

# Anomalous magnetic moment of the muon

Z. Fodor

Penn State, Univ. Wuppertal, FZ Juelich, Univ. Budapest, UCSD

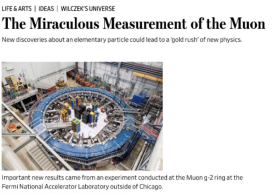
Budapest–Marseille–Wuppertal collaboration (BMW)

Borsanyi, Fodor, Guenther, Hoelbling, Katz, Lellouch, Lippert,  
Miura, Parato, Szabo, Stokes, Toth, Torok, Varnhorst

PSI Colloquium, April 27, 2023



# General interest



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VON HARVEY LEVINE | ACTUALISIERUNG VON JULIAN J. BELL

**Humans humbled by a wobbling particle**  
The world of physics has been hit with a bomb. A tiny, elusive elementary particle has just been found to behave in a way that defies the laws of physics as we know them. The discovery is so surprising that it has caused a major shake-up in the scientific community. It's the first time in over 50 years that a particle has been found to behave in a way that defies the laws of physics as we know them. The discovery is so surprising that it has caused a major shake-up in the scientific community. It's the first time in over 50 years that a particle has been found to behave in a way that defies the laws of physics as we know them.

**Forbes**  
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**The New York Times**  
**A Tiny Particle's Wobble Could Upend the Known Laws of Physics**  
Experiments with particles known as muons suggest that there are forms of matter and energy vital to the nature and evolution of the cosmos that are not yet known to science.

**Le Monde**  
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**Les rotations du muon électrisent la communauté des physiciens**  
Une anomalie dans le comportement magnétique du muon, une particule élémentaire, est-elle le signe que le cadre conceptuel servant à décrire l'univers petit est en train de craquer ? Des résultats contradictoires relancent le débat.

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12. April 2021

**Humans humbled by a wobbling particle**

Important new results came from an experiment conducted at the Muon g-2 ring at the Fermi National Accelerator Laboratory outside of Chicago.

PHOTO: KEVIN MAHUFER/REUTERS/ASSOCIATED PRESS

By Frank Wilczek  
April 13, 2021 12:04 pm ET

**The New York Times**

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Anomalie des Muon  
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VON WANDER LÄNDCHEN  
Aktualisiert am 22.04.2021 15:38

**Forbes**  
APRIL 13, 2021  
**Humans humbled by a wobbling particle**  
The world of physics was rocked last week with news of an experimental result that challenges a pillar of the Standard Model of particle physics. It's a discovery that could lead to new physics. It's a discovery that could lead to new physics. It's a discovery that could lead to new physics.

**The New York Times**  
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12. APRIL 2021 08:14:30 UHR  
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Rebecca Engelke

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SCIENCES - PHYSIQUE

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Par David Rousseau

82 News

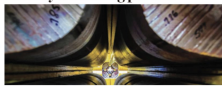
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APRIL 13

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## Humans humbled by a wobbling particle

The discovery of a tiny wobble in the magnetic moment of the muon, a subatomic particle, is shaking up the physics community. It could be a sign that the Standard Model of particle physics is incomplete, or it could be a fluke. But either way, it's a big deal.



Forbes

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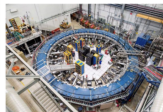
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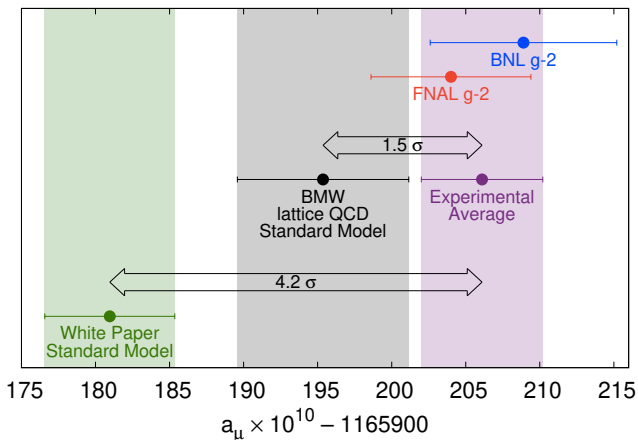
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# Tensions in $(g - 2)_\mu$ : take-home message



[Muon g-2 Theory Initiative, Phys.Rept. 887 (2020) 1-166]

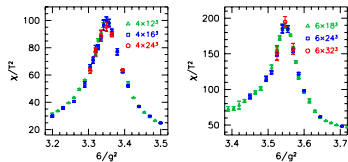
[Budapest–Marseille–Wuppertal-coll., Nature (2021)]

[Muon g-2 coll., Phys. Rev. Lett. 126, 141801 (2021)]

# Lattice QCD: examples

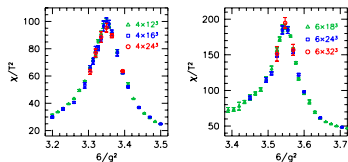
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- Wuppertal–Budapest-collaboration,  
*The order of the quantum chromodynamics transition  
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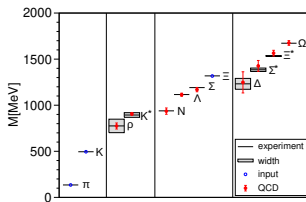


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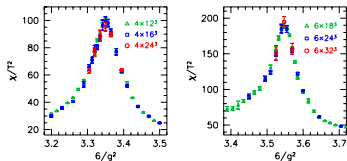


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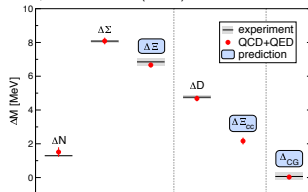


