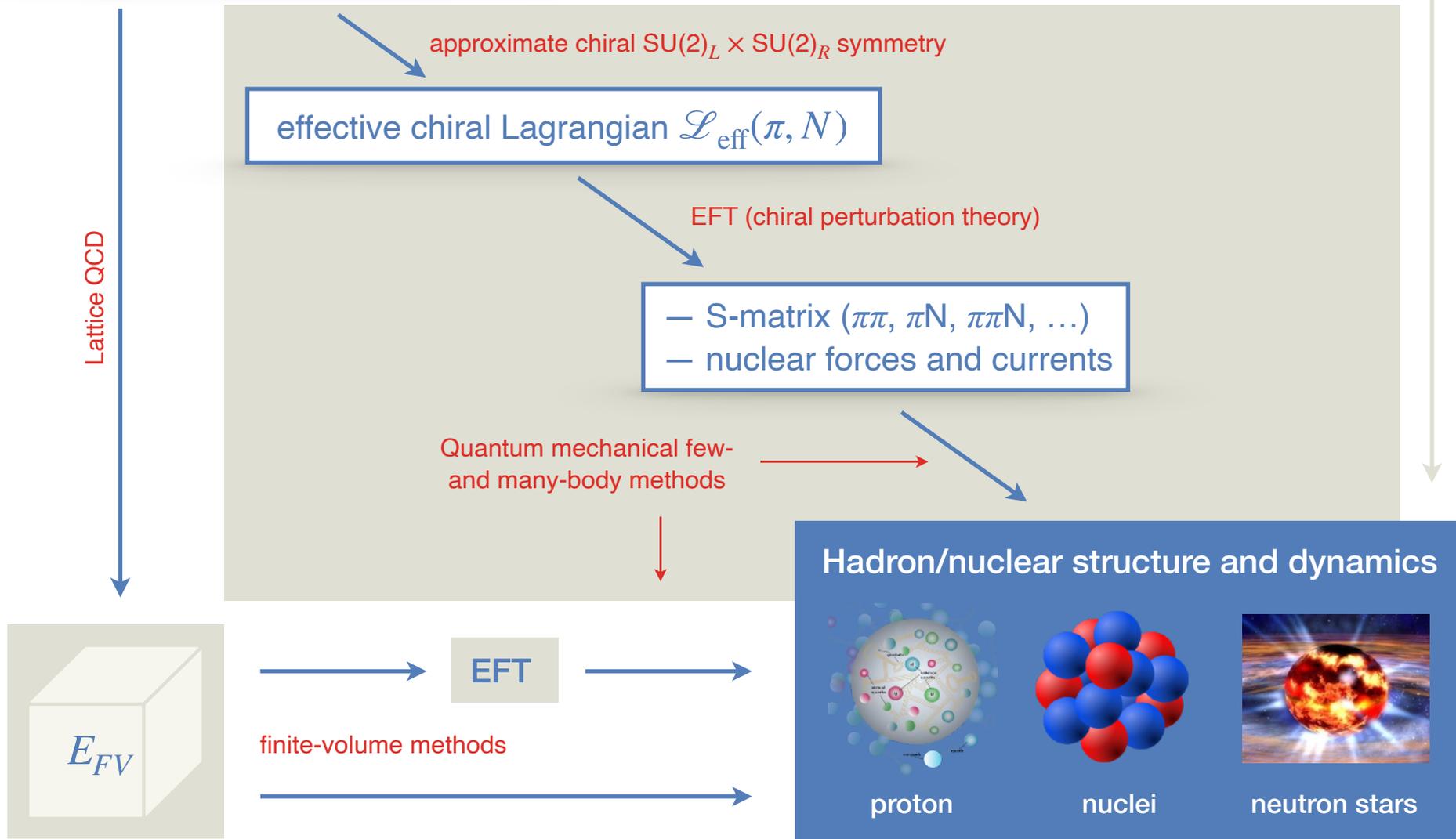
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# From QCD to nuclear physics

The Standard Model (QCD, ...)

Schwinger-Dyson, large- $N_c$ , ...

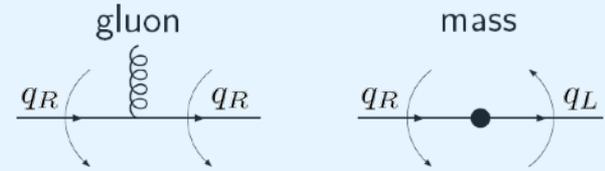


# Chiral perturbation theory

## QCD and the chiral symmetry

$$\begin{aligned} \mathcal{L}_{\text{QCD}} &= -\frac{1}{4}G_a^{\mu\nu}G_{a,\mu\nu} + \bar{q}(i\gamma^\mu D_\mu - \mathcal{M})q \\ &= -\frac{1}{4}G_a^{\mu\nu}G_{a,\mu\nu} + \underbrace{\bar{q}_L i D q_L + \bar{q}_R i D q_R}_{\text{SU}(N_f)_L \times \text{SU}(N_f)_R \text{ invariant}} - \underbrace{q_L \mathcal{M} q_R + q_R \mathcal{M} q_L}_{\text{small for } N_f = 2, (3)} \end{aligned}$$

SSB to  $\text{SU}(N_f)_V \leq \text{SU}(N_f)_L \times \text{SU}(N_f)_R \Rightarrow N_f^2 - 1 \text{ GBs}$



Low-energy QCD dynamics can be described in terms of  $\mathcal{L}_{\text{eff}}[\text{GBs} + \text{matter fields } (N, \Delta, \dots)]$

## Chiral perturbation theory Weinberg, Physica A96 (79) 327; Gasser, Leutwyler, NPB 250 (85) 465; Leutwyler, Annals Phys. 235 (94) 165

QCD in the presence of external sources:  $\mathcal{L} = \mathcal{L}_{\text{QCD}}^0 + \bar{q}(\gamma^\mu \nu_\mu + \gamma_5 \gamma^\mu a_\mu - s - ip)q$

$$\begin{aligned} \langle 0, \text{out} | 0, \text{in} \rangle_{v,a,s,p} &= Z[v, a, s, p] = \int [DG_\mu][Dq][D\bar{q}] e^{i \int d^4x \mathcal{L}(q, \bar{q}, G_{\mu\nu}; v, a, s, p)} \Big|_{\text{low energy}} \\ &= \int \underbrace{[DU]}_{\text{pion fields}} e^{i \int d^4x \mathcal{L}_{\text{eff}}(U; v, a, s, p)} \Big|_{\text{low energy}} \xrightarrow{\text{chiral expansion}} \text{S-matrix} \end{aligned}$$

$Q = \frac{\text{momenta} \sim M_\pi \sim 140 \text{ MeV}}{\text{breakdown scale } \Lambda_b}$

# ChPT for pion-nucleon scattering

Effective chiral Lagrangian:

$$\mathcal{L}_\pi = \mathcal{L}_\pi^{(2)} + \mathcal{L}_\pi^{(4)} + \dots$$

$$\mathcal{L}_{\pi N} = \underbrace{\bar{N} \left( i\gamma^\mu D_\mu[\pi] - m + \frac{g_A}{2} \gamma^\mu \gamma_5 u_\mu[\pi] \right) N}_{\mathcal{L}_{\pi N}^{(1)}} + \underbrace{\sum_i \mathbf{c}_i \bar{N} \hat{O}_i^{(2)}[\pi] N}_{\mathcal{L}_{\pi N}^{(2)}} + \underbrace{\sum_i \mathbf{d}_i \bar{N} \hat{O}_i^{(3)}[\pi] N}_{\mathcal{L}_{\pi N}^{(3)}} + \underbrace{\sum_i \mathbf{e}_i \bar{N} \hat{O}_i^{(4)}[\pi] N}_{\mathcal{L}_{\pi N}^{(4)}} + \dots$$

low-energy constants

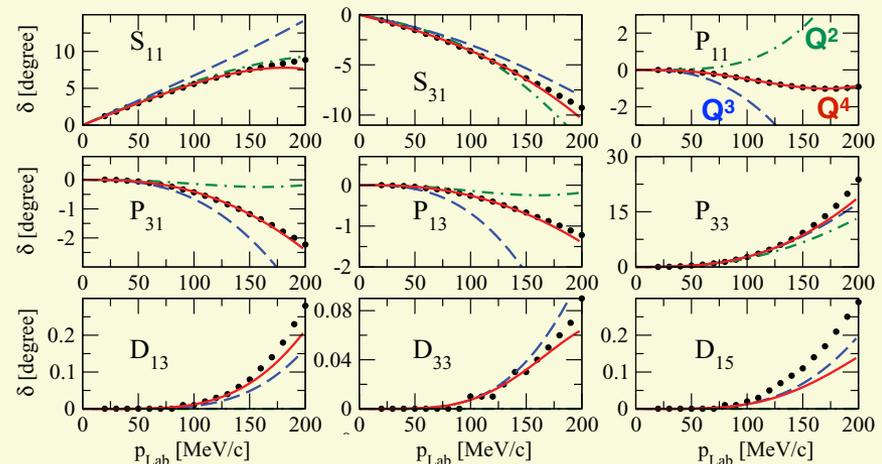
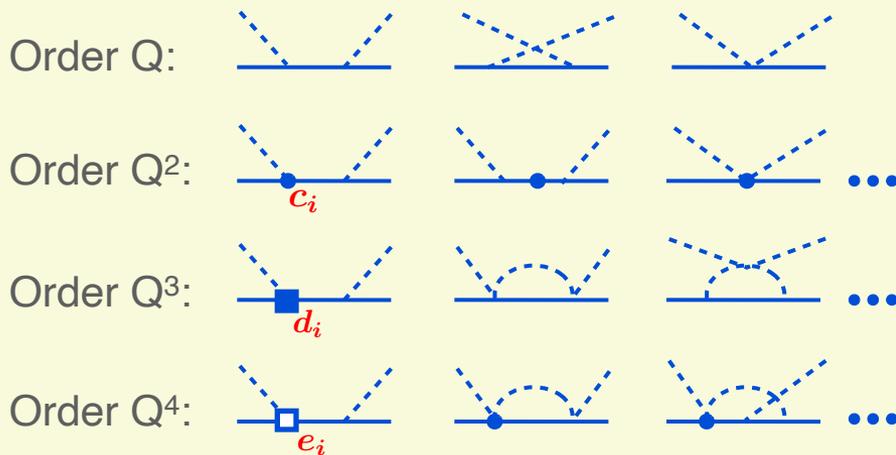
Pion-nucleon scattering amplitude for  $\pi^a(q_1) + N(p_1) \rightarrow \pi^b(q_2) + N(p_2)$ :

