From Effective Field Theories to Gravitational Waves

March 2, 2023

PSI Colloquium

Zvi Bern

ZB, C. Cheung, R. Roiban, C. H. Shen, M. Solon, M. Zeng, arXiv:1901.04424 and arXiv:1908.01493.

ZB, A. Luna, R. Roiban, C. H. Shen, M. Zeng, arXiv:2005.03071

ZB, J. Parra-Martinez, R. Roiban, M. Ruf. C.-H. Shen, M. Solon, M. Zeng, arXiv:2101.07254; arXiv:2112.10750

ZB, Kosmopoulos, Luna, Roiban, Teng, arXiv: 2203.06202

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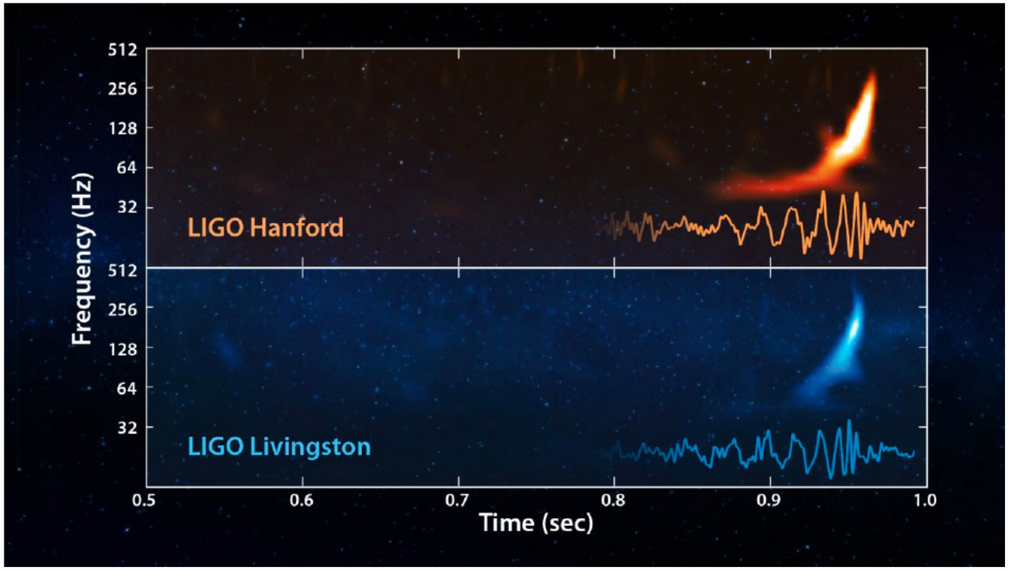
Outline

Era of gravitational-wave astronomy has begun.



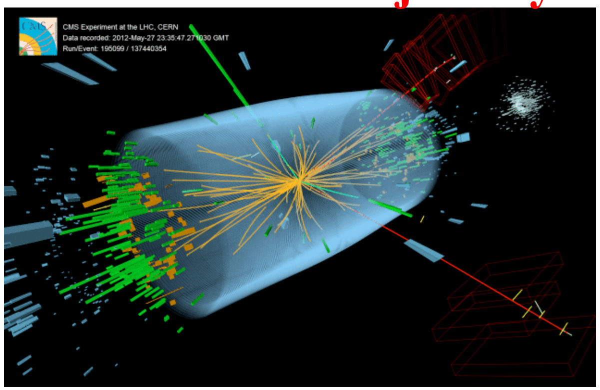
For an instant brighter in gravitational radiation than all the stars in the visible universe are in EM radiation!

How can we, as particle theorists, help out with core mission of LIGO/Virgo?

Can Particle Theory Help with Gravitational Waves?

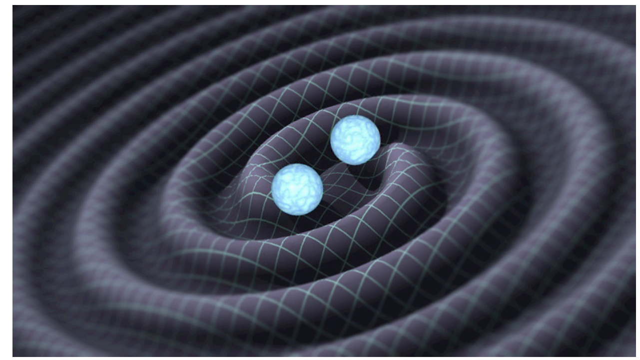
What does particle physics have to do with classical dynamics of astrophysical objects?

unbounded trajectory



gauge theories, QCD, electroweak
quantum field theory

bounded orbit



General Relativity
classical physics

Black holes and neutron stars are point particles as far as long wavelength radiation is concerned.

Iwasaki (1971); **Goldberger, Rothstein (2006)**; Vaydia, Foffa , Porto, Rothstein, Sturani; Kol; Bjerrum-Bohr, Donoghue, Holstein, Plante, Pierre Vanhove; Levi, Steinhoff; Vines etc

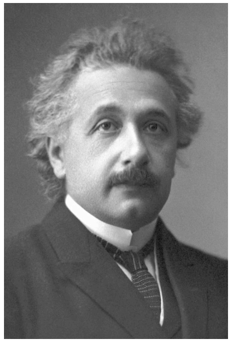
Will explain that EFTs are well suited to push state-of-the-art perturbative calculations for gravitational-wave physics.

Approach to General Relativity

Our appoach does *not* start from usual Einstein Field equations.

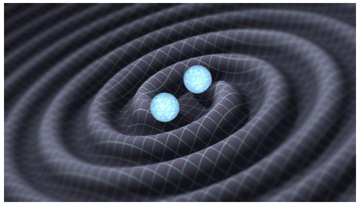
$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

~~**geometry**~~



Gravitons are spin 2 particles

- **Not suited for all problems. Works very well for asymptotically flat space-times in context of perturbation theory.**
- **Well suited for gravitational-wave physics from compact astrophysical objects**



Can Quantum Scattering Help with Gravitational Waves?

In particle physics we are very good at perturbation theory.
Experience with gauge theories and supergravity theories.

Two serious issues for applying this to gravitational waves:

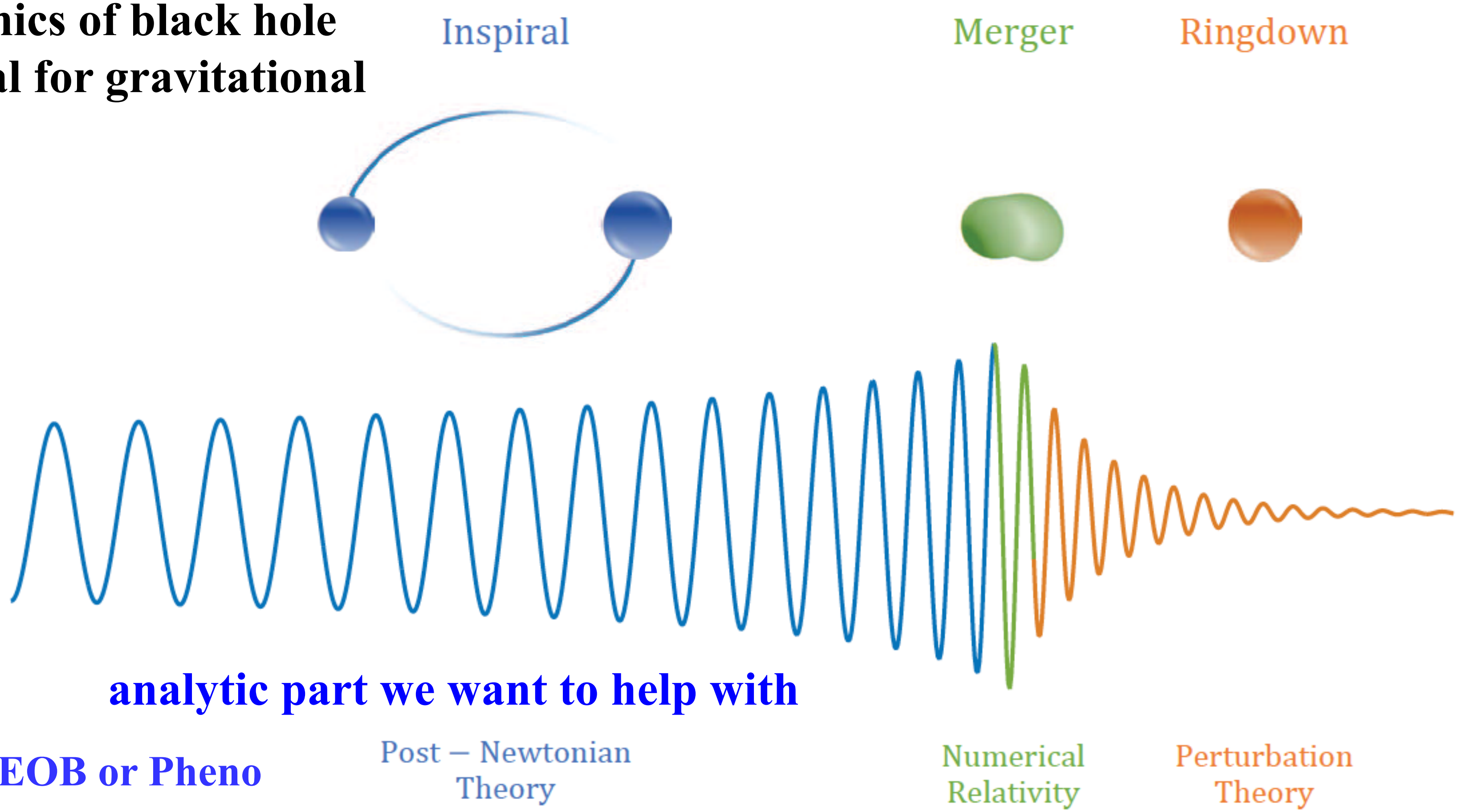
1. **We do quantum *not* classical perturbation theory.**
2. **Scattering process unbounded orbit. Want bounded one for binary black hole gravitational wave emission.**

Two key topics:

- **Modern approach to perturbative gravity.**
- **How do we effectively deal with the above annoying issues?**

Goal: Improve on post-Newtonian Theory

Dynamics of black hole inspiral for gravitational waves.



analytic part we want to help with

PN + EOB or Pheno

Small errors accumulate. Need for high precision.

From Antelis and Moreno, arXiv:1610.03567

Post Newtonian Approximation

For orbital mechanics:

$$v^2 \sim \frac{GM}{R} \ll 1$$

Expand in G and v^2

virial theorem

In center of mass frame:

$$m = m_A + m_B, \quad \nu = \mu/M,$$

$$\mu = m_A m_B/m, \quad P_R = P \cdot \hat{R}$$

$$\frac{H}{\mu} = \frac{P^2}{2} - \frac{Gm}{R}$$ ← Newton

$$+ \frac{1}{c^2}\left\{ -\frac{P^4}{8} + \frac{3\,\nu\,P^4}{8} + \frac{Gm}{R}\left(-\frac{{P_R}^2\,\nu}{2} - \frac{3\,P^2}{2} - \frac{\nu\,P^2}{2}\right) + \frac{G^2m^2}{2R^2}\right\}$$

$$+ \dots$$

1PN: Einstein, Infeld, Hoffmann; Droste, Lorentz

Hamiltonian known to 4PN order.