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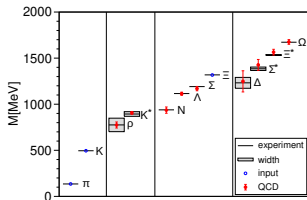
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- Budapest–Marseille–Wuppertal-collaboration, *Ab initio calculation of the neutron-proton mass difference*, Science 347 (2015) 1452-1455

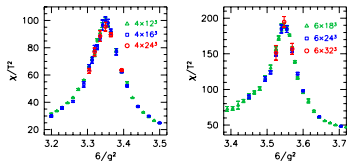


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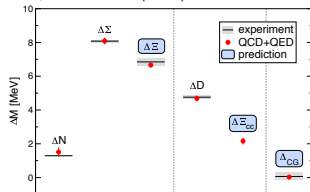


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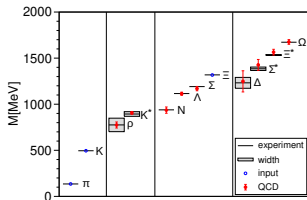
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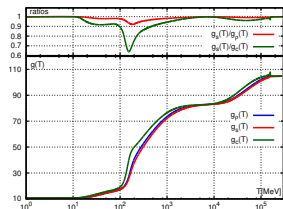
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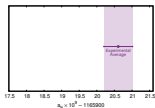
- Wuppertal–Budapest-collaboration, *Lattice QCD for Cosmology*, Nature 539 (2016) 7627, 69-71



# Outline

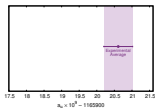
# Outline

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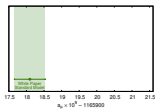


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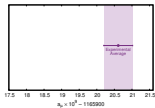


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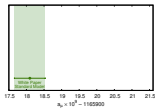


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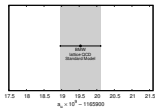
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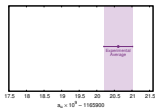


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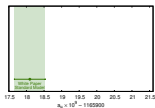


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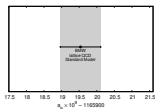
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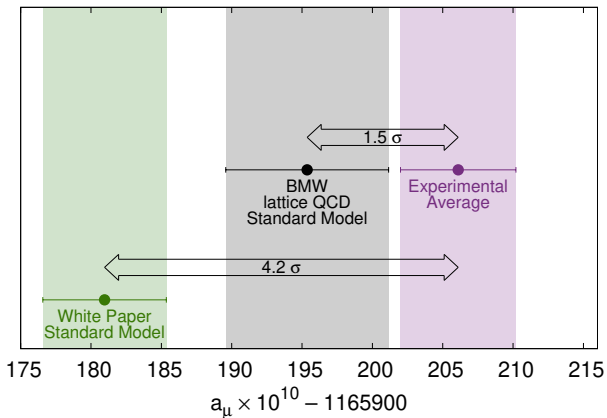
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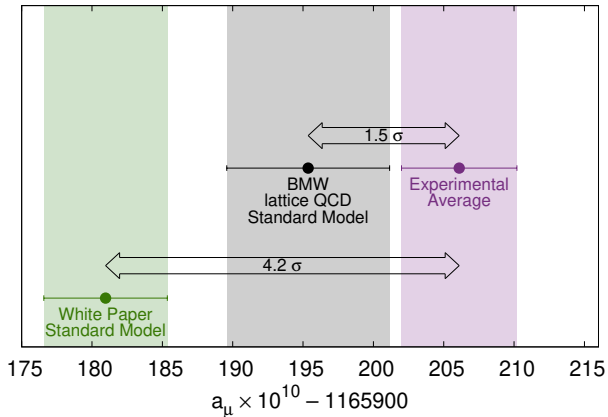
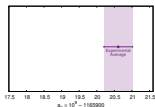


4. Summary



# Outline

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# Experimental result

- Newly announced result at Fermilab

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- Target uncertainty: (1.6)

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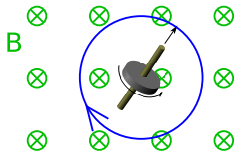
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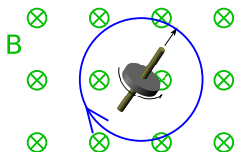
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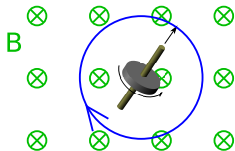


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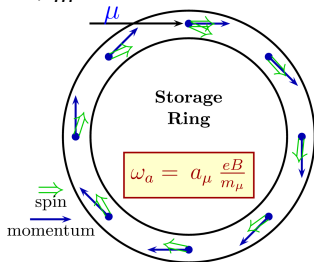
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- Gives  $a_\mu = \frac{g_\mu - 2}{2}$  directly



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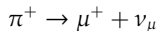
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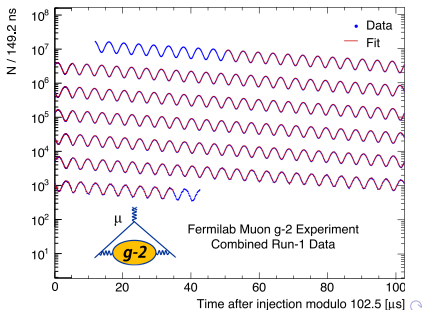
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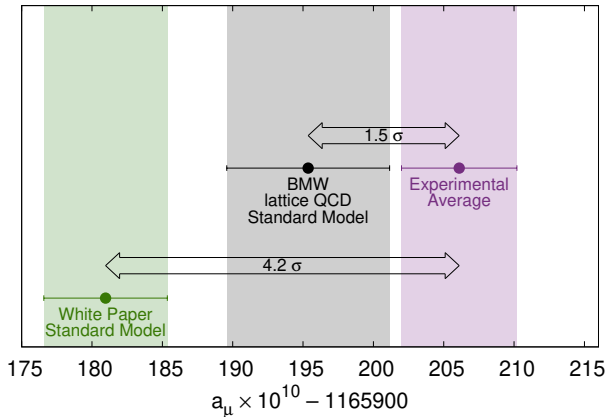
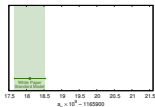