$$T_{\pi N}^{ba} = \frac{E + m}{2m} \left( \delta^{ba} \left[ \underbrace{g^+(\omega, t)}_{\uparrow} + i\vec{\sigma} \cdot \vec{q}_2 \times \vec{q}_1 \underbrace{h^+(\omega, t)}_{\uparrow} \right] + i\epsilon^{bac} \tau^c \left[ \underbrace{g^-(\omega, t)}_{\uparrow} + i\vec{\sigma} \cdot \vec{q}_2 \times \vec{q}_1 \underbrace{h^-(\omega, t)}_{\uparrow} \right] \right)$$

calculated by means of the chiral expansion

## Pion-nucleon scattering up to $Q^4$ in heavy-baryon ChPT

Fettes, Meißner '00; Krebs, Gasparyan, EE '12



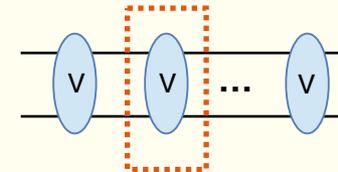
# Chiral EFT for nuclear systems



For few-N, ladder diagrams are enhanced and must be re-summed Weinberg '90, '91

$$T = V + VG_0V + VG_0VG_0V + \dots = V + VG_0T$$

$$G_0 = \frac{m}{\vec{k}^2 - \vec{p}^2 + i\epsilon}$$



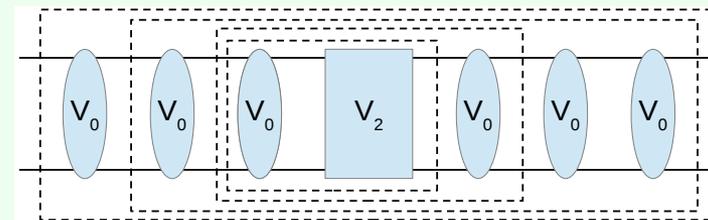
$\Sigma$  few-nucleon irreducible diagrams (no enhancement, ChPT)

(divergent integrals in the Lippmann-Schwinger equation are usually regularized with a cutoff  $\Lambda$ )

**Finite-cutoff EFT** ( $\Lambda \sim \Lambda_b \sim 600 \text{ MeV}$ ) Lepage, EE, Gegelia, Meißner, Reinert, Entem, Machleidt, ...

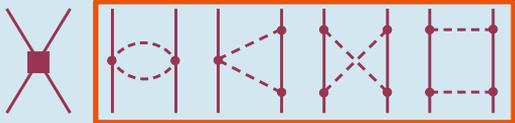
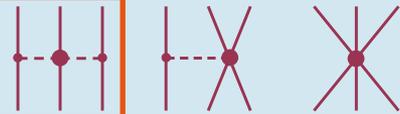
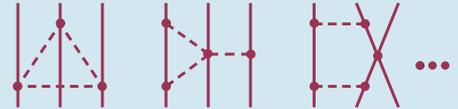
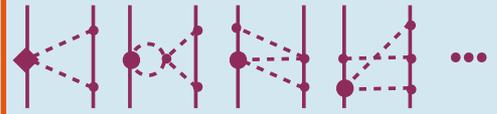
- implicit renormalization (achieved by tuning bare LECs to data)
- approximate  $\Lambda$ -independence of calculated observables has to be verified a posteriori
- **explicit proof of renormalizability** (in the EFT sense) has been given to NLO using the BPHZ formalism

Ashot Gasparyan, EE, PRC 105 (22); PRC 107 (23)



# Chiral expansion of the nuclear forces

(NDA, chiral EFT with pions and nucleons as the only DoF)

	Two-nucleon force	Three-nucleon force	Four-nucleon force
LO ( $Q^0$ )	 <p>Weinberg '90</p>		
NLO ( $Q^2$ )			
N <sup>2</sup> LO ( $Q^3$ )	 <p>van Kolck et al. '94 Friar, Coon '94 Kaiser et al. '97 Epelbaum et al. '98</p>	 <p>van Kolck '94; Epelbaum et al. '02</p>	
N <sup>3</sup> LO ( $Q^4$ )	 <p>Kaiser '00-'02</p>	 <p>Bernard, Epelbaum, Krebs, Meißner '08, '11</p>	 <p>Epelbaum '06, '07</p>
N <sup>4</sup> LO ( $Q^5$ )	 <p>Entem, Kaiser, Machleidt, Nösyk '15</p>	 <p>Girlanda et al. '11, Krebs et al. '11, '13</p>	

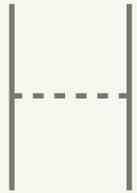
Explains the observed hierarchy of nuclear forces Weinberg, van Kolck, Friar

Chiral dynamics: Long-range interactions are predicted in terms of on-shell amplitudes



# Chiral symmetry and nuclear interactions

Chiral symmetry +  $\pi N$  data = predictions for the large-distance behavior of the nuclear forces.



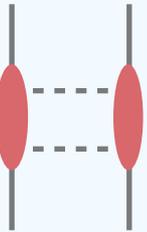
$$\mathcal{L}_{pv} = -\frac{g}{2m_N} \bar{N} \gamma^5 \gamma^\mu \tau N \cdot \partial_\mu \pi$$

~~$$\mathcal{L}_{ps} = -ig \bar{N} \gamma^5 \tau N \cdot \pi$$~~



⇒ the same OPEP (on-shell)

i.e., **not** constrained by  $\chi$  symmetry...



← strongly constrained by  $\chi$  symmetry:  
 $\mathcal{L}_{ps}$  vs.  $\mathcal{L}_{pv}$  matters, also  $\pi\pi$ ,  $\pi\pi N$ ,  
 etc interactions play a role...

Dispersive representation:

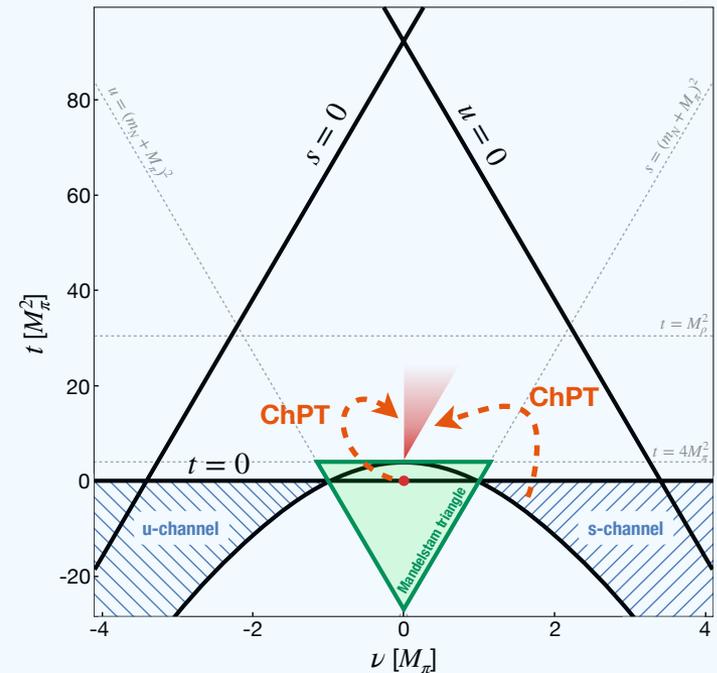
$$V_{2\pi}(q) = \frac{2}{\pi} \int_{2M_\pi}^{\infty} \mu d\mu \frac{\rho(\mu)}{q^2 + \mu^2} + \dots$$

$\rho(\mu)$  can be extracted from (analytically continued)