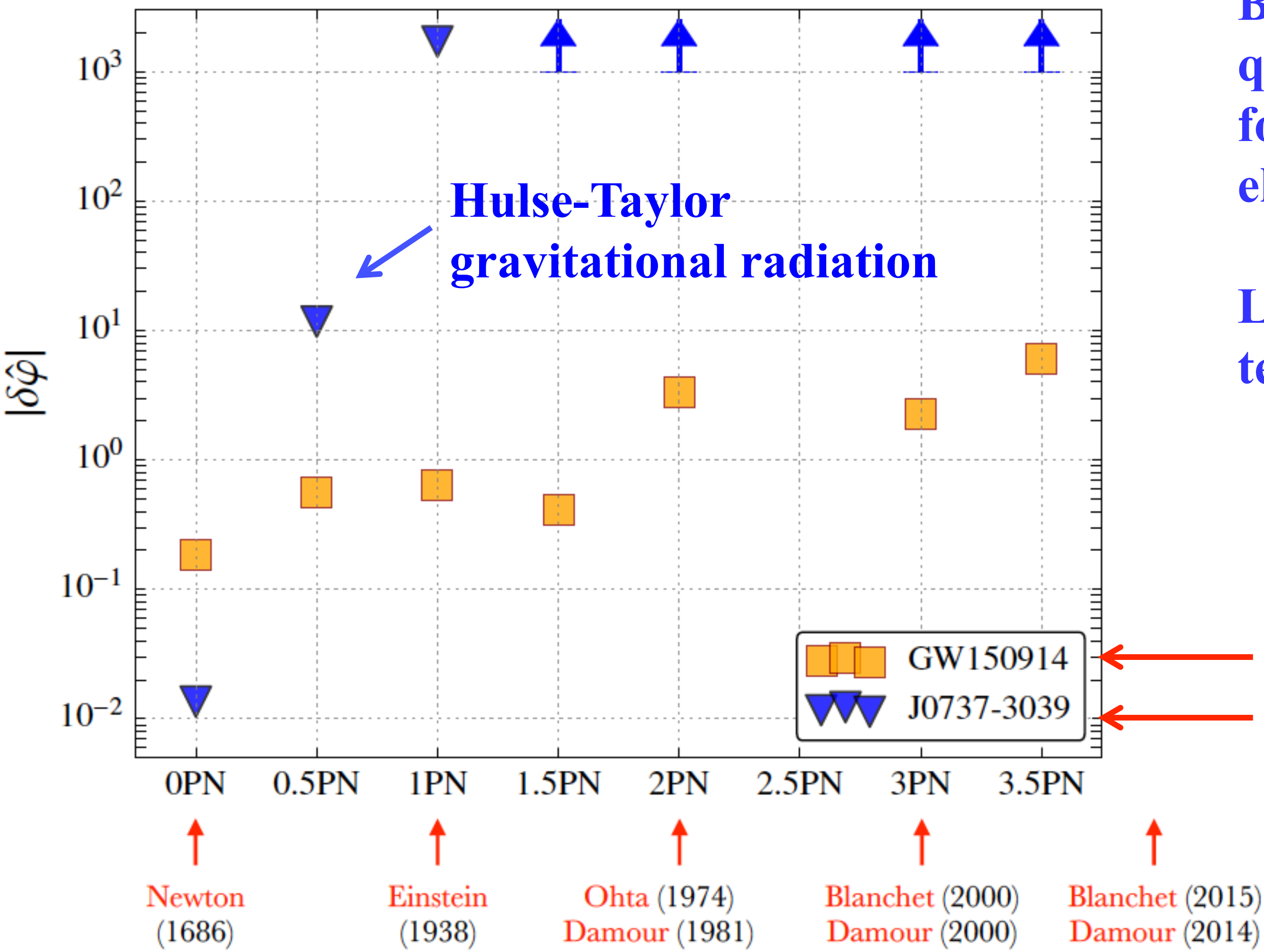
2PN: Ohta, Okamura, Kimura and Hiida.

3PN: Damour, Jaranowski and Schaefer; L. Blanchet and G. Faye.

4PN: Damour, Jaranowski and Schaefer; Foffa (2017), Porto, Rothstein, Sturani (2019).

Importance of higher orders for LIGO/Virgo

LIGO/Virgo Collaboration arXiv:1602.03841



Binary pulsar confirms quadrupole radiation formula and not much else.

LIGO/Virgo tests PN terms from GR

LIGO

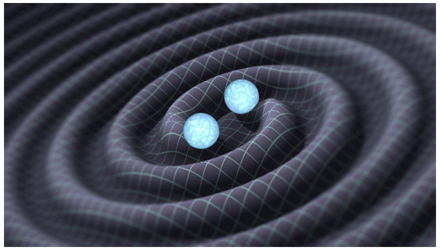
Binary pulsar

LIGO/Virgo sensitive to high PN orders.

Which problem to solve?

ZB, Cheung, Roiban, Shen, Solon, Zeng

Some problems for (analytic) theorists:



1. Spin.
2. Finite size effects.
3. New physics effects.
4. Radiation.
5. High orders in perturbation theory.

Which problem should we solve?

- Needs to be difficult using standard methods.
- Needs to be of direct importance to LIGO theorists.
- Needs to be in a form that can in principle enter LIGO analysis pipeline.

2-body Hamiltonian at 3rd post-Minkowskian order

High-energy gravitational scattering and the general relativistic two-body problem

Thibault Damour*
Institut des Hautes Etudes Scientifiques, 35 route de Chartres, 91440 Bures-sur-Yvette, France
(Dated: October 31, 2017)

A technique for translating the classical scattering function of two gravitationally interacting bod-

tum gra... amplitude of two particles, and we urge amplitude experts to use their novel techniques to compute the 2-loop scattering amplitude of scalar masses, from which one could deduce the third post-Minkowskian effective one-body Hamiltonian

"… and we urge amplitude experts to use their novel techniques to compute the 2-loop scattering amplitude of scalar masses, from which one could deduce the third post-Minkowskian effective one-body Hamiltonian."

Hard to resist an invitation with this kind of clarity!

- **Difficult using standard methods.**
- **Of direct importance to LIGO/Virgo theorists.**
- **Can in principle enter LIGO/Virgo analysis pipeline.**

nals from inspiralling and coalescing binary black holes has been significantly helped, from the theoretical side, by the availability of a large bank of waveform templates, defined [5, 6] within the analytical effective one-body (EOB) formalism [7–11]. ...

mostly based on the post-Newtonian (PN) approach to the general relativistic two-body interaction. The conservative two-body dynamics was derived, successively, at the second post-Newtonian (2PN) [14, 15], third post-

...ntly introduced ... to derive ... from the (gauge-invariant) *scattering function* Φ linking (half) the center of mass (c.m.) classical gravitational scattering angle χ to the total energy, $E_{\rm real} \equiv \sqrt{s}$, and the total angular momentum, J, of the system[1]

with

PN versus PM expansion for conservative two-body dynamics

From Buonanno Amplitudes 2018

$$\mathcal{L} = -Mc^2 + \underbrace{\frac{\mu v^2}{2} + \frac{GM\mu}{r}}_{\downarrow} + \frac{1}{c^2}\left[\dots\right] + \frac{1}{c^4}\left[\dots\right] + \dots$$

$$E(v) = -\frac{\mu}{2}v^2 + \cdots$$

non-spinning compact objects

		0PN	1PN	2PN	3PN	4PN	5PN	...
0PM:	1	v^2	v^4	v^6	v^8	v^{10}	v^{12}	...
1PM:		$1/r$	v^2/r	v^4/r	v^6/r	v^8/r	v^{10}/r	...
2PM:			$1/r^2$	v^2/r^2	v^4/r^2	v^6/r^2	v^8/r^2	...
3PM:				$1/r^3$	v^2/r^3	v^4/r^3	v^6/r^3	...
4PM:					$1/r^4$	v^2/r^4	v^4/r^4	...
...						...	...	

(credit: Justin Vines)

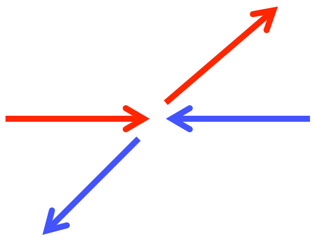
$$1 \to Mc^2, \qquad v^2 \to \frac{v^2}{c^2}, \qquad \frac{1}{r} \to \frac{GM}{rc^2}.$$

current known PN results — current known PM results — overlap between PN & PM results — unknown

- **PM results** (Westfahl 79, Westfahl & Goller 80, Portilla 79-80, Bel et al. 81, Ledvinka et al. 10, Damour 16-17, Guevara 17, Vines 17, Bini & Damour 17-18, Vines in prep)

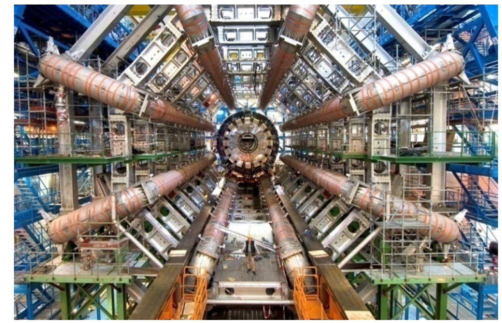
Quantum Field Theory and Scattering Amplitudes

Scattering amplitudes give us quantum mechanical description of events at particle colliders.

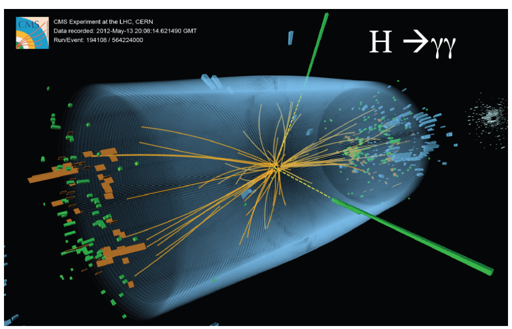


particle scattering

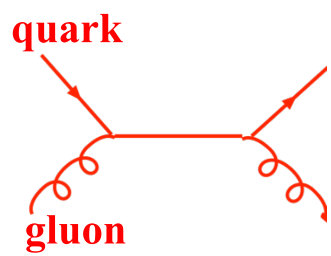
Large Hadron Collider



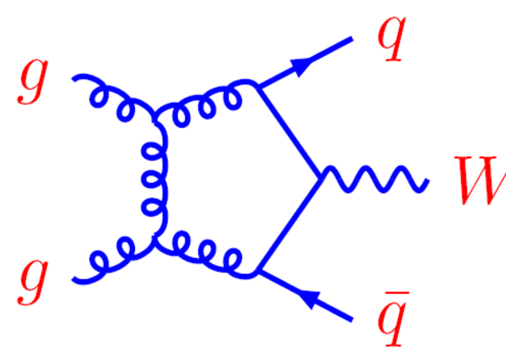
ATLAS Detector



Higgs boson event



Tree Feynman diagram



loop diagram
higher order

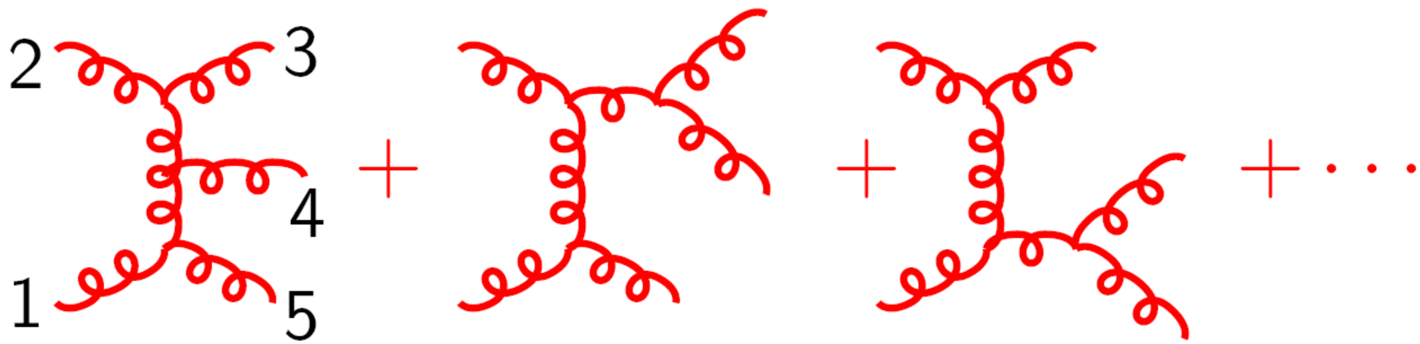
At first sight, does not seem to have much to do with gravitational waves

Tree-level example: Five gluons

Consider strong interactions described by Quantum Chromodynamics

Force carriers in QCD are gluons. Similar to photons of QED except they self interact.