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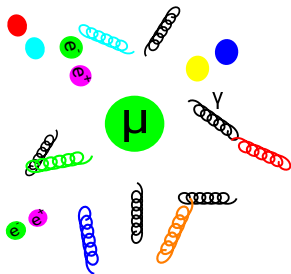


# Outline

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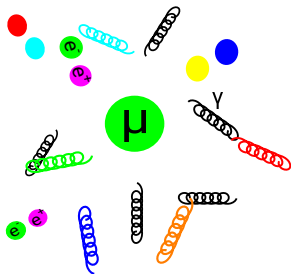
# Theory: Standard Model



Sum over all known physics:



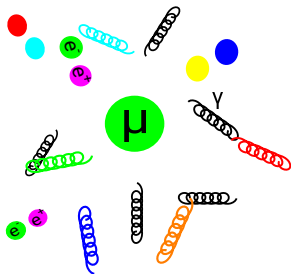
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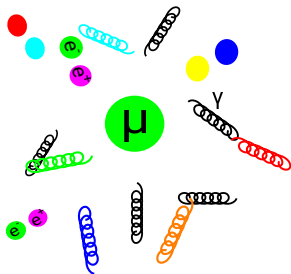
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- 3 strong (QCD): quarks and gluons

# Theory: Standard Model

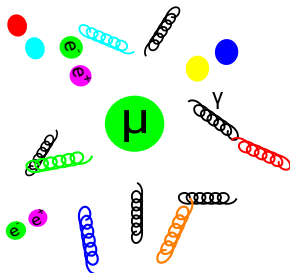


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theory initiative; theory consensus, R-ratio, dispersion relation

# Theory: Standard Model



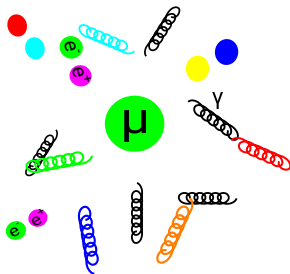
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	$a_\mu \times 10^{-10}$
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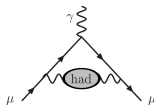
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4.0 out of the 4.3 error comes from [LO hadron vacuum polarisation](#)

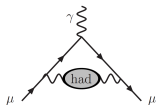
# Hadronic contributions

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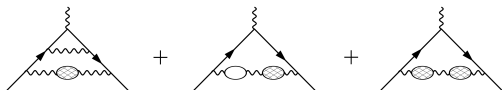


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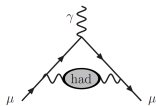


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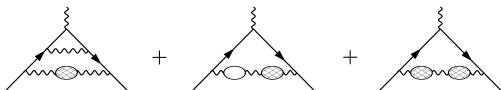


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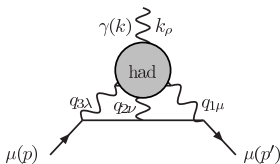
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- pheno  $a_{\mu}^{\text{HLbL}} = 9.2(1.9)$

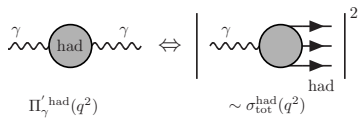
[Colangelo, Hoferichter, Kubis, Stoffer et al '15-'20]

- lattice  $a_{\mu}^{\text{HLbL}} = 7.9(3.1)(1.8)$  or  $10.7(1.5)$

[RBC/UKQCD '19 and Mainz '21]

# HVP from R-ratio

- Optical theorem



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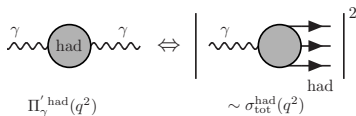
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$$\Pi_{\gamma}^{\text{had}}(q^2) \Leftrightarrow \left| \begin{array}{c} \gamma \text{ enters } \text{had} \text{ circle} \\ \text{had} \text{ exits } \end{array} \right|^2 \sim \sigma_{\text{tot}}^{\text{had}}(q^2)$$

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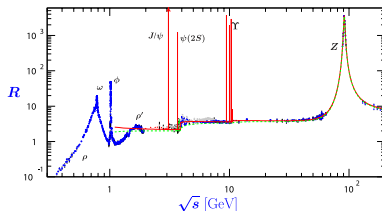
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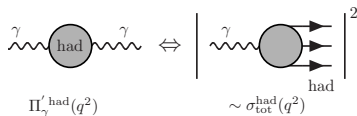
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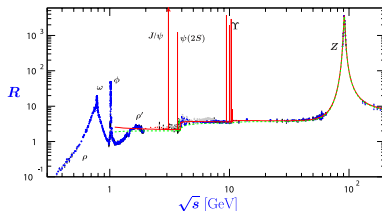
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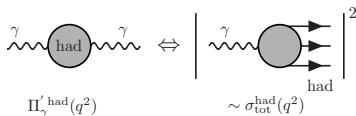
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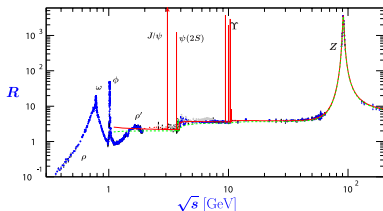
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Systematic uncertainty:  $\approx 4$  times larger than the statistical error (e.g. Davier et al.)