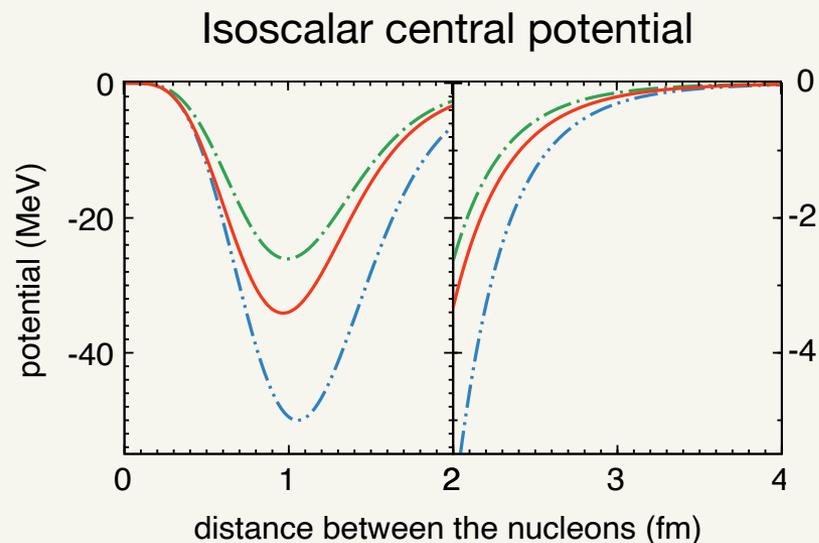
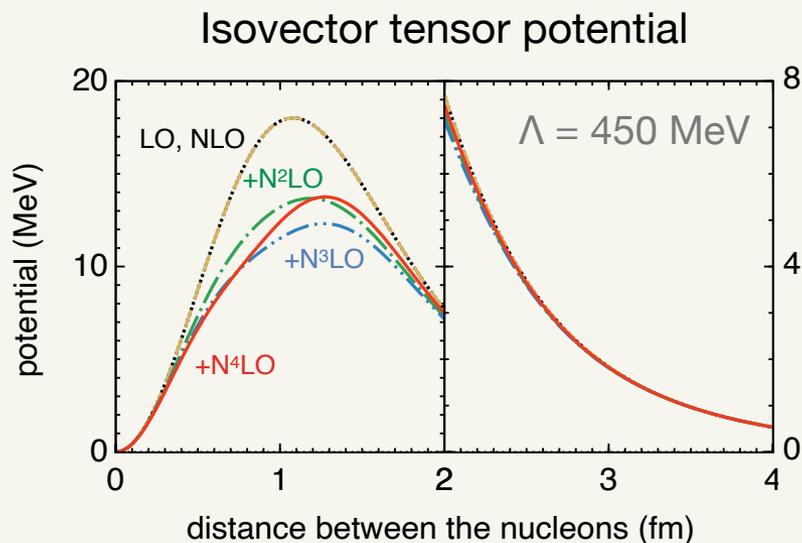
$T_{\pi N}(s, t)$ , which can be calculated in ChPT

⇒ parameter-free predictions for  $V_{2\pi}(r)$  at  $r \gtrsim M_\pi^{-1}$

Mandelstam plane for  $\pi N$  scattering



# Chiral expansion of the multi-pion exchange



- Long-distance behavior of the NN force is a **parameter-free prediction of chiral EFT**
- Agrees with phenomenology (strong intermediate-range attraction from TPEP)
- Reasonable convergence of the chiral expansion (at large  $r$ )
- Short-range interactions parametrized by contacts

Regularization EE, Krebs, Meißner EPJA 51 (15); Phys. Rev. Lett. 115 (15); Reinert, Krebs, EE, EPJA 54 (18)

$$V_{1\pi}(q) = \frac{\alpha}{\vec{q}^2 + M_\pi^2} e^{-\frac{\vec{q}^2 + M_\pi^2}{\Lambda^2}} + \text{subtraction,}$$

$$V_{2\pi}(q) = \frac{2}{\pi} \int_{2M_\pi}^{\infty} d\mu \mu \frac{\rho(\mu)}{\vec{q}^2 + \mu^2} e^{-\frac{\vec{q}^2 + \mu^2}{2\Lambda^2}} + \text{subtractions}$$

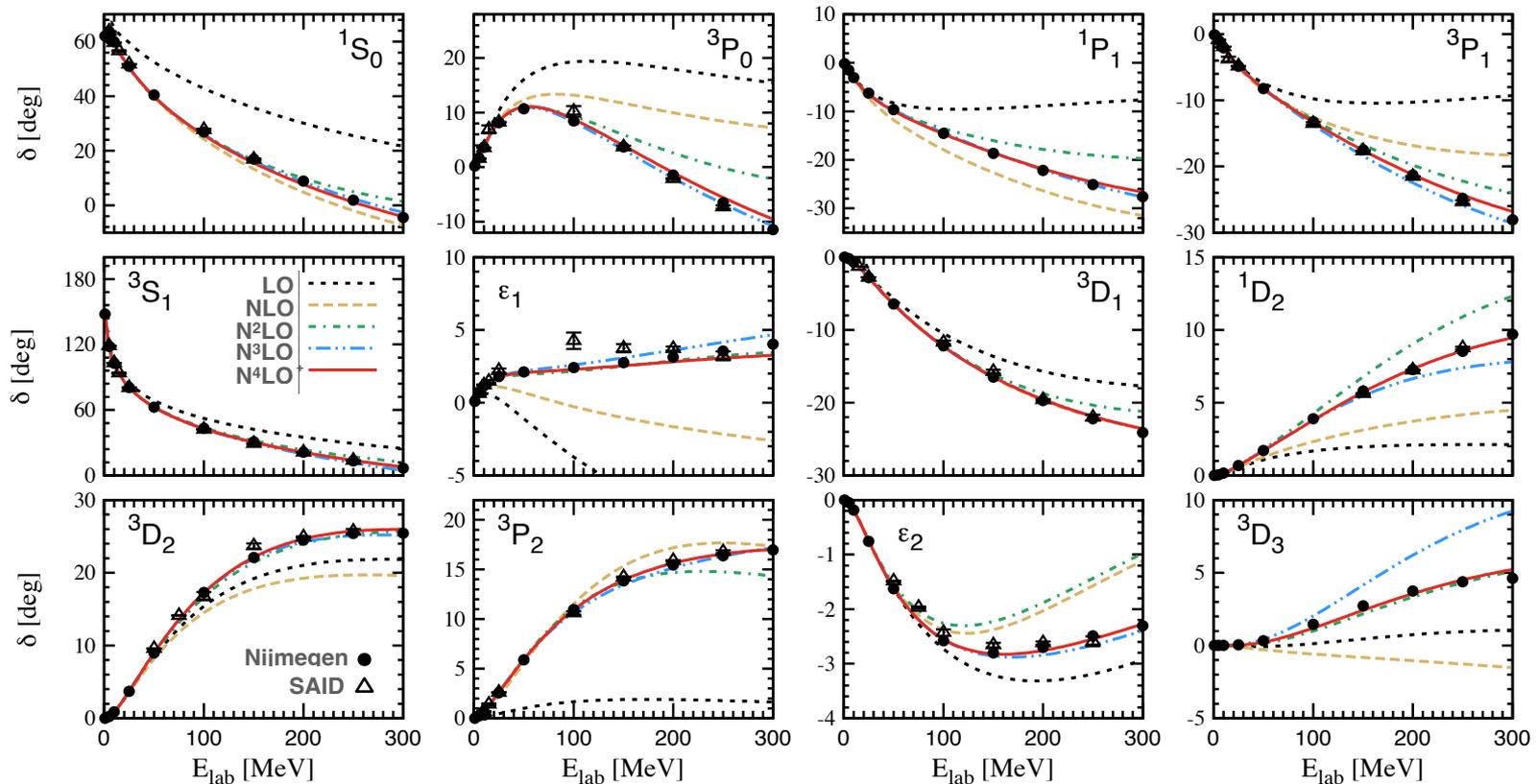
# The two-nucleon system

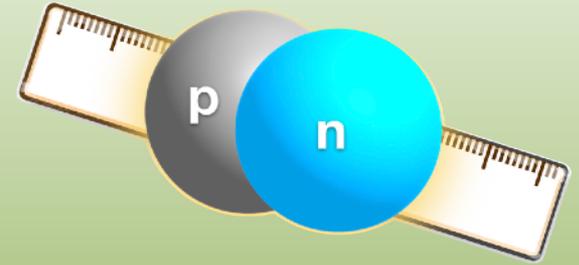
Results for  $\Lambda = 450$  MeV

from: P. Reinert, H. Krebs, EE, EPJA 54 (2018) 88

	LO ( $Q^0$ )	NLO ( $Q^2$ )	N <sup>2</sup> LO ( $Q^3$ )	N <sup>3</sup> LO ( $Q^4$ )	N <sup>4</sup> LO ( $Q^5$ )	N <sup>4</sup> LO <sup>+</sup>
$\chi^2/\text{datum}$ (np, 0 – 300 MeV)	75	14	4.1	2.01	1.16	1.06
$\chi^2/\text{datum}$ (pp, 0 – 300 MeV)	1380	91	41	3.43	1.67	1.00
	2 LECs	+ 7 + 1 IB LECs		+ 12 LECs	+ 1 LEC (np)	+ 4 LECs

Chiral expansion of the neutron-proton phase shifts [ $\Lambda = 450$  MeV]





# Two nucleons: Chiral EFT as a precision tool

- High-precision determination of the  $\pi N$  coupling constants [Reinert, Krebs, EE, PRL 126 \(21\) 092501](#)
- Deuteron structure radius and the neutron size [Filin, Baru, EE, Krebs, Möller, Reinert, PRL 124 \(20\) 082501](#)