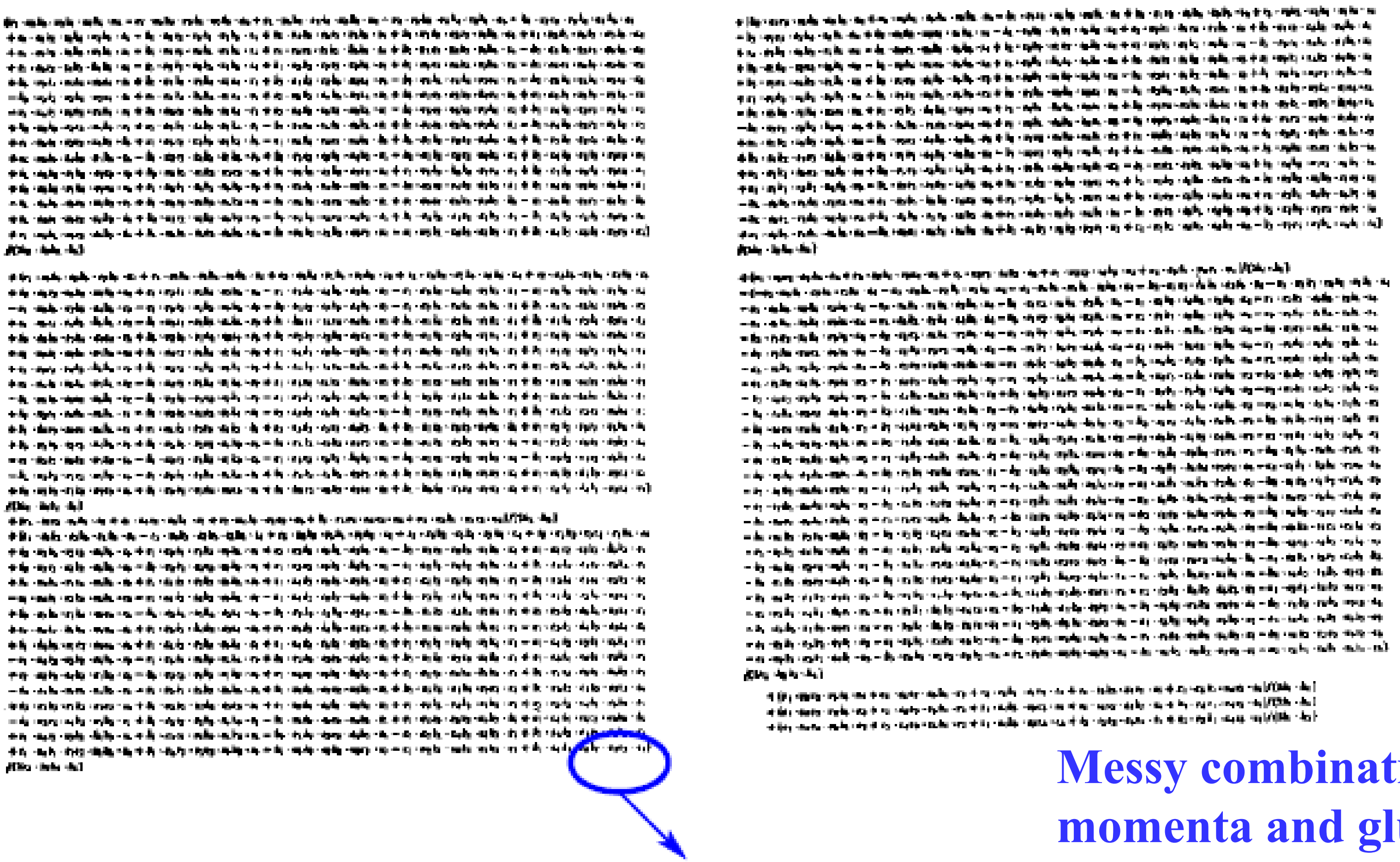
Consider the five-gluon amplitude:



Used in calculation of *pp* ⟶ 3 jets at CERN

If you evaluate this following textbook Feynman rules you find…

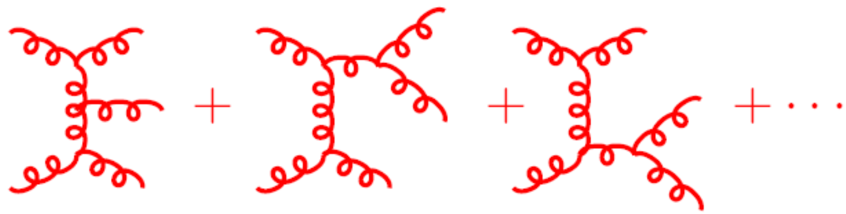
Result of evaluation (actually only a small part of it):



$$k_1 \cdot k_4 \, \varepsilon_2 \cdot k_1 \, \varepsilon_1 \cdot \varepsilon_3 \, \varepsilon_4 \cdot \varepsilon_5$$

Messy combination of momenta and gluon polarization vectors.

Reconsider Five-Gluon Tree



With a little helicity state (circular polarization) magic:

Xu, Zhang and Chang
and many others

$$A_5^{\text{tree}}(1^\pm, 2^+, 3^+, 4^+, 5^+) = 0$$

$$A_5^{\text{tree}}(1^-, 2^-, 3^+, 4^+, 5^+) = i\frac{\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 45\rangle\langle 51\rangle}$$

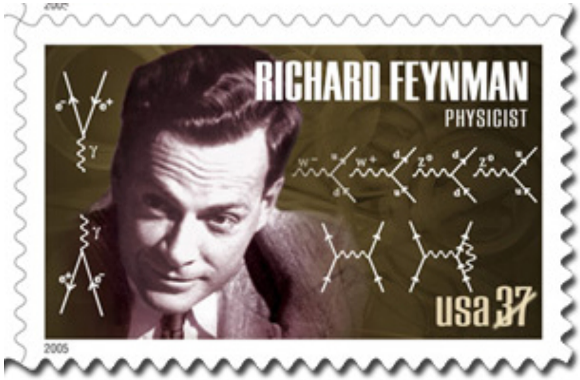
$$A_5^{\text{tree}}(1^-, 2^+, 3^-, 4^+, 5^+) = i\frac{\langle 13\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 45\rangle\langle 51\rangle}$$

Same physical information as on previous page.

Using helicity states are good idea!

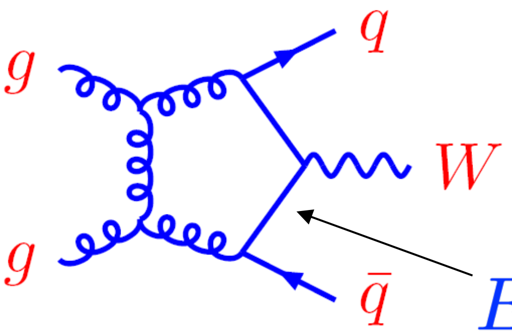
Will be very helpful when we compute gravitational Hamiltonian

Why are Feynman diagrams difficult for high-loop or high-multiplicity processes?



Feynman diagrams involve unphysical gauge-dependent off-shell states.

$$\int \frac{d^3\vec{p}\,dE}{(2\pi)^4}$$



$$E^2 - \vec{p}^{\,2} \neq m^2$$

Individual Feynman diagrams unphysical

Einstein's relation between momentum and energy violated in the loops. **Unphysical states! Not gauge invariant.**

- **Use gauge invariant on-shell physical states.**
- **On-shell formalism. Avoid virtual particles.**
- **Don't violate Einstein's relation!** ZB, Dixon, Dunbar, Kosower (1998)

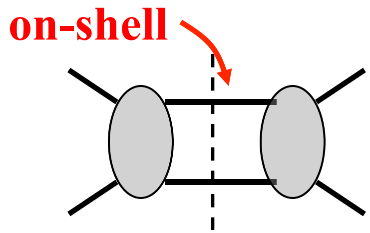
From Tree to Loops: Generalized Unitarity Method

Use tree amplitudes to build higher order (loop) amplitudes.

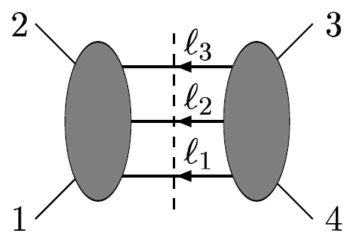
$$E^2 = \vec{p}^{\,2} + m^2$$ ← on-shell

ZB, Dixon, Dunbar and Kosower (1994)

Two-particle cut:

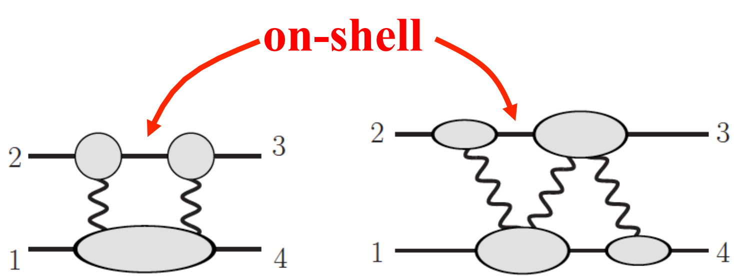


Three-particle cut:



- **Systematic assembly of complete loop amplitudes from tree amplitudes.**
- **Works for any number of particles or loops.**

Generalized unitarity as a practical tool for loops.



ZB, Dixon and Kosower;
ZB, Morgan;
Britto, Cachazo, Feng;
Ossala,Pittau,Papadopoulos;
Ellis, Kunszt, Melnikov;
Forde; Badger;
ZB, Carrasco, Johansson, Kosower
and many others

Idea used in the "NLO revolution" in QCD collider physics.
Want to apply it to gravitational wave problem.

Gravity vs Gauge Theory

Consider the Einstein gravity Lagrangian

$$L_{\text{gravity}} = \frac{2}{\kappa^2}\sqrt{-g}\,R$$

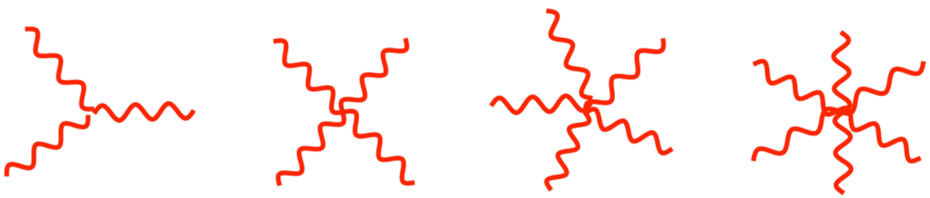
(curvature: R)

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$$

(metric: $g_{\mu\nu}$; Flat-space metric: $\eta_{\mu\nu}$; graviton field: $h_{\mu\nu}$)

$$\kappa^2 = 32\pi G_{\text{Newton}}$$

Infinite number of complicated interactions

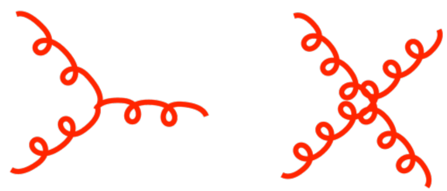


$+ \cdots$

terrible mess

Compare to gauge-theory Lagrangian on which QCD is based

$$L_{\text{YM}} = \frac{1}{g^2}F^2$$



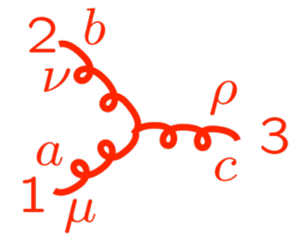
Only three and four point interactions

Gravity seems so much more complicated than gauge theory.