# Tensions in the R-ratio method

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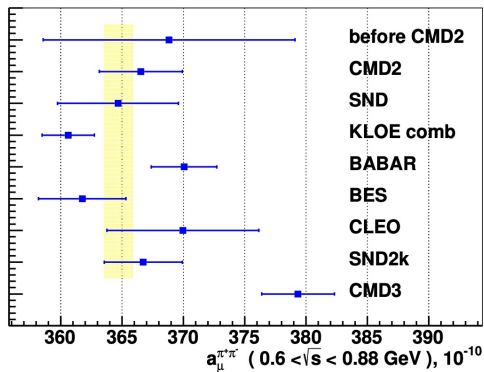
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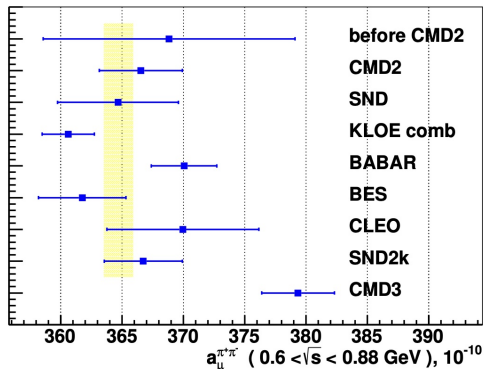
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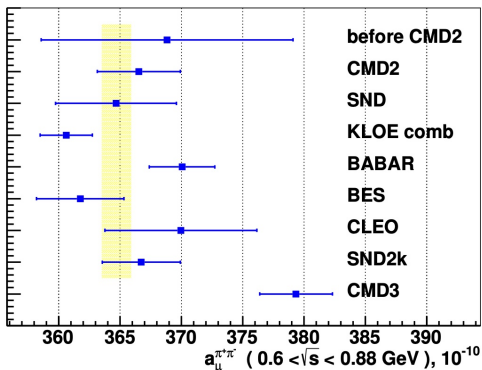


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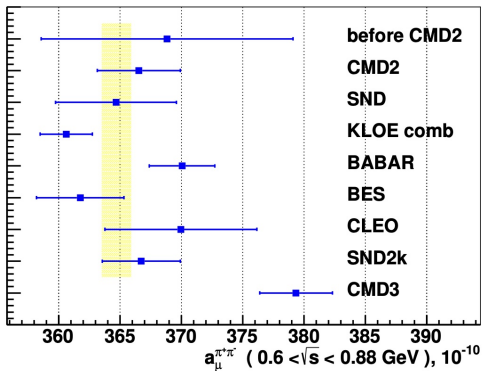
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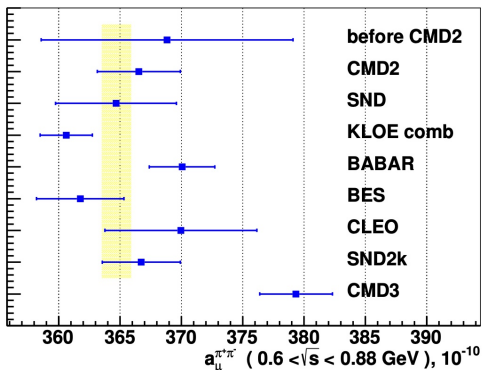
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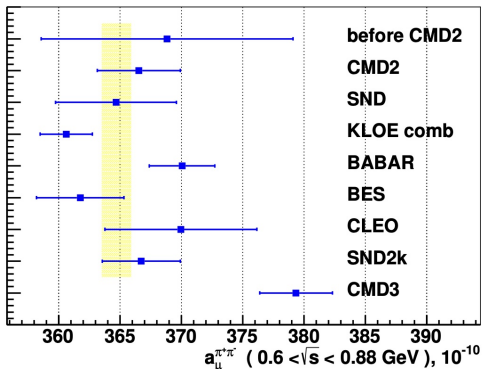
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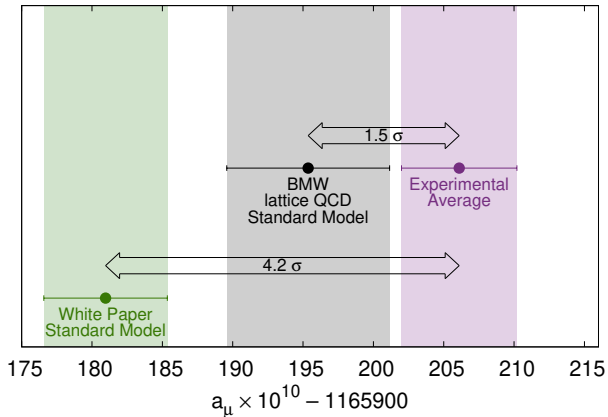
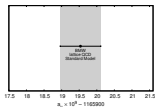
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White Paper must further inflate errors: less chance for new physics?

# Outline

3.



# Lattice QCD

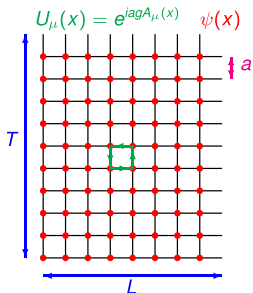
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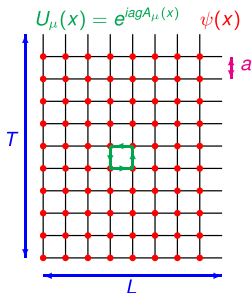
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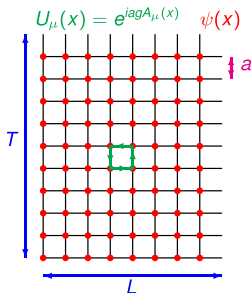
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- To get physical results, need to perform:
  - 1 Chiral limit ( $m_{u/d} \rightarrow m_{phys}$  or use  $m_{phys}$ )
  - 2 Infinite volume limit ( $V \rightarrow \infty$ )  $\rightarrow$  numerically or analytically
  - 3 Continuum limit ( $a \rightarrow 0$ )  $\rightarrow$  min. 3 different  $a$