# The pion-nucleon coupling constants

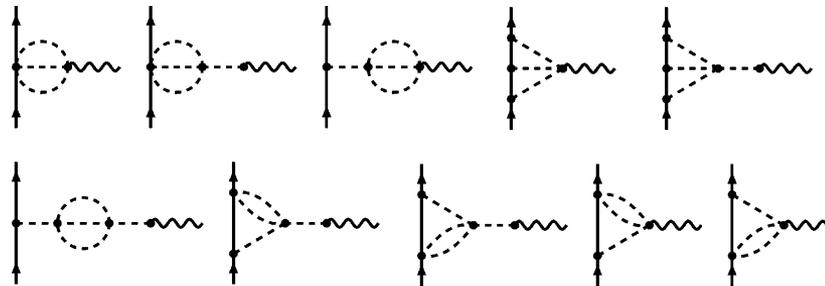
Reinert, Krebs, EE, Phys. Rev. Lett. 126 (2021) 9, 092501

— in the isospin limit:  $\langle N(p') | A_i^\mu(0) | N(p) \rangle = \bar{u}(p) \left[ \underbrace{\gamma^\mu G_A(t)}_{\text{axial form factor}} + \frac{p'^\mu - p^\mu}{2m_N} \underbrace{G_P(t)}_{\text{induced pseudoscalar form factor}} \right] \gamma_5 \frac{\tau_i}{2} u(p)$

— axial FF: a smooth function near  $t = 0$ ; axial charge:  $g_A \equiv G_A(0) = 1.2723(23)$  [PDG]

Goldberger-Treiman relation:  $F_\pi g_{\pi NN} = g_A m_N (1 + \Delta_{GT})$

— induced pseudoscalar FF:  $G_P(t) = 4m_N \frac{g_{\pi NN} F_\pi}{M_\pi^2 - t} - \underbrace{\frac{2}{3} g_A m_N^2 r_A^2}_{\text{non-pole terms}} + \mathcal{O}(t, M_\pi^2)$   
 BKM'94; Kaiser '03



# The pion-nucleon coupling constants

Reinert, Krebs, EE, Phys. Rev. Lett. 126 (2021) 9, 092501

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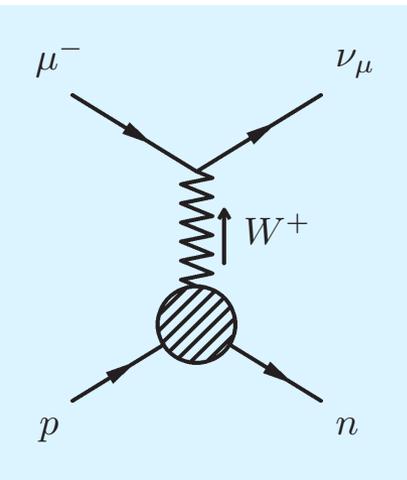
– pseudoscalar coupling:  $\bar{g}_P = \frac{M_\mu}{2m_N} G_P(t) \Big|_{-0.877M_\mu^2}$

MuCap@PSI:  $\Lambda_{\text{singlet}}^{\text{MuCap}} = 715.6(7.4) \text{ s}^{-1}$

$r_A^2 = 0.453(23) \text{ fm}^2 \Rightarrow \bar{g}_P^{\text{MuCap}} = 8.26(49), \quad \bar{g}_P^{\text{ChPT}} = 8.25(7)$  [Hill et al. '18]  
Bodek et al. '08

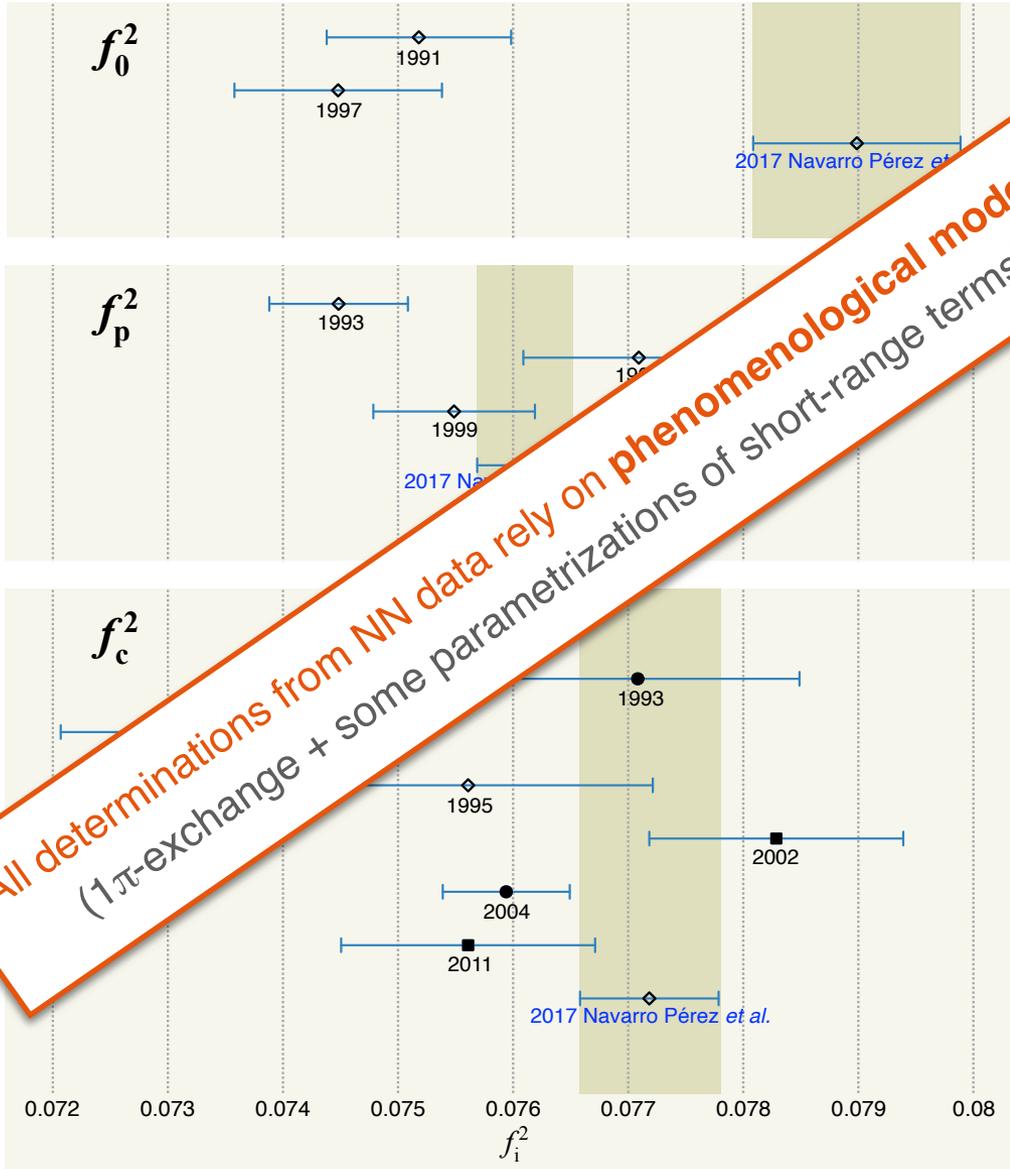
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Meyer et al. '16

calculated using  $g_{\pi NN} = 13.12(10)$  [Baru et al. '11]



# Determination of the $\pi N$ constants

Reinert, Krebs, EE, Phys. Rev. Lett. 126 (2021) 9, 092501



Standard notation ( $f_{\pi NN} = \frac{M_{\pi^\pm}}{2\sqrt{4\pi m_N}} g_{\pi NN}$ ):