Gauge and gravity theories seem rather different.

Three Vertices

Standard perturbative approach:



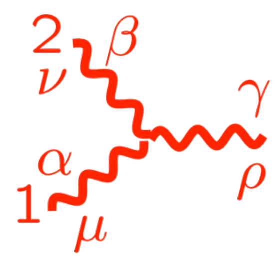
Three-gluon vertex:

$$V_{3\,\mu\nu\sigma}^{abc} = -g f^{abc} (\eta_{\mu\nu}(k_1 - k_2)_\rho + \eta_{\nu\rho}(k_1 - k_2)_\mu + \eta_{\rho\mu}(k_1 - k_2)_\nu)$$

Three-graviton vertex:

$$k_i^2 = E_i^2 - \vec{k}_i^2 \neq 0$$

$$\begin{aligned} G_{3\mu\alpha,\nu\beta,\sigma\gamma}(k_1, k_2, k_3) = \\ \text{sym}[-\tfrac{1}{2} P_3(k_1 \cdot k_2 \eta_{\mu\alpha}\eta_{\nu\beta}\eta_{\sigma\gamma}) - \tfrac{1}{2} P_6(k_{1\nu} k_{1\beta} \eta_{\mu\alpha}\eta_{\sigma\gamma}) + \tfrac{1}{2} P_3(k_1 \cdot k_2 \eta_{\mu\nu}\eta_{\alpha\beta}\eta_{\sigma\gamma}) \\ + P_6(k_1 \cdot k_2 \eta_{\mu\alpha}\eta_{\nu\sigma}\eta_{\beta\gamma}) + 2P_3(k_{1\nu} k_{1\gamma} \eta_{\mu\alpha}\eta_{\beta\sigma}) - P_3(k_{1\beta} k_{2\mu} \eta_{\alpha\nu}\eta_{\sigma\gamma}) \\ + P_3(k_{1\sigma} k_{2\gamma} \eta_{\mu\nu}\eta_{\alpha\beta}) + P_6(k_{1\sigma} k_{1\gamma} \eta_{\mu\nu}\eta_{\alpha\beta}) + 2P_6(k_{1\nu} k_{2\gamma} \eta_{\beta\mu}\eta_{\alpha\sigma}) \\ + 2P_3(k_{1\nu} k_{2\mu} \eta_{\beta\sigma}\eta_{\gamma\alpha}) - 2P_3(k_1 \cdot k_2 \eta_{\alpha\nu}\eta_{\beta\sigma}\eta_{\gamma\mu})] \end{aligned}$$



About 100 terms in three vertex

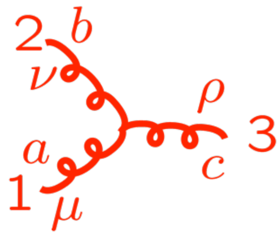
Naïve conclusion: Gravity is a nasty mess.

Simplicity of Gravity Amplitudes

People were looking at gravity amplitudes the wrong way.
On-shell viewpoint much more powerful.

***On-shell* three vertices contains all information:** $E_i^2 - \vec{k}_i^2 = 0$

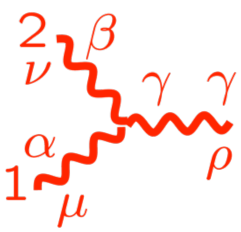
Yang-Mills (QCD) gauge theory:



"color" factor

$$-g f^{abc}(\eta_{\mu\nu}(k_1 - k_2)_\rho + \text{cyclic})$$

Einstein gravity:



$$i\kappa(\eta_{\mu\nu}(k_1 - k_2)_\rho + \text{cyclic}) \times (\eta_{\alpha\beta}(k_1 - k_2)_\gamma + \text{cyclic})$$

"square" of Yang-Mills vertex.

Gravitons are like two gluons!

KLT Relation Between Gravity and Gauge Theory

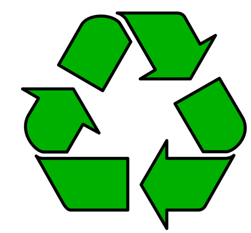
KLT (1985)

Kawai-Lewellen-Tye string relations in low-energy limit:

gravity ↙ gauge-theory color ordered ↙

$$M_4^{\rm tree}(1,2,3,4) = -is_{12} A_4^{\rm tree}(1,2,3,4)\, A_4^{\rm tree}(1,2,4,3)\,,$$

$$M_5^{\rm tree}(1,2,3,4,5) = is_{12}s_{34} A_5^{\rm tree}(1,2,3,4,5)\, A_5^{\rm tree}(2,1,4,3,5) + is_{13}s_{24} A_5^{\rm tree}(1,3,2,4,5)\, A_5^{\rm tree}(3,1,4,2,5)$$



Generalizes to explicit all-leg form. ZB, Dixon, Perelstein, Rozowsky

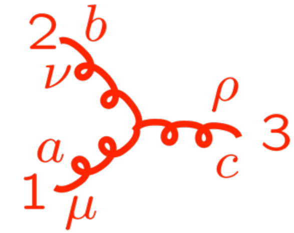
1. **Gravity is derivable from gauge theory.**
2. **Standard Lagrangian methods offer no hint why this is possible.**
3. **It is very general property of gravity.**

Duality Between Color and Kinematics

ZB, Carrasco, Johansson (2007)

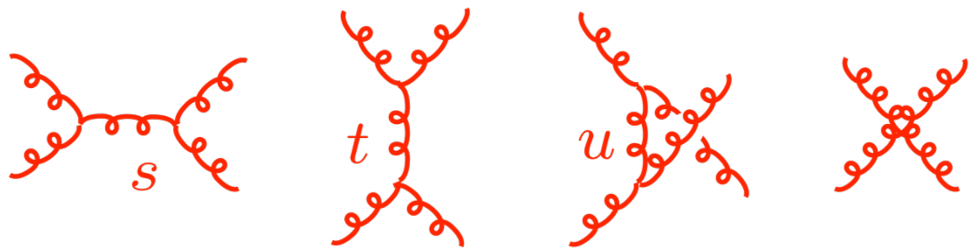
coupling constant → **color factor** ← **momentum dependent kinematic factor**

$$-g f^{abc}(\eta_{\mu\nu}(k_1 - k_2)_\rho + \text{cyclic})$$



Color factors based on a Lie algebra: $[T^a, T^b] = i f^{abc} T^c$

Jacobi Identity $f^{a_1 a_2 b} f^{b a_4 a_3} + f^{a_4 a_2 b} f^{b a_3 a_1} + f^{a_4 a_1 b} f^{b a_2 a_3} = 0$



Use 1 = *s/s* = *t/t* = *u/u* to assign 4-point diagram to others.

$s = (k_1 + k_2)^2 \qquad t = (k_1 + k_4)^2$

$u = (k_1 + k_3)^2$

$$\mathcal{A}_4^{\text{tree}} = g^2\left(\frac{n_s c_s}{s} + \frac{n_t c_t}{t} + \frac{n_u c_u}{u}\right)$$

Color factors satisfy Jacobi identity: $c_u = c_s - c_t$

Numerator factors satisfy similar identity: $n_u = n_s - n_t$

Proven at tree level

ZB, Carrasco, Johansson; Kiermaier; Bjerrum-Bohr, Damgaard, Sondergaard, Vanhove; Cachazo, etc

Duality Between Color and Kinematics

Consider five-point tree amplitude:

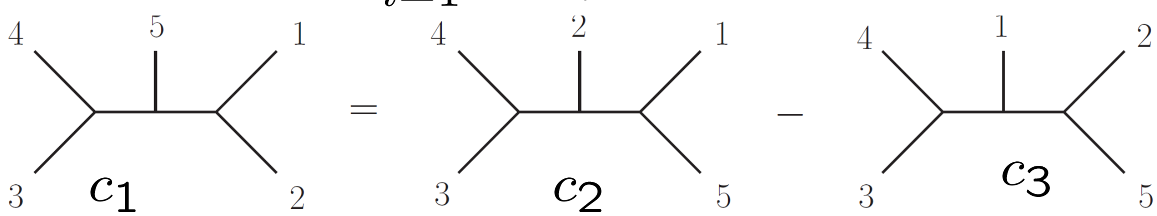
ZB, Carrasco, Johansson (BCJ)

$$\mathcal{A}_5^{\text{tree}} = \sum_{i=1}^{15} \frac{c_i n_i}{\prod_{\alpha_i} p_{\alpha_i}^2}$$

color factor

kinematic numerator factor

Feynman propagators



$$c_1 = f^{a_3 a_4 b} f^{b a_5 c} f^{c a_1 a_2} \qquad c_2 = f^{a_3 a_4 b} f^{b a_2 c} f^{c a_1 a_5} \qquad c_3 = f^{a_3 a_4 b} f^{b a_1 c} f^{c a_2 a_5}$$

$$n_i \sim k_4 \cdot k_5 \, k_2 \cdot \varepsilon_1 \, \varepsilon_2 \cdot \varepsilon_3 \, \varepsilon_4 \cdot \varepsilon_5 + \cdots$$

$$\boxed{c_1 + c_2 + c_3 = 0 \;\Leftrightarrow\; n_1 + n_2 + n_3 = 0}$$

Claim: We can always find a rearrangement so color and kinematics satisfy the *same* algebraic constraint equations.

Very amusing, but why is this interesting?

Gravity from Gauge Theory

ZB, Carrasco, Johansson

gauge theory (QCD): $\mathcal{A}_n^{\text{tree}} = i g^{n-2} \sum_i \frac{c_i\, n_i}{D_i}$

color factor

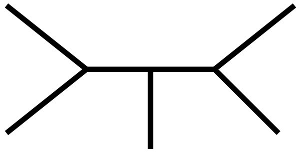
kinematic numerator factor

Feynman propagators

$c_k = c_i - c_j$

$n_k = n_i - n_j$

$c_i \to n_i$



sum over diagrams with only 3 vertices

Einstein gravity: $\mathcal{M}_n^{\text{tree}} = i \kappa^{n-2} \sum_i \frac{n_i^2}{D_i}$

$n_i \sim k_4 \cdot k_5\, k_2 \cdot \varepsilon_1\, \varepsilon_2 \cdot \varepsilon_3\, \varepsilon_4 \cdot \varepsilon_5 + \cdots$

