Challenging QED with atomic Hydrogen

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1. Theory
2. Experiment
3. Comparison
4. More Data

1. Theory

Energy Levels of atomic Hydrogen

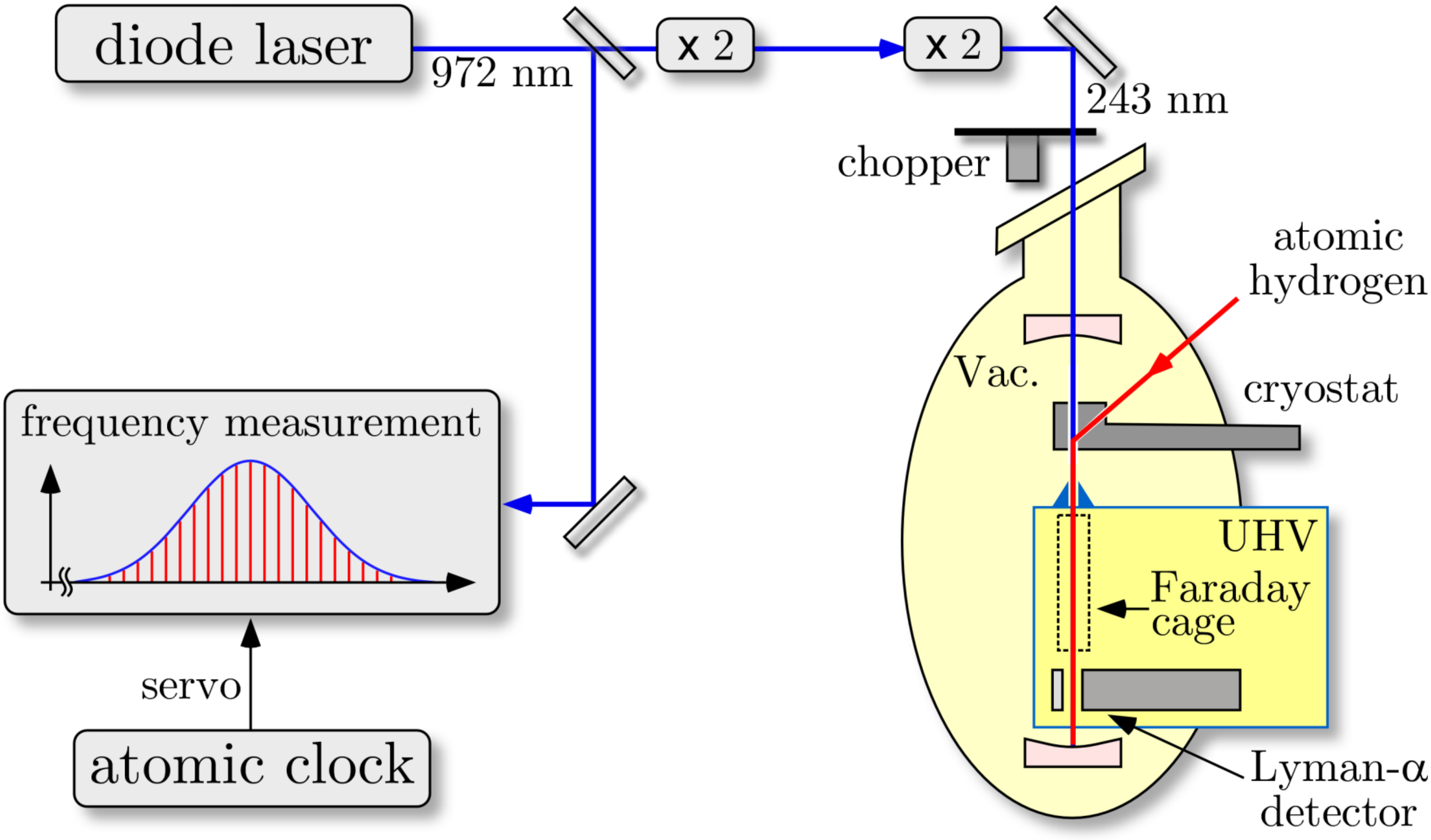
Full recoil and QED in SI units:

$$E_{nlj} = R_\infty \left(-\frac{1}{n^2} + f_{nlj}\left(\alpha, \frac{m_e}{m_p}, \dots\right) + \frac{16\pi^2 m_e^2 c^2 \alpha^2}{3n^3h^2}\, r_p^2 \right)$$