$$f_0^2 = -f_{\pi^0 nn} f_{\pi^0 pp}$$

$$f_p^2 = f_{\pi^0 pp} f_{\pi^0 pp}$$

$$2f_c^2 = f_{\pi^\pm pn} f_{\pi^\pm pn}$$

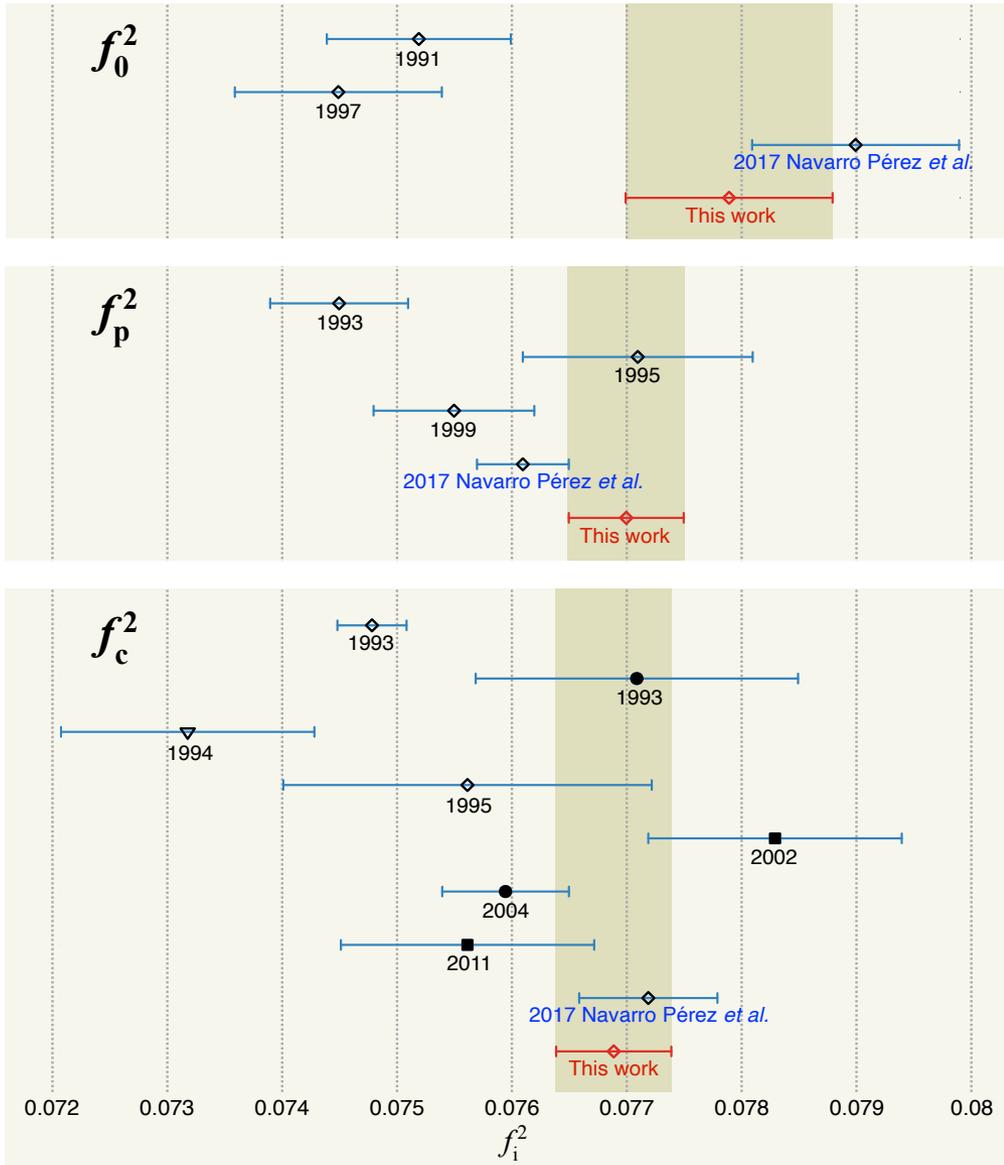
2017 Granada PWA: claimed to find significant charge dependence of the coupling constants:

$$f_0^2 - f_p^2 = 0.0029(10)$$

Navarro Perez et al., PRC 95 (2017) 6, 064001

# Determination of the $\pi N$ constants

Reinert, Krebs, EE, Phys. Rev. Lett. 126 (2021) 9, 092501



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Navarro Perez *et al.*, PRC 95 (2017) 6, 064001

**Our result ( $\chi$ EFT at  $N^4$ LO):**

Bayesian determination; statistical **and systematic** uncertainties.

No evidence for charge dependence of the  $\pi N$  coupling constants

Reinert, Krebs, EE, Phys. Rev. Lett. 126 (2021) 092501

# Determination of the $\pi N$ constants

Reinert, Krebs, EE, Phys. Rev. Lett. 126 (2021) 9, 092501

Our  $g_{\pi NN}$  value corresponding to  $f_c^2$  reads:

$$g_{\pi NN} = 13.23 \pm 0.04$$

Pionic hydrogen exp. at PSI (GMO sum rule)

[Hirtl et al., Eur. Phys. J. A57 (2021) 2, 70]

$$\epsilon_{1s}^{\pi H} + \epsilon_{1s}^{\pi D} : g_{\pi NN} = 13.10 \pm 0.10$$

$$\Gamma_{1s}^{\pi H} : g_{\pi NN} = 13.24 \pm 0.10$$

Impact on the theoretical value of  $\bar{g}_P$ :

$$r_A^2 = 0.453(23) \text{ fm}^2 \Rightarrow \bar{g}_P^{\text{ChPT}} = 8.25(7) \rightarrow 8.33(4)$$

Bodek et al. '08

$$r_A^2 = 0.46(22) \text{ fm}^2 \Rightarrow \bar{g}_P^{\text{ChPT}} = 8.25(25) \rightarrow 8.32(25)$$

Meyer et al. '16

Both values are still in a very good agreement with the MuCap datum:  $\bar{g}_P^{\text{MuCap}} = 8.26(49)$  or  $8.23(83)$

Standard notation ( $f_{\pi NN} = \frac{M_{\pi^\pm}}{2\sqrt{4\pi m_N}} g_{\pi NN}$ ):

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Bayesian determination; statistical and systematic uncertainties.

No evidence for charge dependence of the  $\pi N$  coupling constants

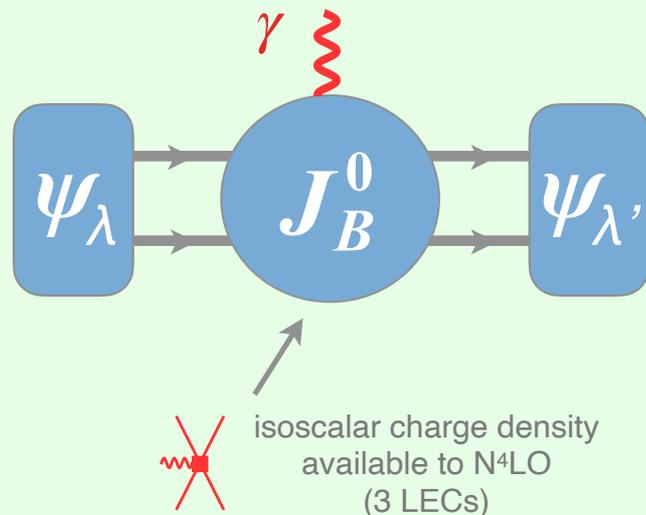
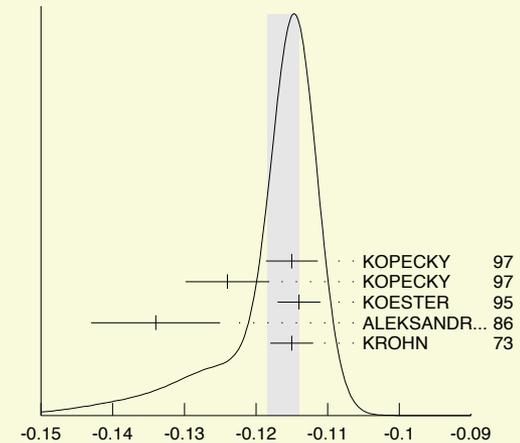
Reinert, Krebs, EE, Phys. Rev. Lett. 126 (2021) 092501

# How big is a neutron?

The proton radius puzzle solved. **What about the neutron radius?**

- no neutron targets; extrapolations of  $G_C^n(Q^2)$  extracted from  $^2\text{H}$  not reliable...
- the only information comes from (fairly old) n-scattering experiments on Pb, Bi, ...

→ PDG value:  $r_n^2 = -0.1161 \pm 0.0022 \text{ fm}^2$



**Idea: accurate calculation of the  $^2\text{H}$  structure radius,** which incorporates all nuclear effects

$$r_d^2 = r_{str}^2 + \left( r_p^2 + \frac{3}{4m_p^2} \right) + r_n^2$$

combined with  $^1\text{H}$ - $^2\text{H}$  isotope shifts data

$$r_d^2 - r_p^2 = 3.82070(31) \text{ fm}^2$$

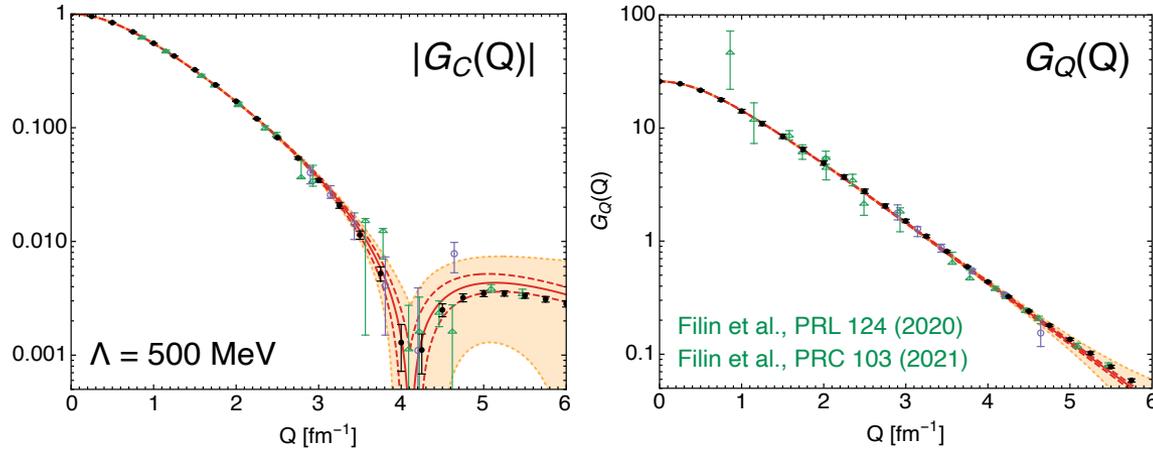
Jentschura et al. '11; Pachucki et al. '18

**can be used to extract  $r_n^2$  !**

# Deuteron charge and quadrupole FFs

Filin, Möller, Baru, EE, Krebs, Reinert, PRL 124 (2020) 082501; PRC 103 (2021) 024313