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Nature 593 (2021) 7857, 51

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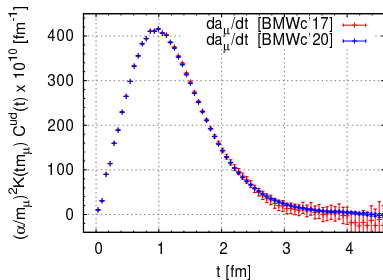
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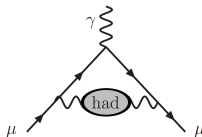
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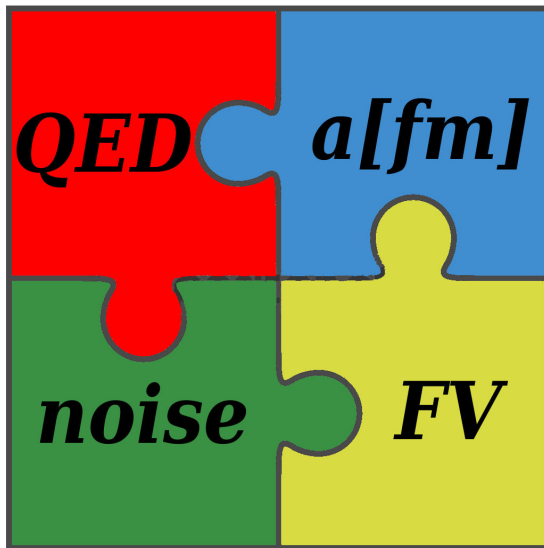
$$a_\mu^{\text{LO-HVP}} = \alpha^2 \int_0^\infty dt K(t) C(t)$$



$K(t)$  describes the leptonic part of diagram



# New challenges



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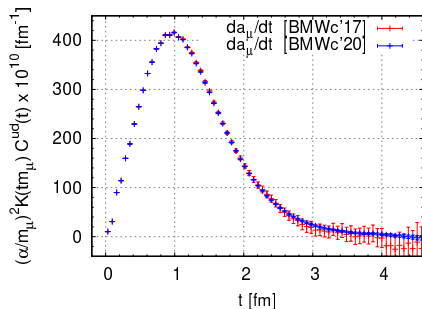
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- 2 For separation of isospin breaking effects:  $w_0$  scale setting
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  - Can be precisely determined on the lattice
  - No experimental value
    - Determine value of  $w_0$  from  $M_\Omega \cdot w_0$

$$w_0 = 0.17236(29)(63)[70] \text{ fm}$$

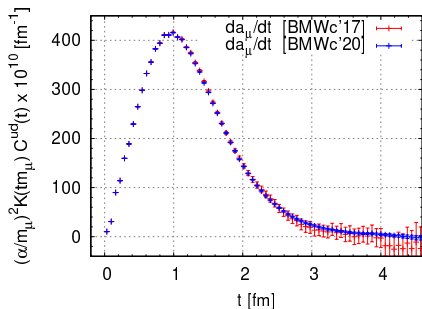
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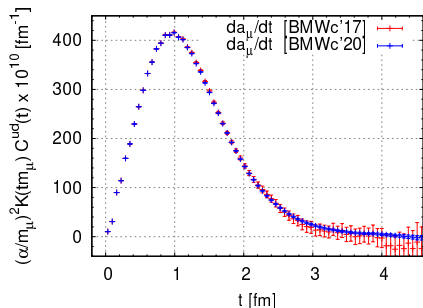


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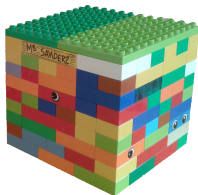
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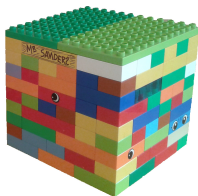


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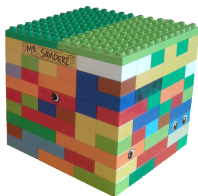
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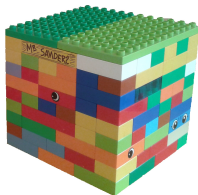
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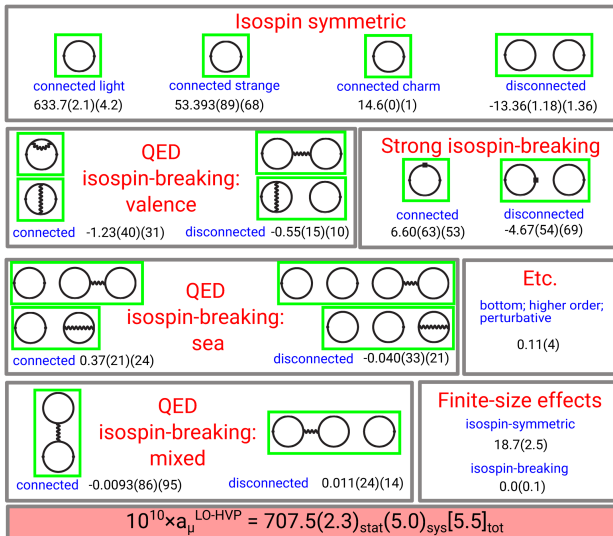
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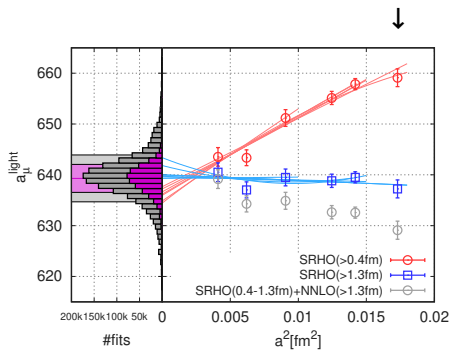
- use models for remnant finite-size effect of “big”  $\sim 0.1\%$