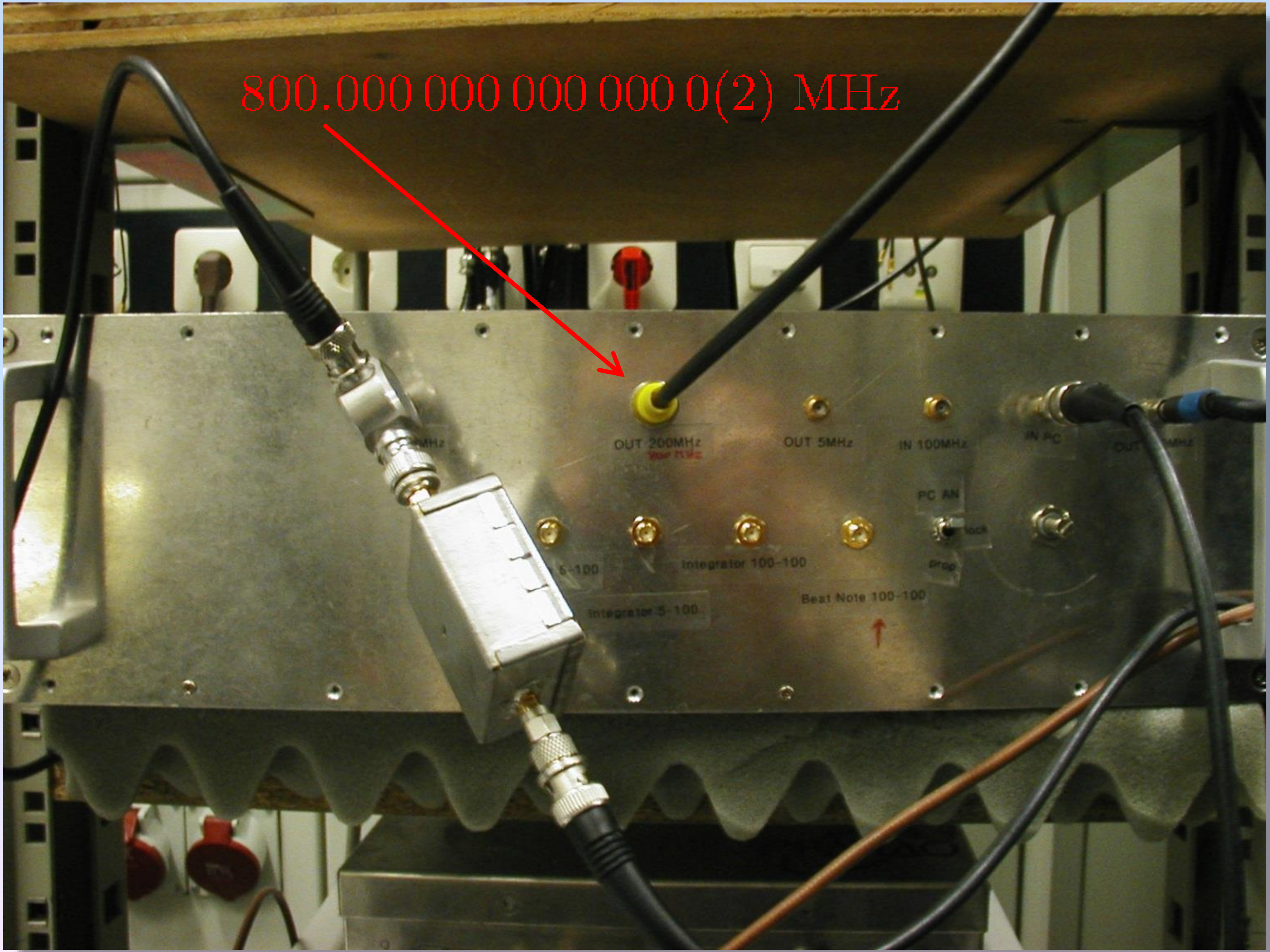
P. Mohr *et al.* Rev. Mod. Phys. 88, 035009 (2016) CODATA 2014
M. Horbatsch and E. A. Hessels, Phys. Rev. A 93, 022513 (2016)
M. Eides *et al.* Theory of Light Hydrogenic Bound States, Springer 2007