## The charge and quadrupole form factors of the deuteron at N<sup>4</sup>LO



The extracted structure radius and quadrupole moment:

$$r_{\text{str}} = 1.9729^{+0.0015}_{-0.0012} \text{ fm}$$

$$Q_d = 0.2854^{+0.0038}_{-0.0017} \text{ fm}^2$$

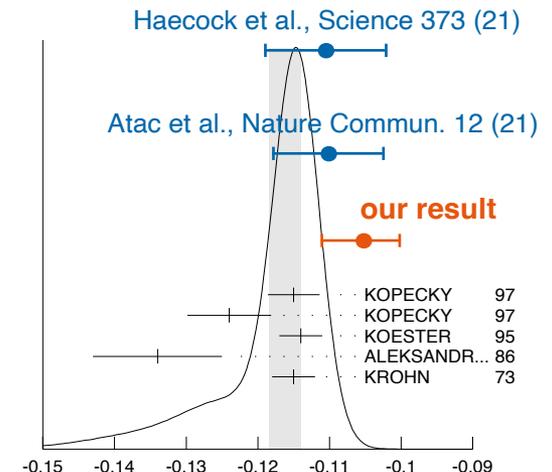
statistical and systematic errors due to the EFT truncation, choice of fitting range and  $\pi$ N LECs

The value of  $Q_d$  is to be compared with  $Q_d^{\text{exp}} = 0.285\,699(15)(18) \text{ fm}^2$  Puchalski et al., PRL 125 (2020)

Combining our result for  $r_{\text{str}}^2 = r_d^2 - r_p^2 - r_n^2 - \frac{3}{4m_p^2}$  with the

$^1\text{H}$ - $^2\text{H}$  isotope shift datum  $r_d^2 - r_p^2 = 3.82070(31) \text{ fm}^2$  leads to the prediction for the neutron radius:

$$r_n^2 = -0.105^{+0.005}_{-0.006} \text{ fm}^2$$



# Three-nucleon force

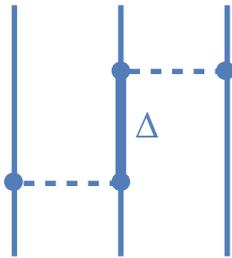


# 3-body force: A frontier in nuclear & atomic physics

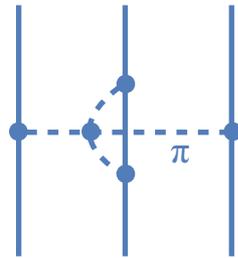
Endo, EE, Naidon, Nishida, Sekiguchi, Takahashi, EPJA 61 (2025) 9

- Three-nucleon forces (3NF) are small **but important** corrections to the dominant NN forces

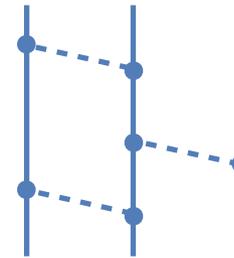
- 3NF mechanisms:



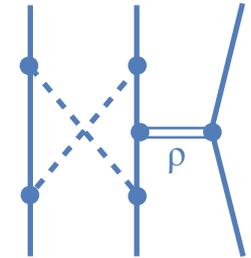
intermediate  $\Delta$ -excitation  
Fujita, Miyazawa '57



multi-pion interactions



off-shell behavior of the  $V_{NN}$   
 $V_{\text{ring}} = \mathcal{A}_{3\pi} - V_{\pi} G_0 V_{\pi} G_0 V_{\pi}$



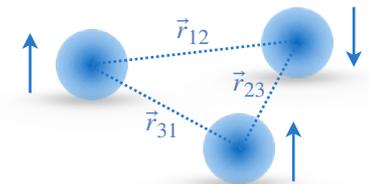
short-range

⇒ 3NF are not directly measurable and depend on the scheme (DoF, off-shell  $V_{NN}$ , ...)

- 3NF have extremely rich and complex structure

– most general **local** 3NF:  $V_{3N} = \sum_{i=1}^{20} O_i f_i(r_{12}, r_{23}, r_{31}) + \text{permutations}$   
EE, Gasparyan, Krebs, Schat '15

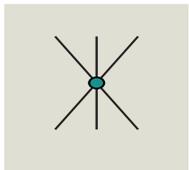
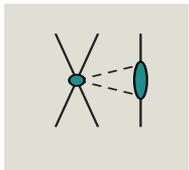
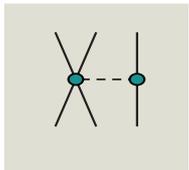
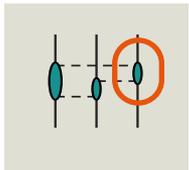
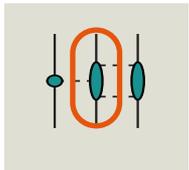
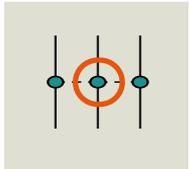
– most general **nonlocal** 3NF: **320 (!)** operators Topolnicki '17



⇒ Guidance from theory indispensable — an opportunity for  $\chi$ EFT!

# 3-body force: A frontier in nuclear & atomic physics

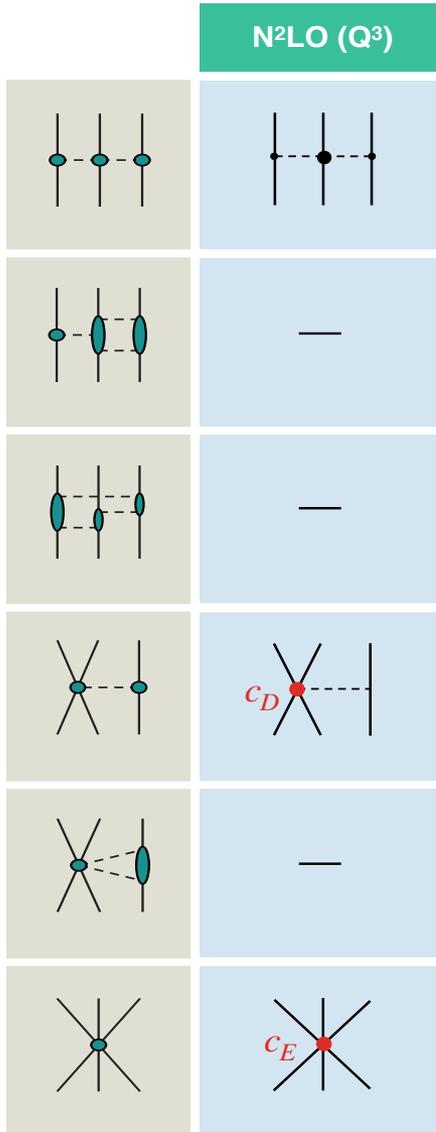
Endo, EE, Naidon, Nishida, Sekiguchi, Takahashi, EPJA 61 (2025) 9



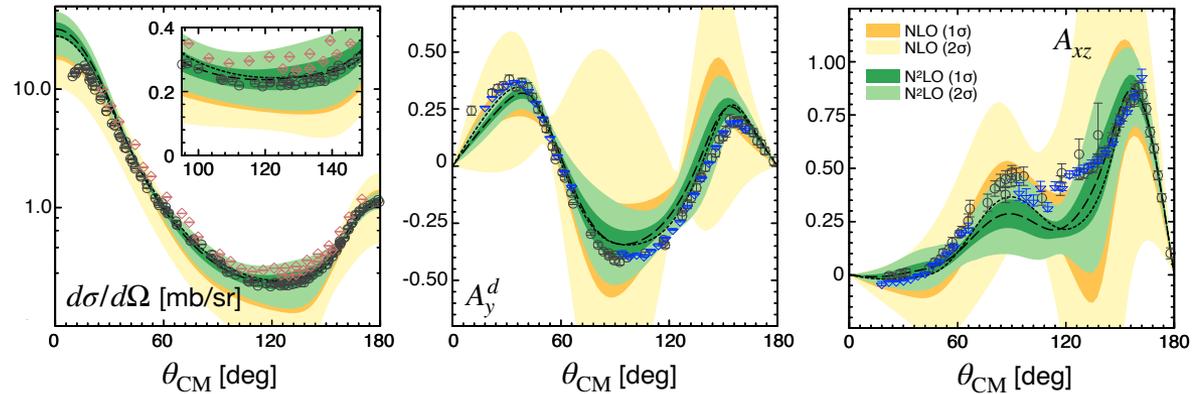
3N potentials at large distance are  
model-independent and parameter-free predictions  
based on  $\chi$  symmetry of QCD + exp. information on  $\pi$ N system

# 3-body force: A frontier in nuclear & atomic physics

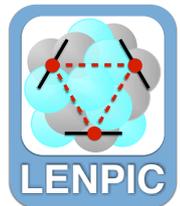
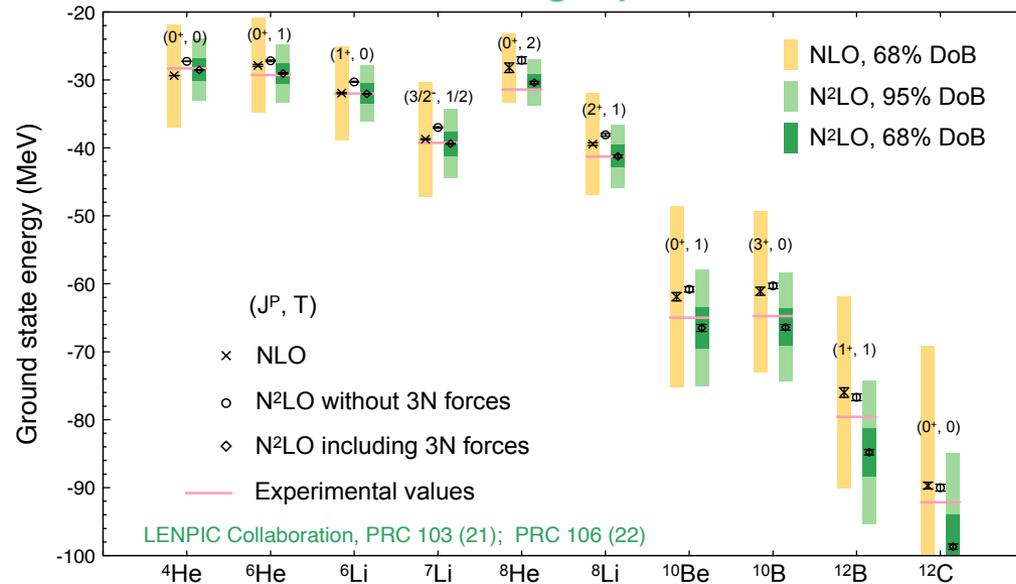
Endo, EE, Naidon, Nishida, Sekiguchi, Takahashi, EPJA 61 (2025) 9