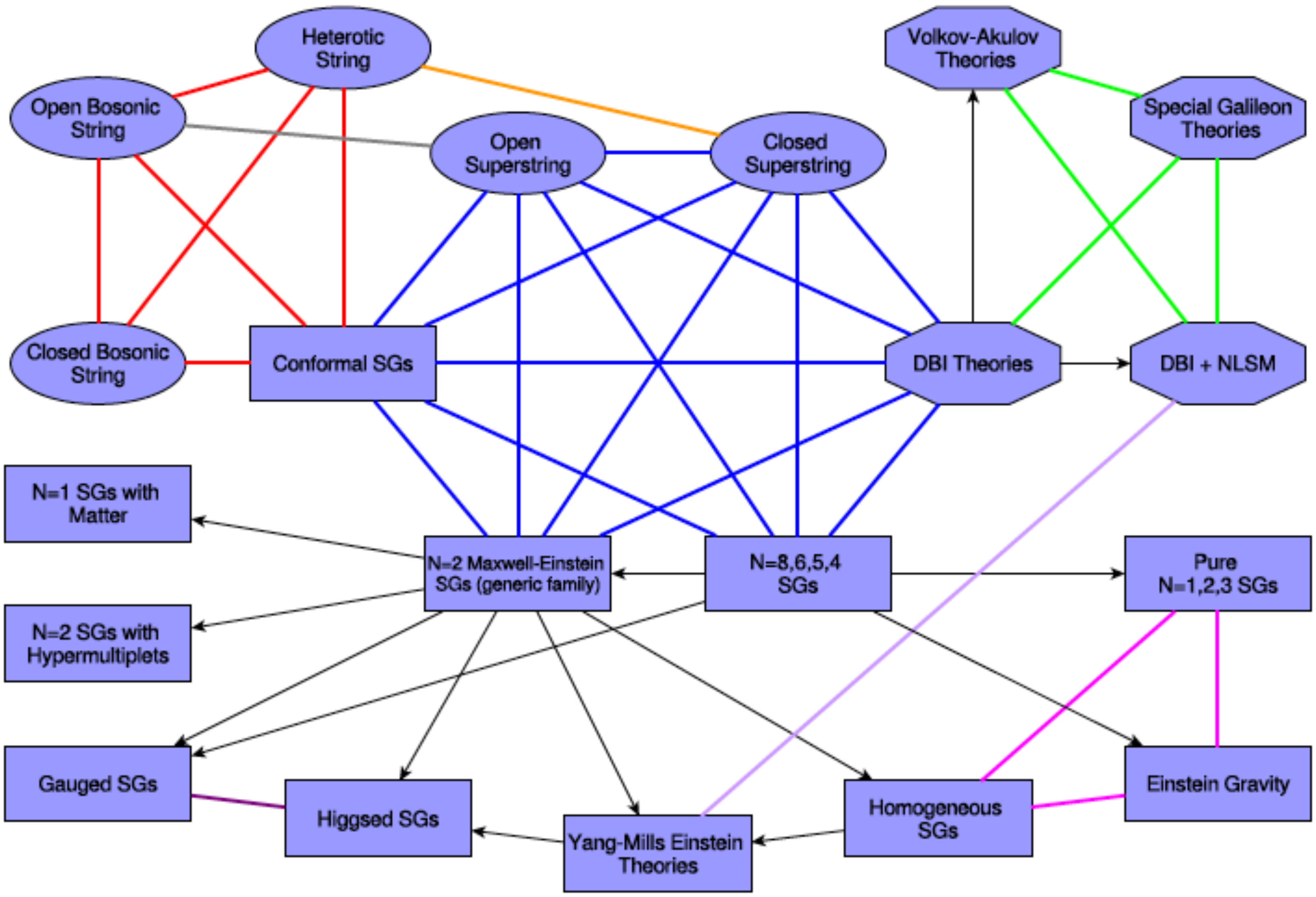
Gravity and gauge theory kinematic numerators are the same!

Underlying physical reason still unclear.

Cries out for a unified description of gravity with gauge theory, presumably along the lines of string theory.

Web of Theories

ZB, Carrasco, Chiodaroli, Johansson, Roiban arXiv:1909.01358, Section 5.



Double copy links various theories through their component theories.

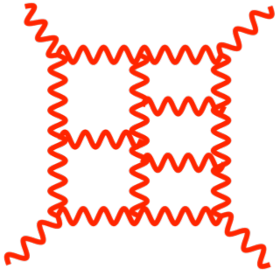
Summary

In a very precise sense:

Gravity ~ (gauge theory) x (gauge theory)

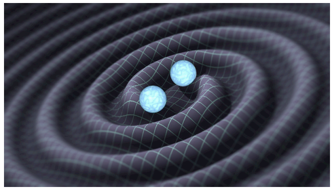
- **Gives us a good way to carry out calculations.**
- **Use it to do difficult calculations, to answer questions of physical interest.**

Examples:

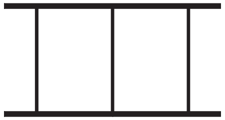


- **5 loop supergravity to study nonrenormalizability of gravity theories.**

ZB, Carrasco, Chen, Edison, Johansson, Roiban, Parra-Martinez, Zeng (2018)



- **G^4 corrections to Newton's potential from GR.**



ZB, Cheung, Roiban, Shen, Solon, Zeng (2019)
ZB, Parra-Martinez, Roiban, Ruf. Shen, Solon, Zeng (2021)

What are we after?

- Replace scattering in General Relativity with a two body potential that is easy to use in bound-state problem.
- Extract physics juice, leaving behind complexity of general relativity.

$$V(\mathbf{r}, \mathbf{p}) = -\frac{G m_1 m_2}{r} + \cdots$$

Just like Newton's potential, except:

- **Compatible with special relativity (all orders in velocity)**
- **Valid through $O(G^4)$.**

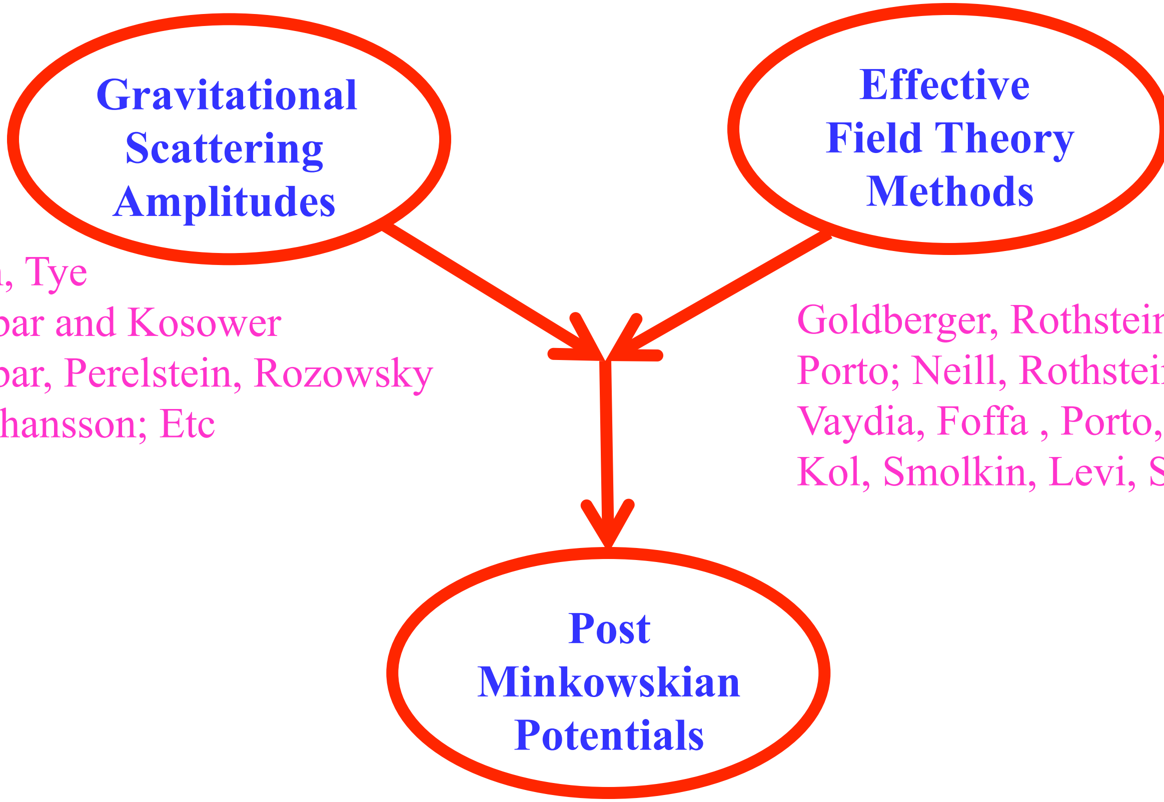
Effective Field Theory Approach

ZB, Cheung, Roiban, Shen, Solon, Zeng

Cheung, Rothstein, Solon (2018)

Amplitudes community

EFT community



Kawai, Lewellen, Tye
ZB, Dixon, Dunbar and Kosower
ZB, Dixon, Dunbar, Perelstein, Rozowsky
ZB, Carrasco, Johansson; Etc

Goldberger, Rothstein;
Porto; Neill, Rothstein;
Vaydia, Foffa , Porto, Rothstein, Sturani;
Kol, Smolkin, Levi, Steinhoff, etc.

Roundabout: **Start with quantum theory and take** $\hbar \to 0$

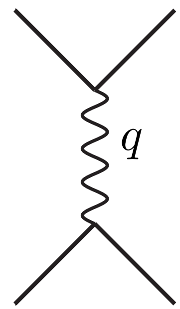
Efficiency: **Almost magical simplifications for gravity amplitudes.**

2 Body Potentials and Amplitudes

Iwasaki; Gupta, Radford; Donoghue; Holstein, Donoghue; Holstein and A. Ross; Bjerrum-Bohr, Donoghue, Vanhove; Neill, Rothstein; Bjerrum-Bohr, Damgaard, Festuccia, Planté. Vanhove; Chueng, Rothstein, Solon; Chung, Huang, Kim, Lee; etc.

Tree-level: Fourier transform gives classical potential.

$$V(r) \sim \int \frac{d^3q}{(2\pi)^3} e^{-i\mathbf{q}\cdot\mathbf{r}} A^{\text{tree}}(\mathbf{q})$$



Newtonian potential follows from Feynman diagrams

Beyond 1 loop things quickly become much less obvious:

- **What I learned in grad school on $\hbar$ and classical limits is wrong! Loops have classical pieces.**
- $1/\hbar^L$ **scaling of at *L* loop.**
- **Double counting and iteration.**
- **Cross terms between $1/\hbar$ and $\hbar$.**

$$e^{iS_{\text{classical}}/\hbar}$$

Piece of loops are classical: Our task is to efficiently extract these pieces.

We harness EFT to clean up confusion

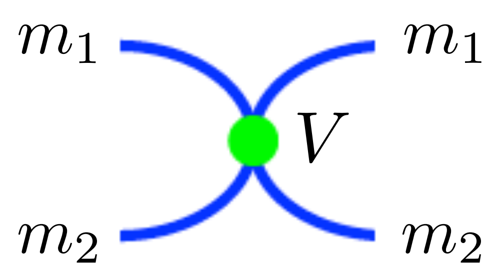
Effective Field Theory is a Clean Approach

Build EFT from which we can read off potential.
Want a Newtonian-like potential,
with GR corrections

Goldberger and Rothstein
Neill, Rothstein
Cheung, Rothstein, Solon (2018)

$$L_{\text{kin}} = \int_{\boldsymbol{k}} A^\dagger(-\boldsymbol{k}) \left(i\partial_t + \sqrt{\boldsymbol{k}^2 + m_A^2} \right) A(\boldsymbol{k}) + \int_{\boldsymbol{k}} B^\dagger(-\boldsymbol{k}) \left(i\partial_t + \sqrt{\boldsymbol{k}^2 + m_B^2} \right) B(\boldsymbol{k})$$

***A*, *B* scalars represents spinless black holes**



$$L_{\text{int}} = -\int_{\boldsymbol{k},\boldsymbol{k}'} V(\boldsymbol{k},\boldsymbol{k}')\, A^\dagger(\boldsymbol{k}') A(\boldsymbol{k}) B^\dagger(-\boldsymbol{k}') B(-\boldsymbol{k})$$

potential we want to obtain

$$H(\boldsymbol{p},\boldsymbol{r}) = \sqrt{\boldsymbol{p}^2 + m_1^2} + \sqrt{\boldsymbol{p}^2 + m_2^2} + V(\boldsymbol{p},\boldsymbol{r})$$

2 body Hamiltonian in c.o.m. frame.

Match amplitudes of this theory to the full theory in classical limit to extract a classical potential of the type Newton would like.

Our LIGO/Virgo theory friends want Hamiltonians.