2. Experiment

Hydrogen Spectrometer

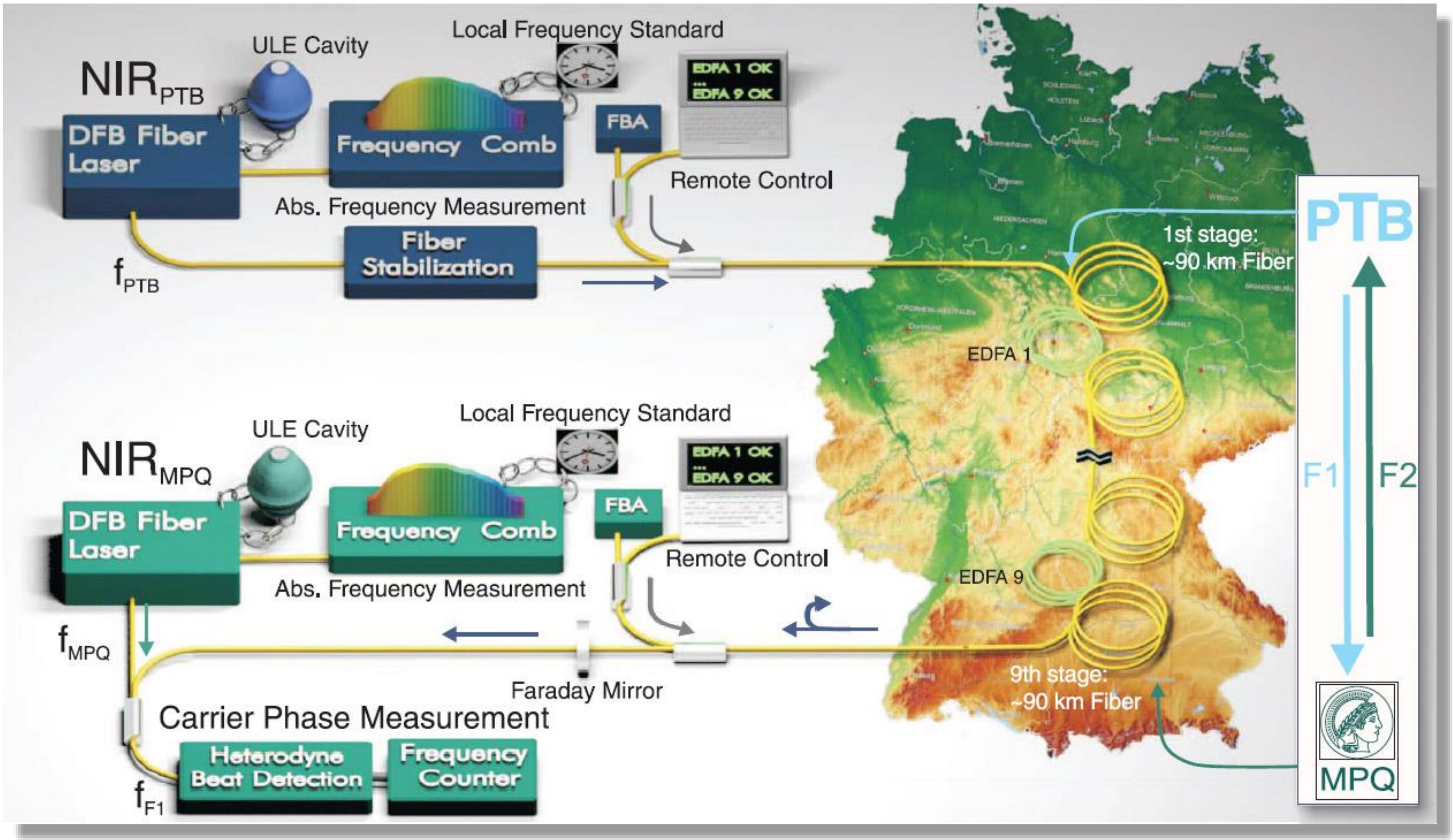


LNE-SYRTE Fountain Clock at MPQ



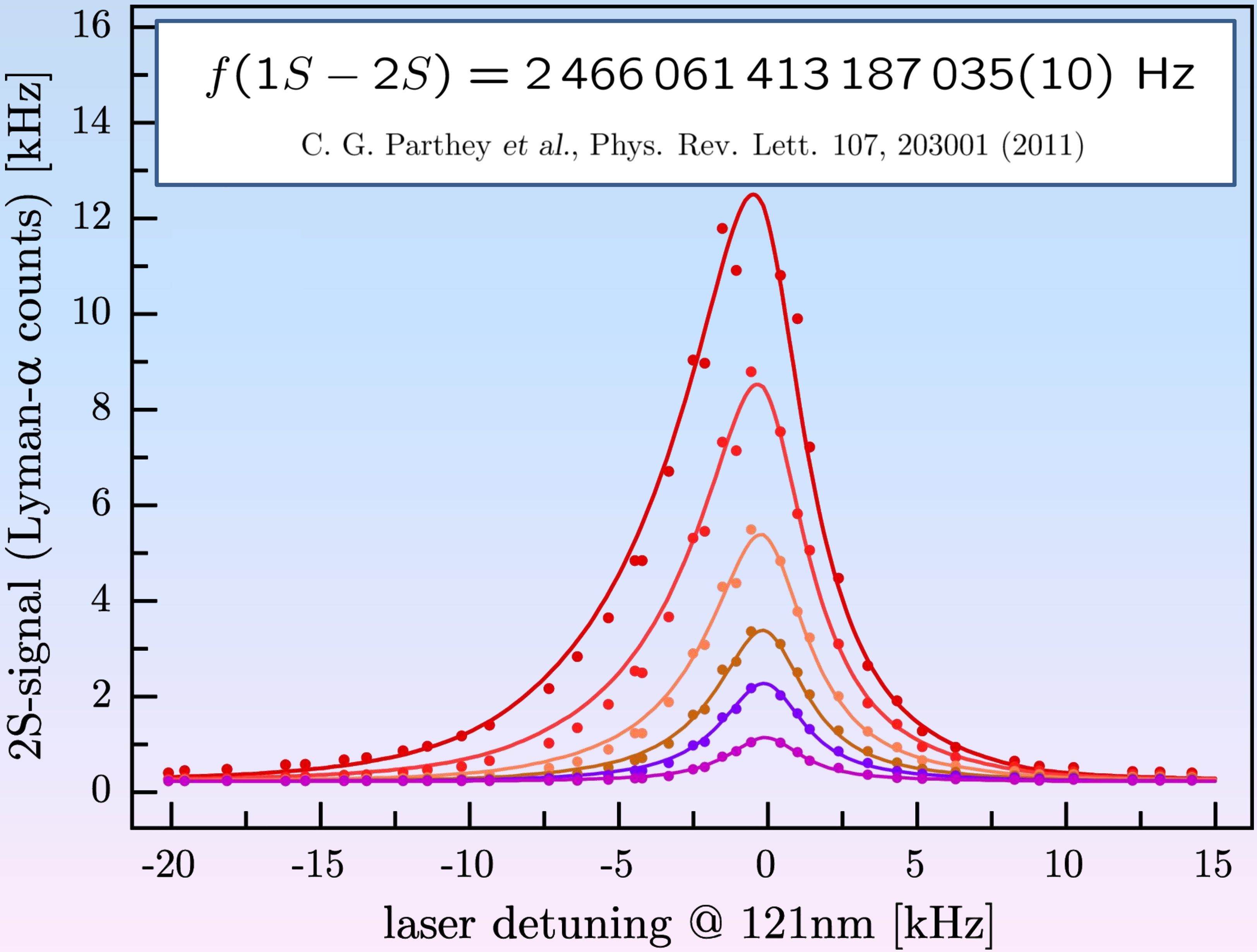
J. Guena *et al.*, IEEE Trans. Ultrason. Ferroelectr. Freq. Control 59, 391 (2012)

920 km Fiber Link



K. Predehl *et al.*, Science 336, 441 (2012); A. Matveev *et al.* PRL 110, 230801 (2013)

Second Order Doppler



3. Comparison

Constants and Parameters

$$E_{nlj} = R_\infty \left(-\frac{1}{n^2} + f_{nlj}\left(\alpha, \frac{m_e}{m_p}, \ldots\right) + \frac{16\pi^2 m_e^2 c^2 \alpha^2}{3n^3 h^2}\, r_p^2 \right)$$

Parameters α, m_e/m_p, R_∞ and r_p

- determination of α from electron g-factor [1]:

$$g-2 = A_1\alpha + A_2\alpha^2 + A_3\alpha^3 + \dots \text{ (891 Feynman diagrams)}$$

- determination of α from atomic recoil shift [2]:

$$R_\infty = \frac{\alpha^2 c^2 m_e}{2h} \Rightarrow \alpha^2 = 4R_\infty \frac{f_{rec}}{f_{opt}} \frac{m_{atom}}{m_e}$$

- determination of m_e/m_p from cyclotron frequencies [3]:

$$\omega_c = \frac{qB}{m} \Rightarrow \frac{m_e}{m_p} = \frac{\omega_c(m_p)}{\omega_c(m_e)}$$

Penning trap

2 effective parameters left to us: R_∞ and r_p.