## Elastic Nd scattering at 135 MeV



## Predictions for light p-shell nuclei

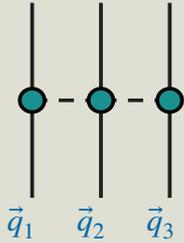


# 3-body force: A frontier in nuclear & atomic physics

Endo, EE, Naidon, Nishida, Sekiguchi, Takahashi, EPJA 61 (2025) 9

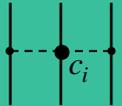
	N <sup>2</sup> LO (Q <sup>3</sup> )	N <sup>3</sup> LO (Q <sup>4</sup> )	N <sup>4</sup> LO (Q <sup>5</sup> )
		+ ... Ishikawa, Robilotta '08; Bernard, EE, Krebs, Meißner '08	+ ... Krebs, Gasparyan, EE '12
	—	+ ... Bernard, EE, Krebs, Meißner '08	+ ... Krebs, Gasparyan, EE '13
	—	+ ... Bernard, EE, Krebs, Meißner '08	+ ... Krebs, Gasparyan, EE '13
	$c_D$	+ ... Bernard, EE, Krebs, Meißner '11	+ ...
	—	+ ... Bernard, EE, Krebs, Meißner '11	+ ...
	$c_E$	—	<b>13 LECs</b> Girlanda, Kievski, Viviani '11

# Example: $2\pi$ -exchange 3NF



$$V_{3N} = \frac{\vec{\sigma}_1 \cdot \vec{q}_1 \vec{\sigma}_3 \cdot \vec{q}_3}{(q_1^2 + M_\pi^2)(q_3^2 + M_\pi^2)} \left[ \tau_1 \cdot \tau_3 \mathcal{A}(q_2) + \tau_1 \times \tau_3 \cdot \tau_2 \vec{q}_1 \times \vec{q}_3 \cdot \vec{\sigma}_2 \mathcal{B}(q_2) \right] + \text{short-range terms} + \text{permutations}$$

**N<sup>2</sup>LO (Q<sup>3</sup>)**



$$\mathcal{A}^{(3)} = \frac{g_A^2}{8F_\pi^4} \left[ (2c_3 - 4c_1)M_\pi^2 + c_3q_2^2 \right], \quad \mathcal{B}^{(3)} = \frac{g_A^2 c_4}{8F_\pi^4}$$

**N<sup>3</sup>LO (Q<sup>4</sup>)**

Bernard, EE, Krebs, Meißner '08



$$\mathcal{A}^{(4)} = \frac{g_A^4}{256\pi F_\pi^6} \left[ (4g_A^2 + 1) M_\pi^3 + 2(g_A^2 + 1) M_\pi q_2^2 + A(q_2) (2M_\pi^4 + 5M_\pi^2 q_2^2 + 2q_2^4) \right]$$

$$\mathcal{B}^{(4)} = -\frac{g_A^4}{256\pi F_\pi^6} \left[ A(q_2) (4M_\pi^2 + q_2^2) + (2g_A^2 + 1) M_\pi \right]$$

$\uparrow \frac{1}{2q_2} \arctan \frac{q_2}{2M_\pi}$

calculated using DimReg

**N<sup>4</sup>LO (Q<sup>5</sup>)**

Krebs, Gasparyan, EE '12



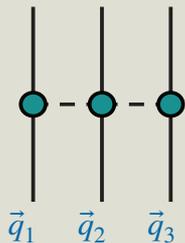
$$\mathcal{A}^{(5)} = \frac{g_A^2 (M_\pi^2 + 2q_2^2)}{4608\pi^2 F_\pi^6} \left\{ [6c_1 - 2c_2 - 3c_3 - 2(6c_1 - c_2 - 3c_3)L(q_2)] 12M_\pi^2 - q_2^2 [5c_2 + 18c_3 - 6L(q_2)(c_2 + 6c_3)] \right\} + \frac{g_A^2 \bar{e}_{14}}{2F_\pi^4} (2M_\pi^2 + q_2^2)^2$$

$\uparrow \frac{\sqrt{q_2^2 + 4M_\pi^2}}{q_2} \log \frac{\sqrt{q_2^2 + 4M_\pi^2} + q_2}{2M_\pi}$

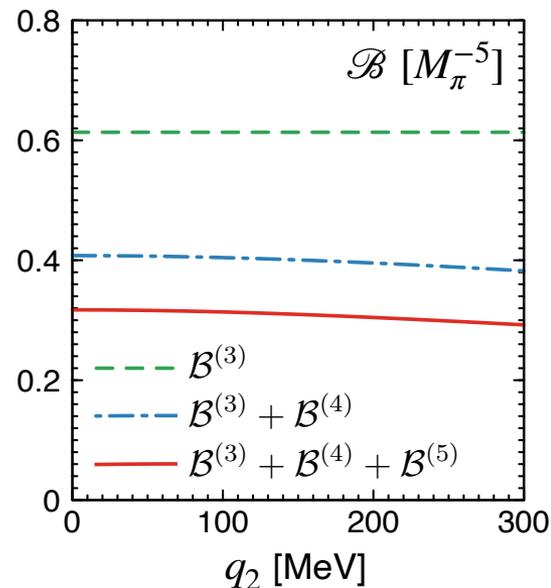
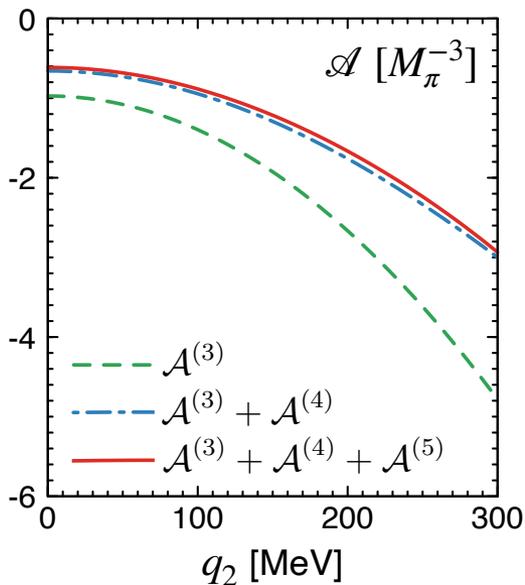
$$\mathcal{B}^{(5)} = \frac{g_A^2 \bar{e}_{17}}{2F_\pi^4} (2M_\pi^2 + q_2^2) - \frac{g_A^2 c_4}{2304\pi^2 F_\pi^6} \left\{ q_2^2 [5 - 6L(q_2)] + 12M_\pi^2 [2 + 9g_A^2 - 2L(q_2)] \right\}$$

calculated using DimReg

# Example: $2\pi$ -exchange 3NF



$$V_{3N} = \frac{\vec{\sigma}_1 \cdot \vec{q}_1 \vec{\sigma}_3 \cdot \vec{q}_3}{(q_1^2 + M_\pi^2)(q_3^2 + M_\pi^2)} \left[ \tau_1 \cdot \tau_3 \mathcal{A}(q_2) + \tau_1 \times \tau_3 \cdot \tau_2 \vec{q}_1 \times \vec{q}_3 \cdot \vec{\sigma}_2 \mathcal{B}(q_2) \right] + \text{short-range terms} + \text{permutations}$$



— the results are only meaningful (converged) at small momenta  $\Rightarrow$  cutoff needed

Problem: Mixing DimReg with Cutoff violates chiral symmetry

**DANGER:** momentum cutoff for pions breaks chiral symmetry!



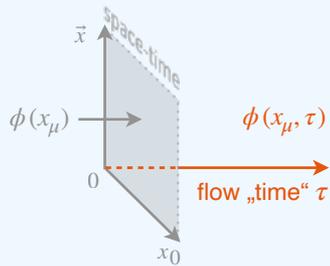
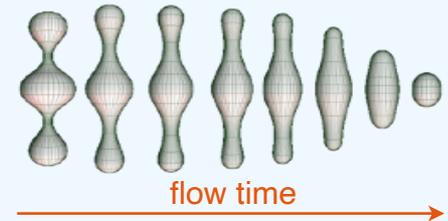
# Chiral gradient flow

Krebs, EE, PRC 110 (2024) 044004

Gradient flows: methods for smoothing manifolds

(e.g., Ricci flow used in the proof of the Poincaré conjecture)