EFT Matching

Cheung, Rothstein, Solon

full general relativity
(complicated)

Amplitude methods
double copy

↓

tree amplitude

generalized
unitarity
$\hbar \to 0$

↓

loop integrand

loop
integration

↓

GR loop amplitude

identical physics

$=$

effective theory
(simpler)

build
ansatz

↓

Potential ***V(r)***

Feynman
diagrams

↓

loop integrand

loop
integration

↓

EFT loop amplitude

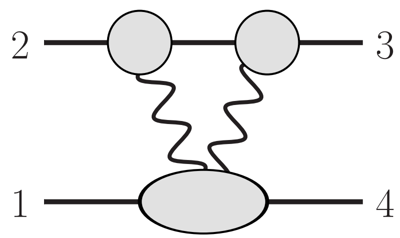
Roundabout, but robust mean to extracts potential.
New methods bypass this, directly giving radial action.

General Relativity: Unitarity + Double Copy

- **Long-range force: Two matter lines must be separated by on-shell propagators.**
- **Classical potential: 1 matter line per loop is cut (on-shell).**

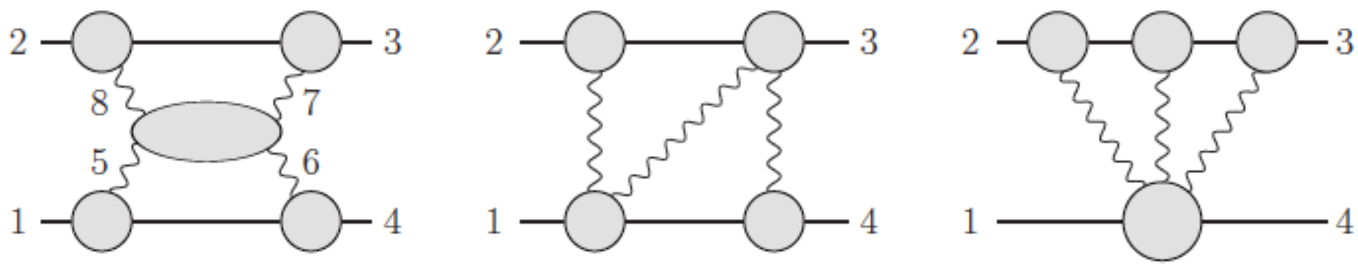
Neill and Rothstein ; Bjerrum-Bohr, Damgaard, Festuccia, Planté, Vanhove; Cheung, Rothstein, Solon

Only independent unitarity cut for 2 PM.

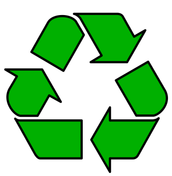


Treat exposed lines on-shell (long range).
Pieces we want are simple!

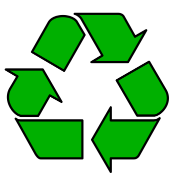
Independent generalized unitarity cuts for 3 PM.



Our amplitude tools fit perfectly with extracting pieces we want.

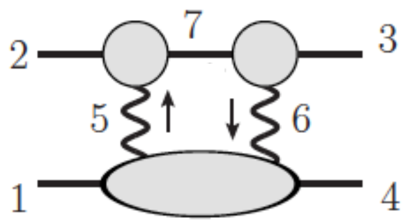


gravity



loops

Generalized Unitarity Cuts



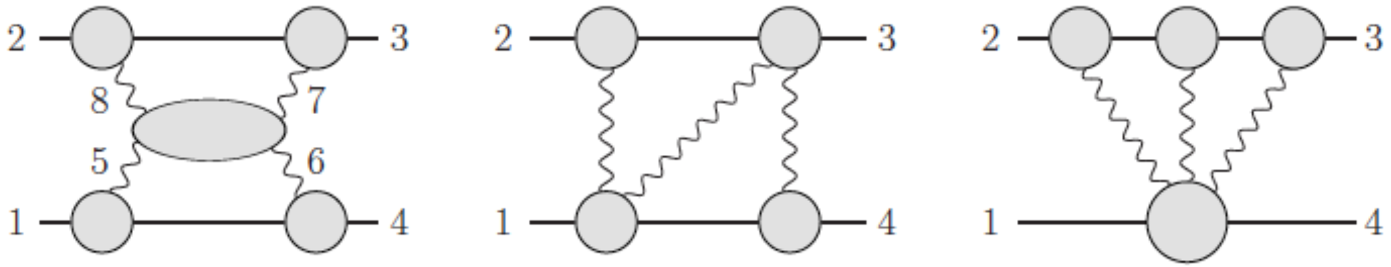
2nd post-Minkowkian order

$$
\begin{aligned}
C_{\rm GR} &= \sum_{h_5,h_6=\pm} M_3^{\rm tree}(3^s,6^{h_6},-7^s)\, M_3^{\rm tree}(7^s,-5^{h_5},2^s)\, M_4^{\rm tree}(1^s,5^{-h_5},-6^{-h_6},4^s) \\
&= \sum_{h_5,h_6=\pm} it[A_3^{\rm tree}(3^s,6^{h_6},-7^s)\, A_3^{\rm tree}(7^s,-5^{h_5},2^s)\, A_4^{\rm tree}(1^s,5^{-h_5},-6^{-h_6},4^s)] \\
&\qquad \times [A_3^{\rm tree}(3^s,6^{h_6},-7^s)\, A_3^{\rm tree}(7^s,-5^{h_5},2^s)\, A_4^{\rm tree}(4^s,5^{-h_5},-6^{-h_6},1^s)]
\end{aligned}
$$

Problem of computing the generalized cuts in gravity is reduced to multiplying and summing gauge-theory tree amplitudes.

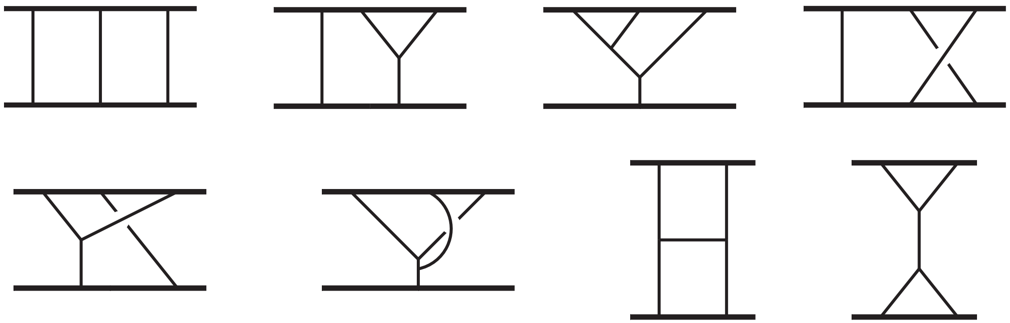
This is then fed into integration and EFT matching

Two Loops and 3 PM



- More complicated than one loop, but no problem.
- To interface easily with EFT approach, rearrange unitarity cuts into conventional-looking diagrams.

Integrand organized into 8 independent diagrams that may contribute in classical limit:



Integrate using methods of Cheung, Rothstein and Solon.

Amplitude in Conservative Classical Potential Limit

ZB, Cheung, Roiban, Shen, Solon, Zeng (BCRSSZ)

The O(G^3) or 3PM conservative terms are:

$$\mathcal{M}_3 = \frac{\pi G^3 \nu^2 m^4 \log \boldsymbol{q}^2}{6\gamma^2 \xi}\Bigg[3 - 6\nu + 206\nu\sigma - 54\sigma^2 + 108\nu\sigma^2 + 4\nu\sigma^3 - \frac{48\nu\left(3 + 12\sigma^2 - 4\sigma^4\right)\operatorname{arcsinh}\sqrt{\frac{\sigma-1}{2}}}{\sqrt{\sigma^2-1}} - \frac{18\nu\gamma\left(1-2\sigma^2\right)\left(1-5\sigma^2\right)}{(1+\gamma)(1+\sigma)}\Bigg] + \frac{8\pi^3 G^3 \nu^4 m^6}{\gamma^4 \xi}\Big[3\gamma\left(1-2\sigma^2\right)\left(1-5\sigma^2\right)F_1 - 32 m^2 \nu^2 \left(1-2\sigma^2\right)^3 F_2\Big]$$

$$m = m_A + m_B, \qquad \mu = m_A m_B/m, \qquad \nu = \mu/m, \qquad \gamma = E/m,$$
$$\xi = E_1 E_2/E^2, \qquad E = E_1 + E_2, \qquad \sigma = p_1 \cdot p_2 / m_1 m_2,$$

- **Amplitude remarkably compact.**
- **Arcsinh and the appearance of a mass singularity is new and robust feature. Cancels mass singularity of real radiation, as expected from KLN theorem.**
  Di Vecchia, Heissenberg, Russo, Veneziano; Damour
- **IR finite parts of amplitude directly connected to scattering angle.**
  Expanded on by Kälin, Porto; Bjerrum-Bohr, Cristofoli, Damgaard
- **Derived conservative scattering angle has simple mass dependence.**
  Observed by Antonelli, Buonanno, Steinhoff, van de Meent, Vines (1901.07102)
  Comprehensive understanding: Damour

Conservative 3PM Hamiltonian

ZB, Cheung, Roiban, Shen, Solon, Zeng (2019)

The 3PM Hamiltonian:

$$H(\boldsymbol{p},\boldsymbol{r}) = \sqrt{\boldsymbol{p}^2+m_1^2} + \sqrt{\boldsymbol{p}^2+m_2^2} + V(\boldsymbol{p},\boldsymbol{r})$$

$$V(\boldsymbol{p},\boldsymbol{r}) = \sum_{i=1}^{3} c_i(\boldsymbol{p}^2)\left(\frac{G}{|\boldsymbol{r}|}\right)^i ,$$

Newton in here

$$c_1 = \frac{\nu^2 m^2}{\gamma^2\xi}\left(1-2\sigma^2\right), \qquad c_2 = \frac{\nu^2 m^3}{\gamma^2\xi}\left[\frac{3}{4}\left(1-5\sigma^2\right) - \frac{4\nu\sigma\left(1-2\sigma^2\right)}{\gamma\xi} - \frac{\nu^2(1-\xi)\left(1-2\sigma^2\right)^2}{2\gamma^3\xi^2}\right],$$

$$\begin{aligned} c_3 = \frac{\nu^2 m^4}{\gamma^2\xi}\Bigg[&\frac{1}{12}\left(3-6\nu+206\nu\sigma-54\sigma^2+108\nu\sigma^2+4\nu\sigma^3\right) - \frac{4\nu\left(3+12\sigma^2-4\sigma^4\right)\operatorname{arcsinh}\sqrt{\frac{\sigma-1}{2}}}{\sqrt{\sigma^2-1}} \\ &- \frac{3\nu\gamma\left(1-2\sigma^2\right)\left(1-5\sigma^2\right)}{2(1+\gamma)(1+\sigma)} - \frac{3\nu\sigma\left(7-20\sigma^2\right)}{2\gamma\xi} - \frac{\nu^2\left(3+8\gamma-3\xi-15\sigma^2-80\gamma\sigma^2+15\xi\sigma^2\right)\left(1-2\sigma^2\right)}{4\gamma^3\xi^2} \\ &+ \frac{2\nu^3(3-4\xi)\sigma\left(1-2\sigma^2\right)^2}{\gamma^4\xi^3} + \frac{\nu^4(1-2\xi)\left(1-2\sigma^2\right)^3}{2\gamma^6\xi^4}\Bigg], \end{aligned}$$

$$m = m_A + m_B, \qquad \mu = m_A m_B/m, \qquad \nu = \mu/m, \qquad \gamma = E/m,$$

$$\xi = E_1 E_2/E^2, \qquad E = E_1 + E_2, \qquad \sigma = p_1\cdot p_2/m_1 m_2,$$

How do we know it is right?