# Isospin breaking effects

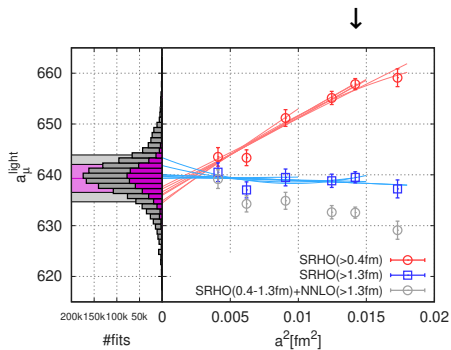
- Include leading order IB effects:  $O(e^2)$ ,  $O(\delta m)$



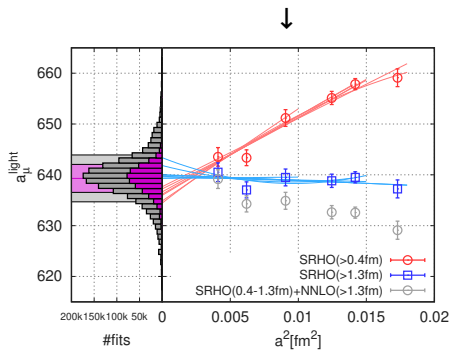
# Continuum limit



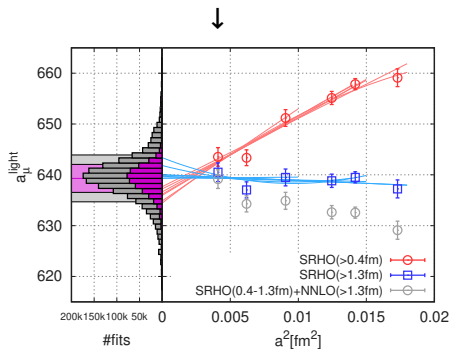
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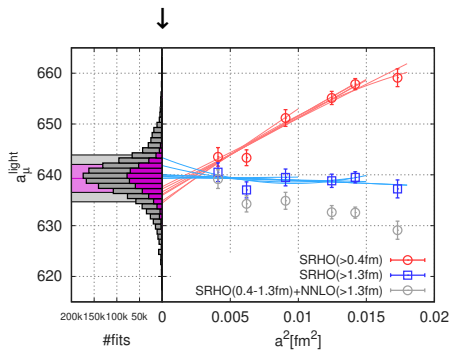
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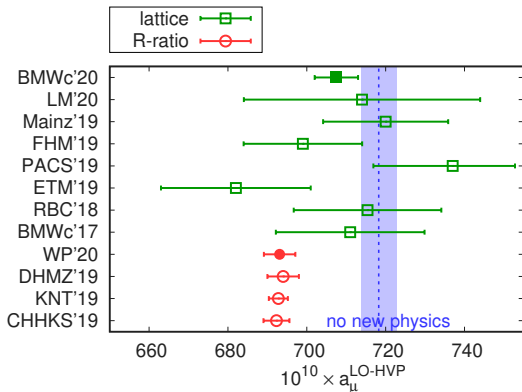
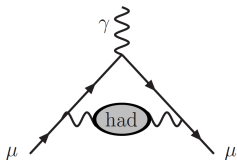
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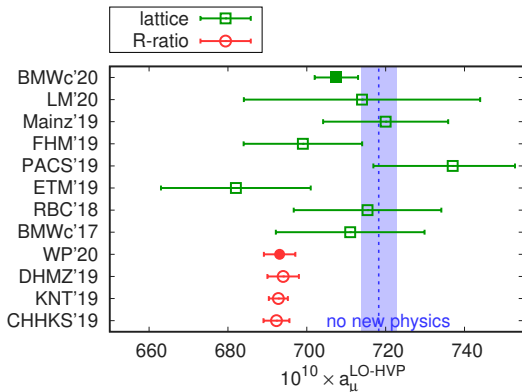
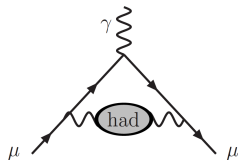
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## Final result for LO-HVP (hadronic vacuum polarization)

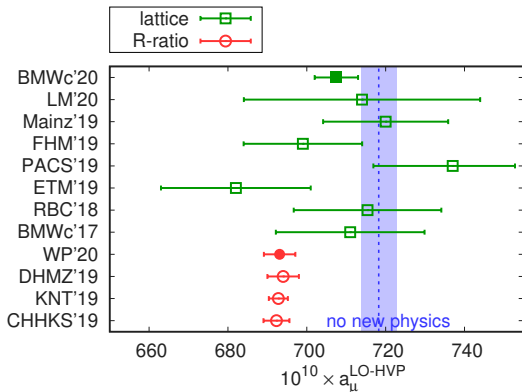
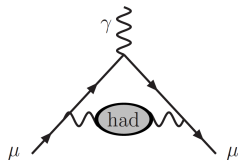


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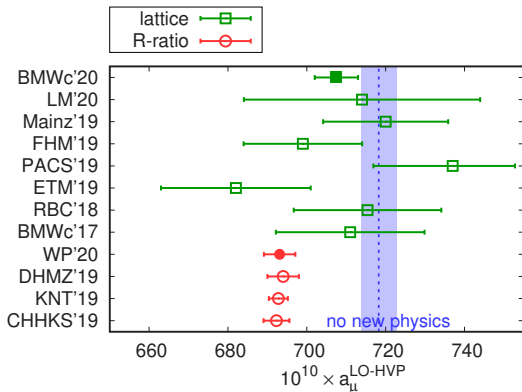
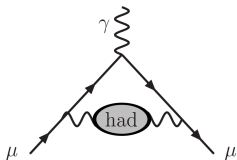
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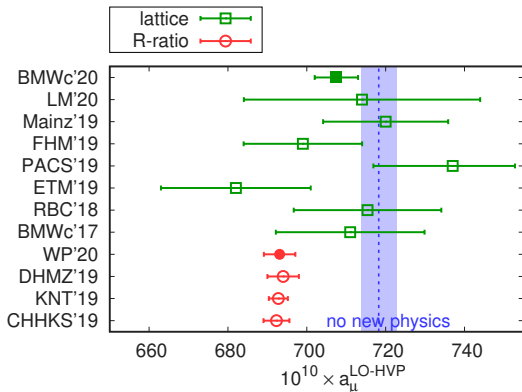
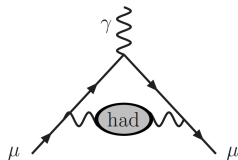
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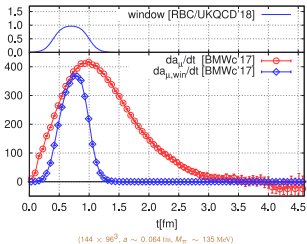