Gradient flow as a regulator in field theory



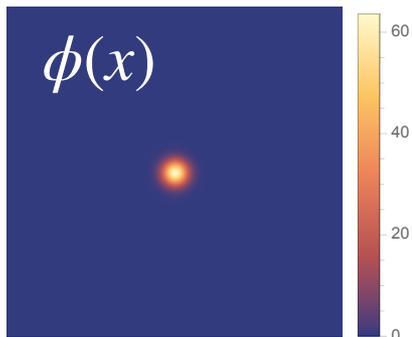
$$\text{Flow equation: } \frac{\partial}{\partial \tau} \phi(x, \tau) = - \left. \frac{\delta S[\phi]}{\delta \phi(x)} \right|_{\phi(x) \rightarrow \phi(x, \tau)}$$

subject to the boundary condition  $\phi(x, 0) = \phi(x)$

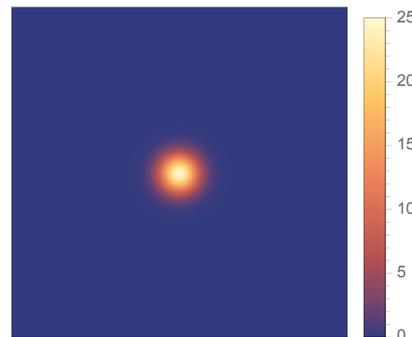
Free scalar field:

$$G(x, \tau) = \frac{\theta(\tau)}{16\pi^2 \tau^2} e^{-\frac{x^2 + 4M^2 \tau^2}{4\tau}}$$

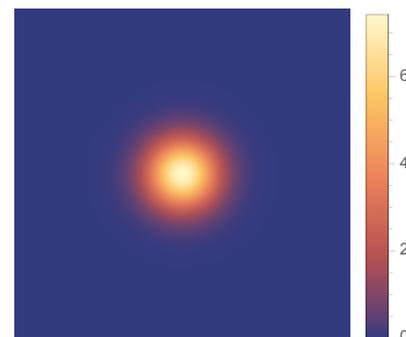
$$[\partial_\tau - (\partial_\mu^x \partial_\mu^x - M^2)] \phi(x, \tau) = 0 \quad \Rightarrow \quad \phi(x, \tau) = \int \underbrace{d^4 y G(x - y, \tau)}_{\text{heat kernel}} \phi(y) \quad \Rightarrow \quad \tilde{\phi}(q, \tau) = e^{-\tau(q^2 + M^2)} \tilde{\phi}(q)$$



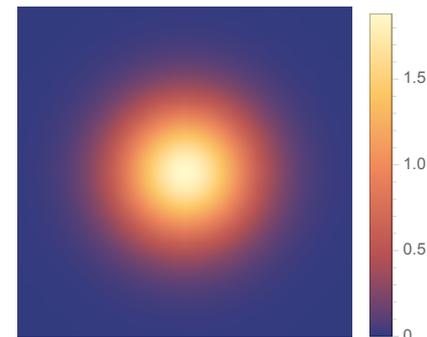
$\tau = 0$



$\tau = 1$



$\tau = 2$



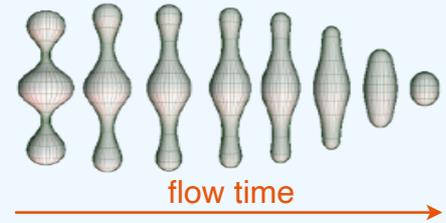
$\tau = 4$

# Chiral gradient flow

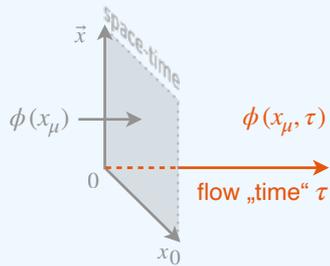
Krebs, EE, PRC 110 (2024) 044004

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$$G(x, \tau) = \frac{\theta(\tau)}{16\pi^2 \tau^2} e^{-\frac{x^2 + 4M^2 \tau^2}{4\tau}}$$

YM gradient flow Narayanan, Neuberger '06, Lüscher, Weisz '11:  $\partial_\tau A_\mu(x, \tau) = D_\nu G_{\nu\mu}(x, \tau)$  ← extensively used in LQCD

Chiral gradient flow Krebs, EE, PRC 110 (2024) 044004

$$\text{Generalize } U(x), U(x) \rightarrow RU(x)L^\dagger \text{ to } W(x, \tau): \quad \partial_\tau W = - \underbrace{i \overline{w} \text{EOM}(\tau)}_{\sqrt{W}} w, \quad W(x, 0) = U(x)$$

$$[D_\mu, w_\mu] + \frac{i}{2} \chi_-(\tau) - \frac{i}{4} \text{Tr} \chi_-(\tau)$$

We have proven  $\forall \tau: W(x, \tau) \in \text{SU}(2), W(x, \tau) \rightarrow RW(x, \tau)L^\dagger$

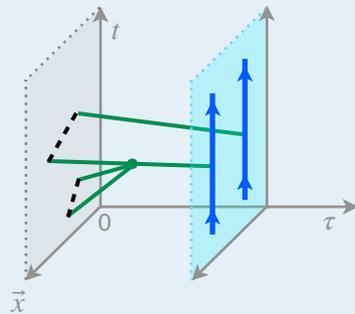
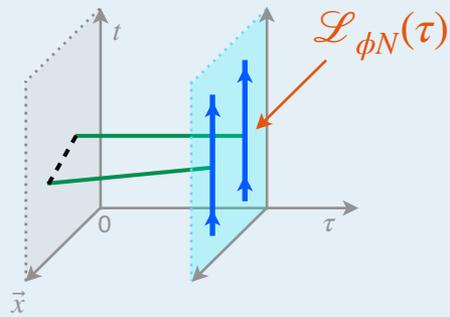
# Nuclear forces using chiral gradient flow

Krebs, EE, PRC 110 (2024) 044004

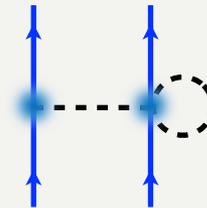
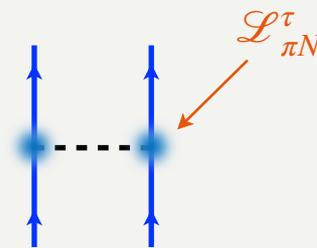
Regularization is achieved by requiring N to „live“ at a fixed  $\tau$ :  $\mathcal{L}_{\pi N} \rightarrow \mathcal{L}_{\phi N}(\tau) = \mathcal{L}_{\pi N} \Big|_{U \rightarrow W(\tau)}$

Notice: chiral symmetry manifest since  $W(\tau) \rightarrow RW(\tau)L^\dagger$  for all  $\tau$ .

## Local field theory in 5d



## Smeared (non-local) theory in 4d



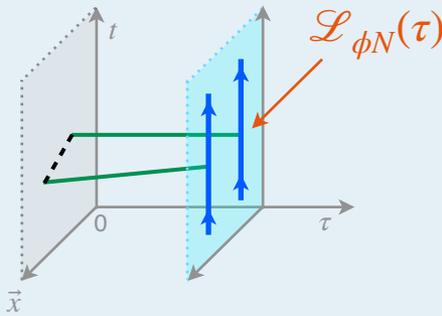
# Nuclear forces using chiral gradient flow

Krebs, EE, PRC 110 (2024) 044004

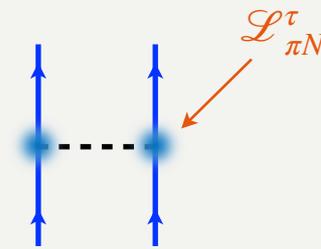
Regularization is achieved by requiring  $N$  to „live“ at a fixed  $\tau$ :  $\mathcal{L}_{\pi N} \rightarrow \mathcal{L}_{\phi N}(\tau) = \mathcal{L}_{\pi N} \Big|_{U \rightarrow W(\tau)}$

Notice: chiral symmetry manifest since  $W(\tau) \rightarrow RW(\tau)L^\dagger$  for all  $\tau$ .

## Local field theory in 5d



## Smeared (non-local) theory in 4d



Nuclear forces (and currents) can be derived from the nonlocal (smeared)  $\mathcal{L}_{\text{eff}}$  using the new path-integral approach Krebs, EE, PRC 110 (2024) 044003:

$$Z[\eta^\dagger, \eta] = A \int \mathcal{D}N^\dagger \mathcal{D}N \mathcal{D}\pi \exp\left(iS_{\text{eff}}^\Lambda + i \int d^4x [\eta^\dagger N + N^\dagger \eta]\right)$$

nonlocal redefinitions of  $N, N^\dagger$   
loops from functional determinant  $\rightarrow$

$$A \int \mathcal{D}\tilde{N}^\dagger \mathcal{D}\tilde{N} \exp\left(iS_{\text{eff}, N}^\Lambda + i \int d^4x [\eta^\dagger \tilde{N} + \tilde{N}^\dagger \eta]\right)$$

instantaneous

# Summary and outlook

- The chiral symmetry of QCD and its breaking pattern play the key role for understanding low-energy nuclear physics
- Chiral EFT has already been developed into a precision tool in the NN sector!

## Frontiers and challenges:

- Precision physics beyond the 2N system (the 3NF challenge)
  - **high-precision 3NFs** (gradient flow method) and 3N scattering
  - precision test of chiral EFT for nuclear forces & electroweak currents in nuclei
  - ab-initio theory for heavier nuclei and reactions
- Chiral EFT as a tool to deal with nuclear effects (SM and BSM): PV, EDM,  $0\nu\beta\beta$ ,...
- EFT and lattice-QCD

Thank you for your attention