Original checks:

- **Compared to 4PN Hamiltonians after canonical transformation**
  Damour, Jaranowski, Schäfer; Bernard, Blanchet, Bohé, Faye, Marsat
- **In test mass limit, $m_1 \ll m_2$, matches Schwarzschild Hamiltonian**

Thibault Damour seriously questioned correctness.
Specific corrections proposed. Damour, arXiv:1912.02139v1

New calculations confirm our 3PM result:

1. **Subsequent papers confirm our result in 6PN overlap.** Blümlein, Maier, Marquard, Schäfer; Bini, Damour, Geralico
2. **New calculations reproducing our 3PM result.** Cheung and Solon; Kälin, Liu, Porto
3. **Scattering angle check.** ZB, Ita, Parra-Martinez, Ruf
4. **Adding real radiation removes mass singularity.** Di Vecchia, Heissenberg, Russo, Veneziano; Damour

3PM results have passed highly nontrivial checks and careful scrutiny.

How do we know it is right?

Primary check:

ZB, Cheung, Roiban, Shen, Solon, Zeng

Compare to 4PN Hamiltonian of Damour, Jaranowski, Schäfer, used in high precision template constructions.

Need canonical transformation in overlap:

preserve Poisson bracket

$$(\boldsymbol{r}, \boldsymbol{p}) \to (\boldsymbol{R}, \boldsymbol{P}) = (A\,\boldsymbol{r} + B\,\boldsymbol{p}, C\,\boldsymbol{p} + D\,\boldsymbol{r})$$

$$A = 1 - \frac{Gm\nu}{2|\boldsymbol{r}|} + \cdots\,, \quad B = \frac{G(1-2/\nu)}{4m|\boldsymbol{r}|}\boldsymbol{p}\cdot\boldsymbol{r} + \cdots$$

$$C = 1 + \frac{Gm\nu}{2|\boldsymbol{r}|} + \cdots\,, \quad D = -\frac{Gm\nu}{2|\boldsymbol{r}|^3}\boldsymbol{p}\cdot\boldsymbol{r} + \cdots\,,$$

Our Hamiltonian equivalent to 4PN Hamiltonian on overlap.

4 PN Hamiltonian

Damour, Jaranowski, Schaefer

$$\mathbf{n} = \hat{\mathbf{r}}$$

$$\widehat{H}_{\mathrm{N}}(\mathbf{r},\mathbf{p}) = \frac{\mathbf{p}^2}{2} - \frac{1}{r},$$

$$c^2\,\widehat{H}_{1\mathrm{PN}}(\mathbf{r},\mathbf{p}) = \frac{1}{8}(3\nu-1)(\mathbf{p}^2)^2 - \frac{1}{2}\Big\{(3+\nu)\mathbf{p}^2 + \nu(\mathbf{n}\cdot\mathbf{p})^2\Big\}\frac{1}{r} + \frac{1}{2r^2},$$

$$c^4\,\widehat{H}_{2\mathrm{PN}}(\mathbf{r},\mathbf{p}) = \frac{1}{16}\left(1-5\nu+5\nu^2\right)(\mathbf{p}^2)^3 + \frac{1}{8}\Big\{\left(5-20\nu-3\nu^2\right)(\mathbf{p}^2)^2 - 2\nu^2(\mathbf{n}\cdot\mathbf{p})^2\mathbf{p}^2 - 3\nu^2(\mathbf{n}\cdot\mathbf{p})^4\Big\}\frac{1}{r}$$
$$+ \frac{1}{2}\Big\{(5+8\nu)\mathbf{p}^2 + 3\nu(\mathbf{n}\cdot\mathbf{p})^2\Big\}\frac{1}{r^2} - \frac{1}{4}(1+3\nu)\frac{1}{r^3},$$

$$c^6\,\widehat{H}_{3\mathrm{PN}}(\mathbf{r},\mathbf{p}) = \frac{1}{128}\left(-5+35\nu-70\nu^2+35\nu^3\right)(\mathbf{p}^2)^4 + \frac{1}{16}\Big\{\left(-7+42\nu-53\nu^2-5\nu^3\right)(\mathbf{p}^2)^3$$
$$+ (2-3\nu)\nu^2(\mathbf{n}\cdot\mathbf{p})^2(\mathbf{p}^2)^2 + 3(1-\nu)\nu^2(\mathbf{n}\cdot\mathbf{p})^4\mathbf{p}^2 - 5\nu^3(\mathbf{n}\cdot\mathbf{p})^6\Big\}\frac{1}{r}$$
$$+ \Big\{\frac{1}{16}\left(-27+136\nu+109\nu^2\right)(\mathbf{p}^2)^2 + \frac{1}{16}(17+30\nu)\nu(\mathbf{n}\cdot\mathbf{p})^2\mathbf{p}^2 + \frac{1}{12}(5+43\nu)\nu(\mathbf{n}\cdot\mathbf{p})^4\Big\}\frac{1}{r^2}$$
$$+ \Big\{\Big(-\frac{25}{8} + \Big(\frac{\pi^2}{64} - \frac{335}{48}\Big)\nu - \frac{23\nu^2}{8}\Big)\mathbf{p}^2 + \Big(-\frac{85}{16} - \frac{3\pi^2}{64} - \frac{7\nu}{4}\Big)\nu(\mathbf{n}\cdot\mathbf{p})^2\Big\}\frac{1}{r^3} + \Big\{\frac{1}{8} + \Big(\frac{109}{12} - \frac{21}{32}\pi^2\Big)\nu\Big\}\frac{1}{r^4},$$

G^4

4 PN Hamiltonian

Damour, Jaranowski, Schaefer

$$\mathbf{n} = \hat{\mathbf{r}}$$

$$
\begin{aligned}
c^8 \widehat{H}^{\rm local}_{\rm 4PN}(\mathbf{r},\mathbf{p}) = {} & \left(\frac{7}{256} - \frac{63}{256}\nu + \frac{189}{256}\nu^2 - \frac{105}{128}\nu^3 + \frac{63}{256}\nu^4\right)(\mathbf{p}^2)^5 \\
& + \left\{\frac{45}{128}(\mathbf{p}^2)^4 - \frac{45}{16}(\mathbf{p}^2)^4\,\nu + \left(\frac{423}{64}(\mathbf{p}^2)^4 - \frac{3}{32}(\mathbf{n}\cdot\mathbf{p})^2(\mathbf{p}^2)^3 - \frac{9}{64}(\mathbf{n}\cdot\mathbf{p})^4(\mathbf{p}^2)^2\right)\nu^2 \right. \\
& + \left(-\frac{1013}{256}(\mathbf{p}^2)^4 + \frac{23}{64}(\mathbf{n}\cdot\mathbf{p})^2(\mathbf{p}^2)^3 + \frac{69}{128}(\mathbf{n}\cdot\mathbf{p})^4(\mathbf{p}^2)^2 - \frac{5}{64}(\mathbf{n}\cdot\mathbf{p})^6\mathbf{p}^2 + \frac{35}{256}(\mathbf{n}\cdot\mathbf{p})^8\right)\nu^3 \\
& \left. + \left(-\frac{35}{128}(\mathbf{p}^2)^4 - \frac{5}{32}(\mathbf{n}\cdot\mathbf{p})^2(\mathbf{p}^2)^3 - \frac{9}{64}(\mathbf{n}\cdot\mathbf{p})^4(\mathbf{p}^2)^2 - \frac{5}{32}(\mathbf{n}\cdot\mathbf{p})^6\mathbf{p}^2 - \frac{35}{128}(\mathbf{n}\cdot\mathbf{p})^8\right)\nu^4\right\}\frac{1}{r} \\
& + \left\{\frac{13}{8}(\mathbf{p}^2)^3 + \left(-\frac{791}{64}(\mathbf{p}^2)^3 + \frac{49}{16}(\mathbf{n}\cdot\mathbf{p})^2(\mathbf{p}^2)^2 - \frac{889}{192}(\mathbf{n}\cdot\mathbf{p})^4\mathbf{p}^2 + \frac{369}{160}(\mathbf{n}\cdot\mathbf{p})^6\right)\nu \right. \\
& + \left(\frac{4857}{256}(\mathbf{p}^2)^3 - \frac{545}{64}(\mathbf{n}\cdot\mathbf{p})^2(\mathbf{p}^2)^2 + \frac{9475}{768}(\mathbf{n}\cdot\mathbf{p})^4\mathbf{p}^2 - \frac{1151}{128}(\mathbf{n}\cdot\mathbf{p})^6\right)\nu^2 \\
& \left. + \left(\frac{2335}{256}(\mathbf{p}^2)^3 + \frac{1135}{256}(\mathbf{n}\cdot\mathbf{p})^2(\mathbf{p}^2)^2 - \frac{1649}{768}(\mathbf{n}\cdot\mathbf{p})^4\mathbf{p}^2 + \frac{10353}{1280}(\mathbf{n}\cdot\mathbf{p})^6\right)\nu^3\right\}\frac{1}{r^2} \\
& + \left\{\frac{105}{32}(\mathbf{p}^2)^2 + \left(\left(\frac{2749\pi^2}{8192} - \frac{589189}{19200}\right)(\mathbf{p}^2)^2 + \left(\frac{63347}{1600} - \frac{1059\pi^2}{1024}\right)(\mathbf{n}\cdot\mathbf{p})^2\mathbf{p}^2 + \left(\frac{375\pi^2}{8192} - \frac{23533}{1280}\right)(\mathbf{n}\cdot\mathbf{p})^4\right)\nu \right. \\
& + \left(\left(\frac{18491\pi^2}{16384} - \frac{1189789}{28800}\right)(\mathbf{p}^2)^2 + \left(-\frac{127}{3} - \frac{4035\pi^2}{2048}\right)(\mathbf{n}\cdot\mathbf{p})^2\mathbf{p}^2 + \left(\frac{57563}{1920} - \frac{38655\pi^2}{16384}\right)(\mathbf{n}\cdot\mathbf{p})^4\right)\nu^2 \\
& \left. + \left(-\frac{553}{128}(\mathbf{p}^2)^2 - \frac{225}{64}(\mathbf{n}\cdot\mathbf{p})^2\mathbf{p}^2 - \frac{381}{128}(\mathbf{n}\cdot\mathbf{p})^4\right)\nu^3\right\}\frac{1}{r^3} \\
& + \left\{\frac{105}{32}\mathbf{p}^2 + \left(\left(\frac{185761}{19200} - \frac{21837\pi^2}{8192}\right)\mathbf{p}^2 + \left(\frac{3401779}{57600} - \frac{28691\pi^2}{24576}\right)(\mathbf{n}\cdot\mathbf{p})^2\right)\nu \right. \\
& \left. + \left(\left(\frac{672811}{19200} - \frac{158177\pi^2}{49152}\right)\mathbf{p}^2 + \left(\frac{110099\pi^2}{49152} - \frac{21827}{3840}\right)(\mathbf{n}\cdot\mathbf{p})^2\right)\nu^2\right\}\frac{1}{r^4} \quad \longleftarrow G^4 \\
& + \left\{-\frac{1}{16} + \left(\frac{6237\pi^2}{1024} - \frac{169199}{2400}\right)\nu + \left(\frac{7403\pi^2}{3072} - \frac{1256}{45}\right)\nu^2\right\}\frac{1}{r^5}. \quad \longleftarrow G^5
\end{aligned}
$$

After canonical transformation we match all but G^4 and G^5 terms

Mess is partly due to gauge choice.