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- CMD3 is also 15 units larger than the White Paper: spot on

# Window observable

- Restrict correlator to window between  $t_1 = 0.4$  fm and  $t_2 = 1.0$  fm

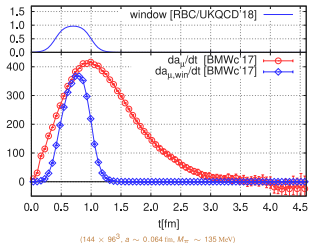
[RBC/UKQCD'18]



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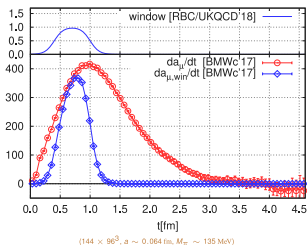


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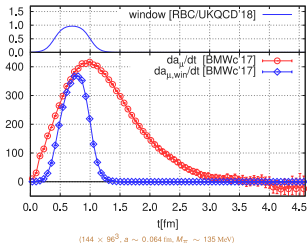


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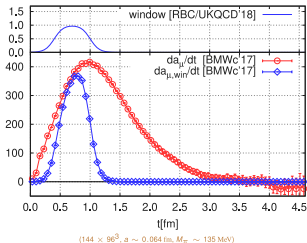
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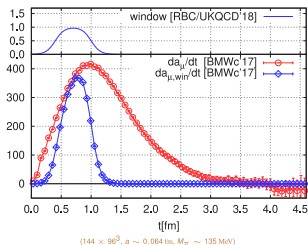
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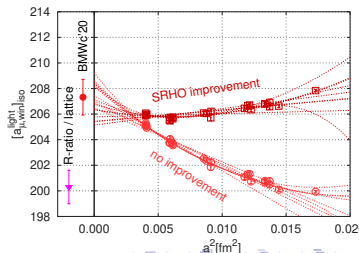


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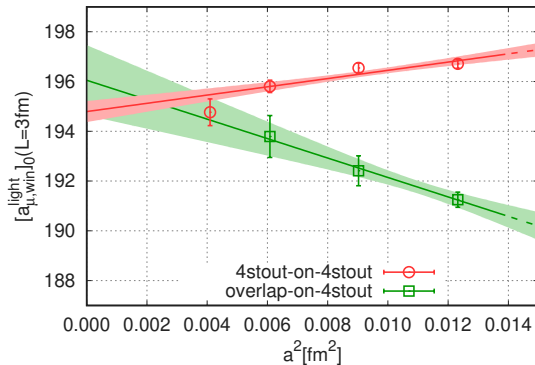
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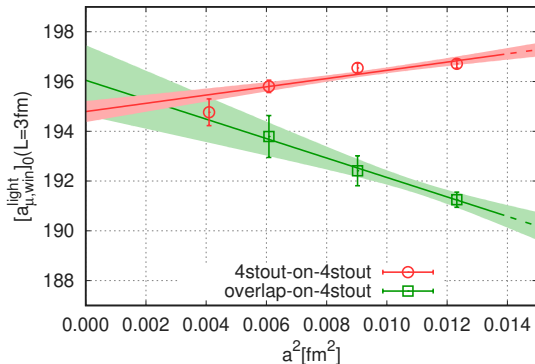
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# Crosscheck – overlap

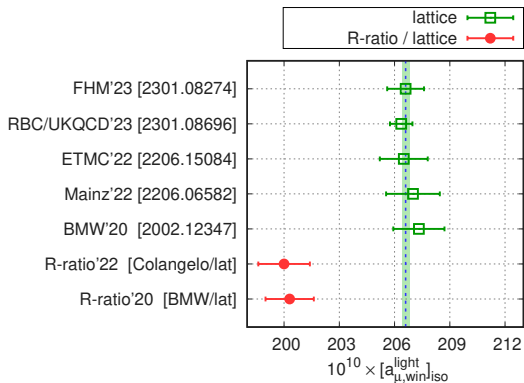


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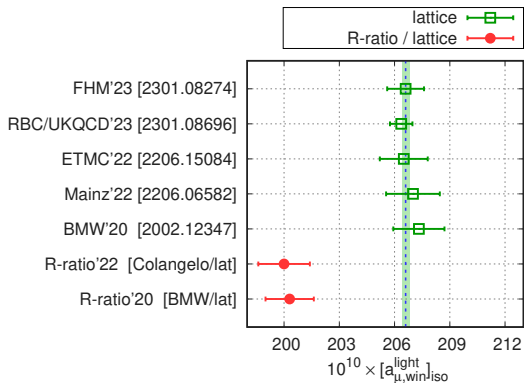


- compute  $a_{\mu,win}$  with overlap valence
- local current instead of conserved  $\rightarrow$  had to compute  $Z_V$
- cont. limit in  $L = 3$  fm box consistent with staggered valence

# Tension in the window observables

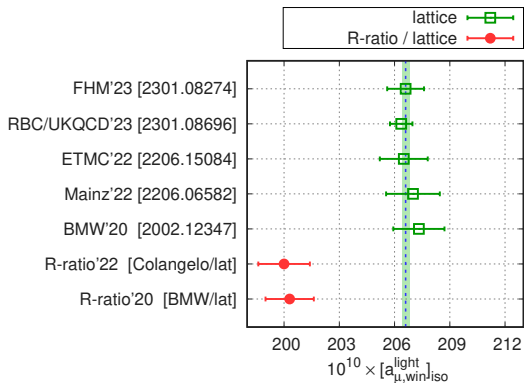


# Tension in the window observables



5 fully independent results  
 most of them: blinded(\*)  
 all agree with each other

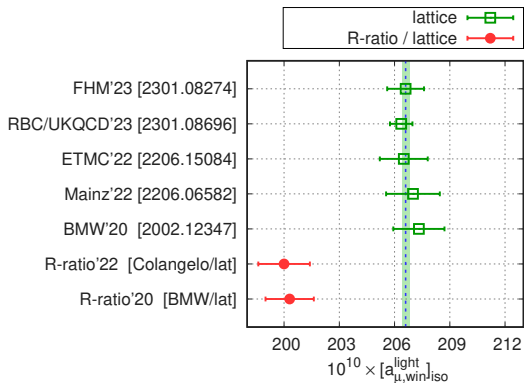
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as for the R-ratio

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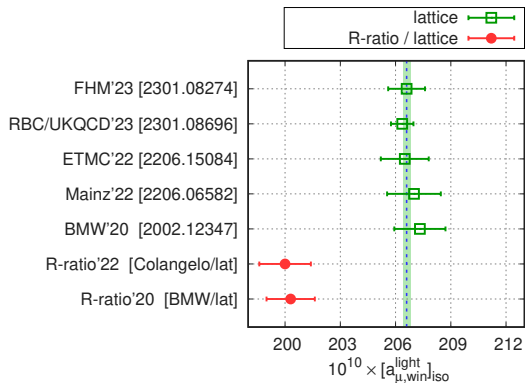
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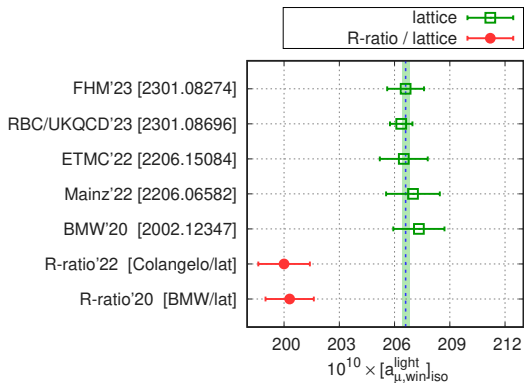
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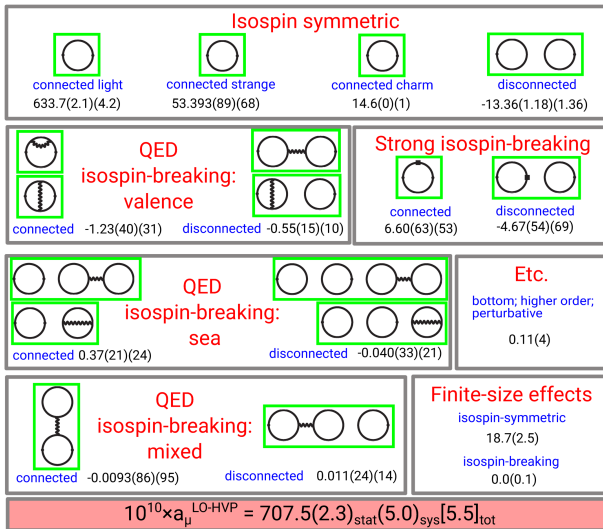
QCD compared with QCD

either new physics  
or underestimated errors

# Outline

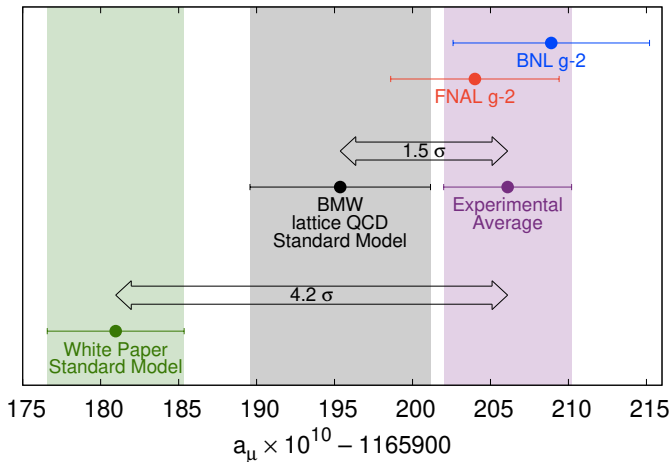
## 5. Summary

## Final result



# Tension: take-home message #1 full g-2

Systematic/statistical error ratios: lattice  $\approx 2$ ; R-ratio  $\approx 4$



# Tension: take-home message #2 lattice/ $e^+e^-$ window

about 4.4–4.9–5.1 $\sigma$  tensions for distance & energy regions

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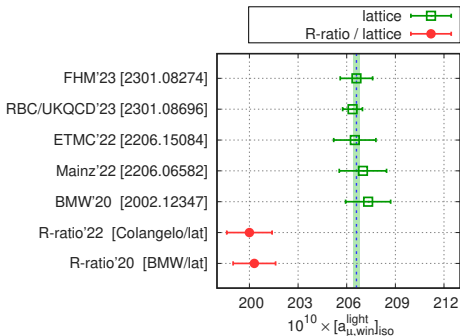
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