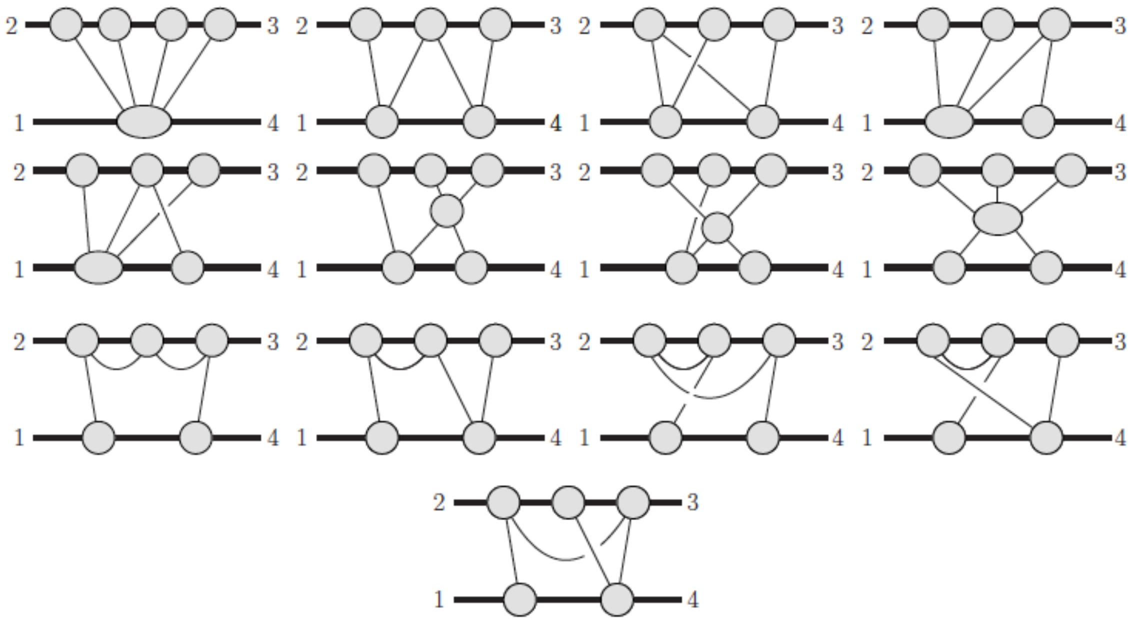
Ours is all orders in p at G^3

Our goal is to not only find new powerful methods but to clean up results

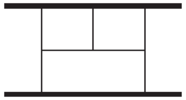
Higher Order Scalability: $O(G^4)$

Methods scale well to higher orders

ZB, Parra-Martinez, Roiban, Ruf, Shen, Solon, Zeng (2022)



Conservative Contribution O(G^4)



ZB, Parra-Martinez, Roiban, Ruf, Shen, Solon, Zeng (2022)

O(G^4) amplitude — **test particle** — **1st self force** — **Iteration. No need to compute**

$$\mathcal{M}_4^{\rm cons} = G^4 M^7 \nu^2 |\boldsymbol{q}| \pi^2 \left[\mathcal{M}_4^{\rm p} + \nu\left(4\mathcal{M}_4^{\rm t}\log\left(\tfrac{p_\infty}{2}\right) + \mathcal{M}_4^{\pi^2} + \mathcal{M}_4^{\rm rem}\right)\right] + \int_\ell \frac{\tilde I_{r,1}^4}{Z_1Z_2Z_3} + \int_\ell \frac{\tilde I_{r,1}^2 \tilde I_{r,2}}{Z_1Z_2} + \int_\ell \frac{\tilde I_{r,1}\tilde I_{r,3}}{Z_1} + \int_\ell \frac{\tilde I_{r,2}^2}{Z_1}$$

tail effect

$$D = 4 - 2\epsilon$$

$$\mathcal{M}_4^{\rm t} = r_1 + r_2 \log\left(\tfrac{\sigma+1}{2}\right) + r_3 \frac{\operatorname{arccosh}(\sigma)}{\sqrt{\sigma^2-1}}, \qquad \mathcal{M}_4^{\rm p} = -\frac{35\left(1 - 18\sigma^2 + 33\sigma^4\right)}{8\left(\sigma^2-1\right)}$$

$$\mathcal{M}_4^{\pi^2} = r_4\pi^2 + r_5\,\mathrm{K}\left(\tfrac{\sigma-1}{\sigma+1}\right)\mathrm{E}\left(\tfrac{\sigma-1}{\sigma+1}\right) + r_6\,\mathrm{K}^2\left(\tfrac{\sigma-1}{\sigma+1}\right) + r_7\,\mathrm{E}^2\left(\tfrac{\sigma-1}{\sigma+1}\right),$$

← elliptic

$$\begin{aligned}\mathcal{M}_4^{\rm rem} = {} & r_8 + r_9 \log\left(\tfrac{\sigma+1}{2}\right) + r_{10}\frac{\operatorname{arccosh}(\sigma)}{\sqrt{\sigma^2-1}} + r_{11}\log(\sigma) + r_{12}\log^2\left(\tfrac{\sigma+1}{2}\right) + r_{13}\frac{\operatorname{arccosh}(\sigma)}{\sqrt{\sigma^2-1}}\log\left(\tfrac{\sigma+1}{2}\right) + r_{14}\frac{\operatorname{arccosh}^2(\sigma)}{\sigma^2-1} \\ & + r_{15}\,\mathrm{Li}_2\left(\tfrac{1-\sigma}{2}\right) + r_{16}\,\mathrm{Li}_2\left(\tfrac{1-\sigma}{1+\sigma}\right) + r_{17}\frac{1}{\sqrt{\sigma^2-1}}\left[\mathrm{Li}_2\left(-\sqrt{\tfrac{\sigma-1}{\sigma+1}}\right) - \mathrm{Li}_2\left(\sqrt{\tfrac{\sigma-1}{\sigma+1}}\right)\right].\end{aligned}$$

$$\nu = m_1 m_2/(m_1+m_2)^2 \qquad \sigma = p_1\cdot p_2/m_1 m_2,$$

r_{ij} rational coefficients

This is *complete* conservative contribution.

$$\mathcal{M}_4^{\rm radgrav,f} = \frac{12044}{75}p_\infty^2 + \frac{212077}{3675}p_\infty^4 + \frac{115917979}{793800}p_\infty^6 - \frac{9823091209}{76839840}p_\infty^8 + \frac{115240251793703}{1038874636800}p_\infty^{10} + \cdots$$

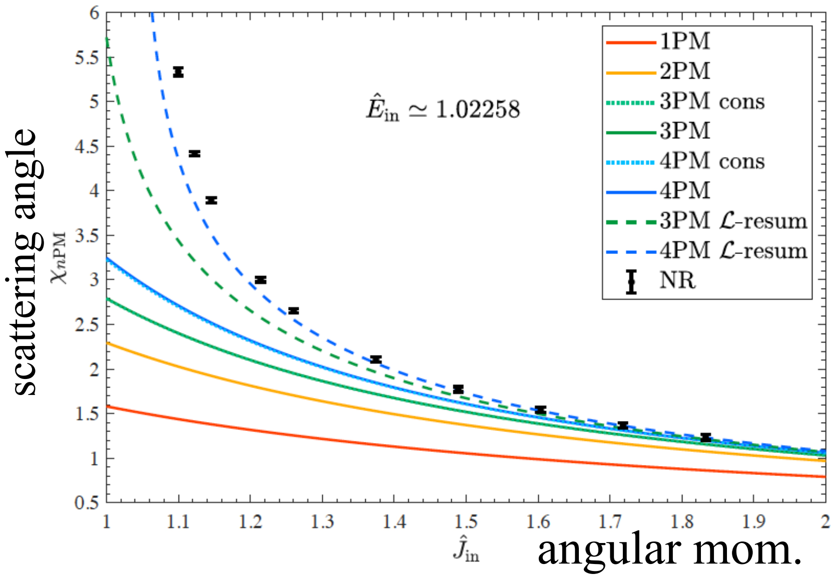
First 3 terms match 6PN results of Bini, Damour, Geralico!

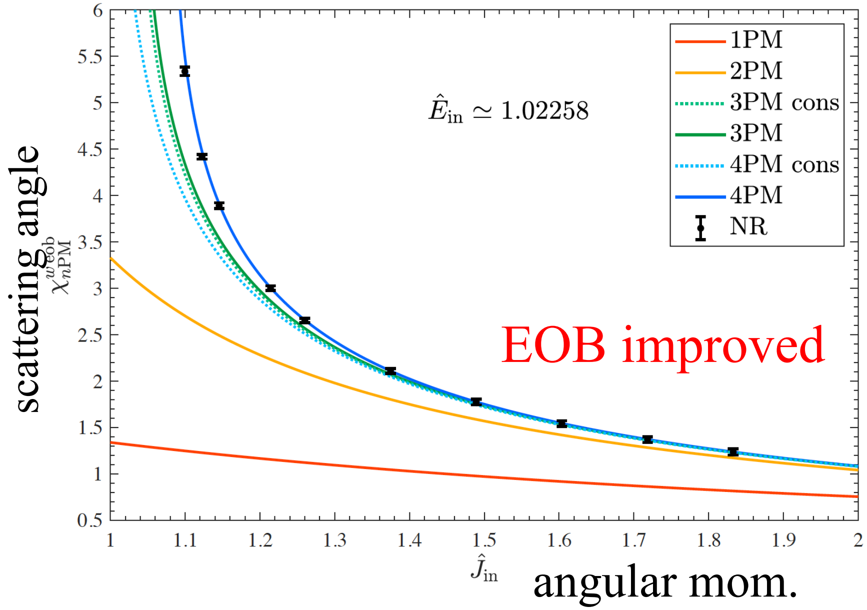
- Some potential disagreement with Blumlein, Maier, Marquard, Schafer; Foffa, Sturani.
- Our PN colleagues are working hard to track down the origin of disagreement.

Comparison with Numerical Relativity

Khalil, Buonanno, Vines, Steinhoff; Damour and Rettego

An improved comparison using new dissipative results:





Plot uses

4PM Conservative: ZB, Parra-Martinez, Roiban, Ruf. Shen, Solon, Zeng
4PM Dissipative: Manohar, Shen and Ridgeway; Dlapa, Kalen, Lui, Neef, Porto
NR: Damour,. Guercilena, Hinder, Hopper, Nagar, Rezzolla

Surprisingly good agreement with numerical relativity!
Great motivation for going to 5 PM order.

Outlook

Field theory methods have a lot of promise and their use has already been tested for a variety of problems.

- **Pushing state of the art for high orders in G.**
  ZB, Cheung, Roiban, Parra-Martinez, Ruf, Shen, Solon, Zeng
- **Radiation.** Cristofoli, Gonzo, Kosower, O'Connell; Herrmann, Parra-Martinez, Ruf, Zeng; Di Vecchia, Heissenberg, Russo, Veneziano
- **Finite size effects.** Cheung and Solon; Haddad and Helset; Kälin, Liu, Porto; Cheung, Shah, Solon; ZB, Parra-Martinez, Roiban, Sawyer, Shen
- **Spin.** Vaidya; Geuvara, O'Connell, Vines; Chung, Huang, Kim, Lee; ZB, Luna, Roiban, Shen, Zeng; Kosmopoulos, Luna; Febres Cordero, Kraus, Lin, Ruf, Zeng

The standard quantities of interest for the inspiral phase can all be computed in this formalism. Formalism is far from exhausted.

Summary

- EFTs and amplitudes give us new ways to think about problems of current interest in general relativity.
- Double-copy idea gives a unified framework for gravity and gauge theory.
- Combining with EFT methods gives a powerful tool for gravitational-wave physics in language LIGO/Virgo can use.
- Pushed state of the art: $O(G^4)$ Newtonian-like conservative part.
- Methods nowhere close to exhausted.
- Higher orders in G, resummations in G, spin, finite-size effects, radiation obvious paths to pursue.

Expect many more advances in coming years, not only for gravitational-wave physics, but more generally for understanding gravity and its relation to the other forces via double copy.

Further Reading

Double Copy:

Z. Bern, J. J. Carrasco, M. Chiodaroli, H. Johansson, R. Roiban.
"The Duality Between Color and Kinematics and its Applications"
arXiv:1909.01358.

Black Hole Physics from Amplitudes:

Z. Bern, C. Cheung, R. Roiban, Chia-Hsien Shen, M. P. Solon, M. Zeng
"Black Hole Binary Dynamics from the Double Copy and Effective Theory"
arXiv:1908.01493