[1] D. Hanneke *et al.* PRL 100, 120801 (2008).
[2] R. H. Parker *et al.* Science 360, 191 (2018).
[3] D. L. Farnham *et al.* PRL 75, 3598 (1995).

Constants and Parameters

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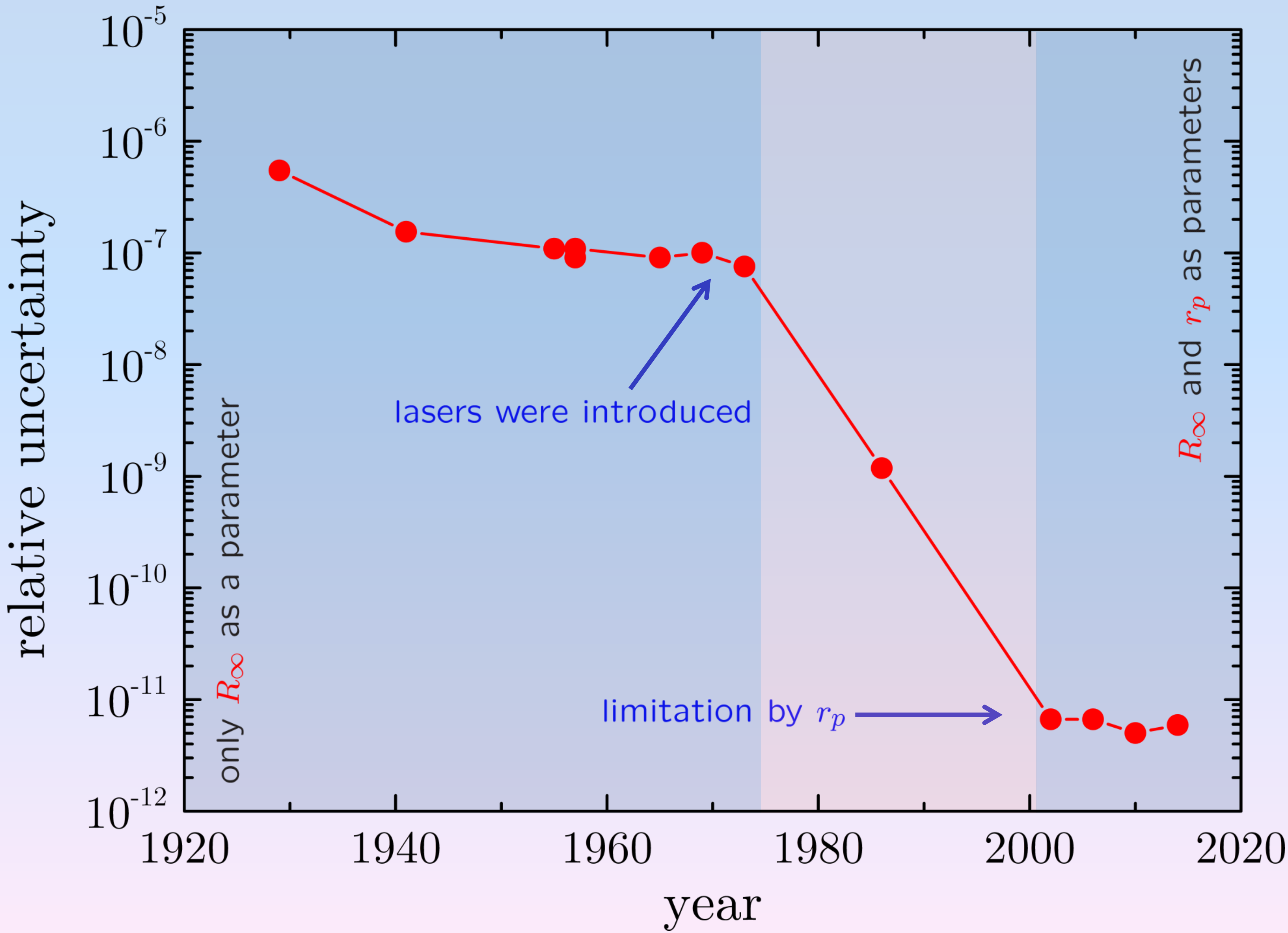
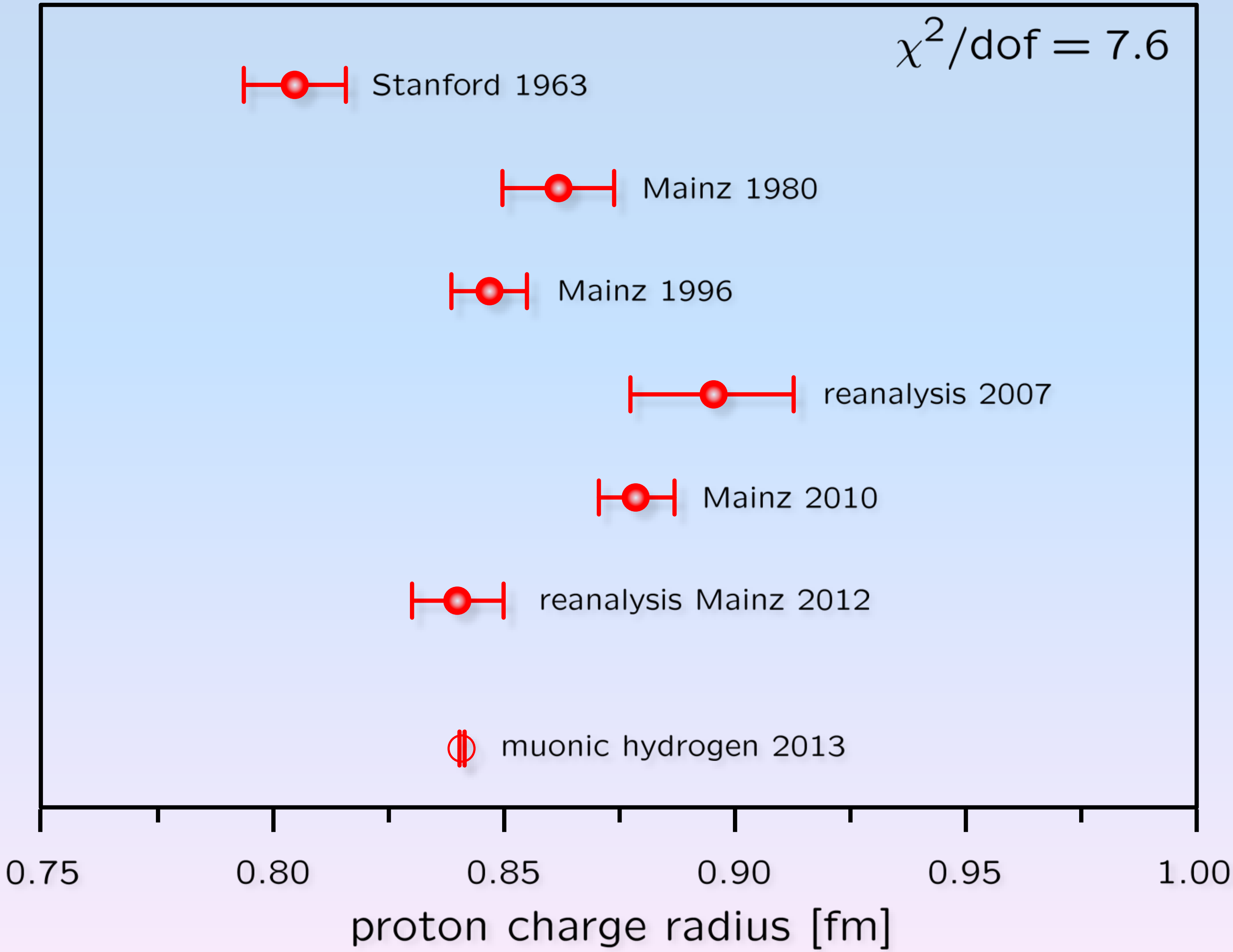
- effective number of parameters depends on the requested accuarcy
- α, m_e/m_p and m_e/h are obtained from other experiments
- R_∞ and r_p are left as adjustable parameters
- need two transitions to determine the values of R_∞ and r_p
- need more than two transitions to test QED
- QED test: check consitency of obtained parameter values
- muonic hydrogen: much more sensitive to r_p when $m_e \to m_\mu \approx 200 \times m_e$

History of the Rydberg Constant

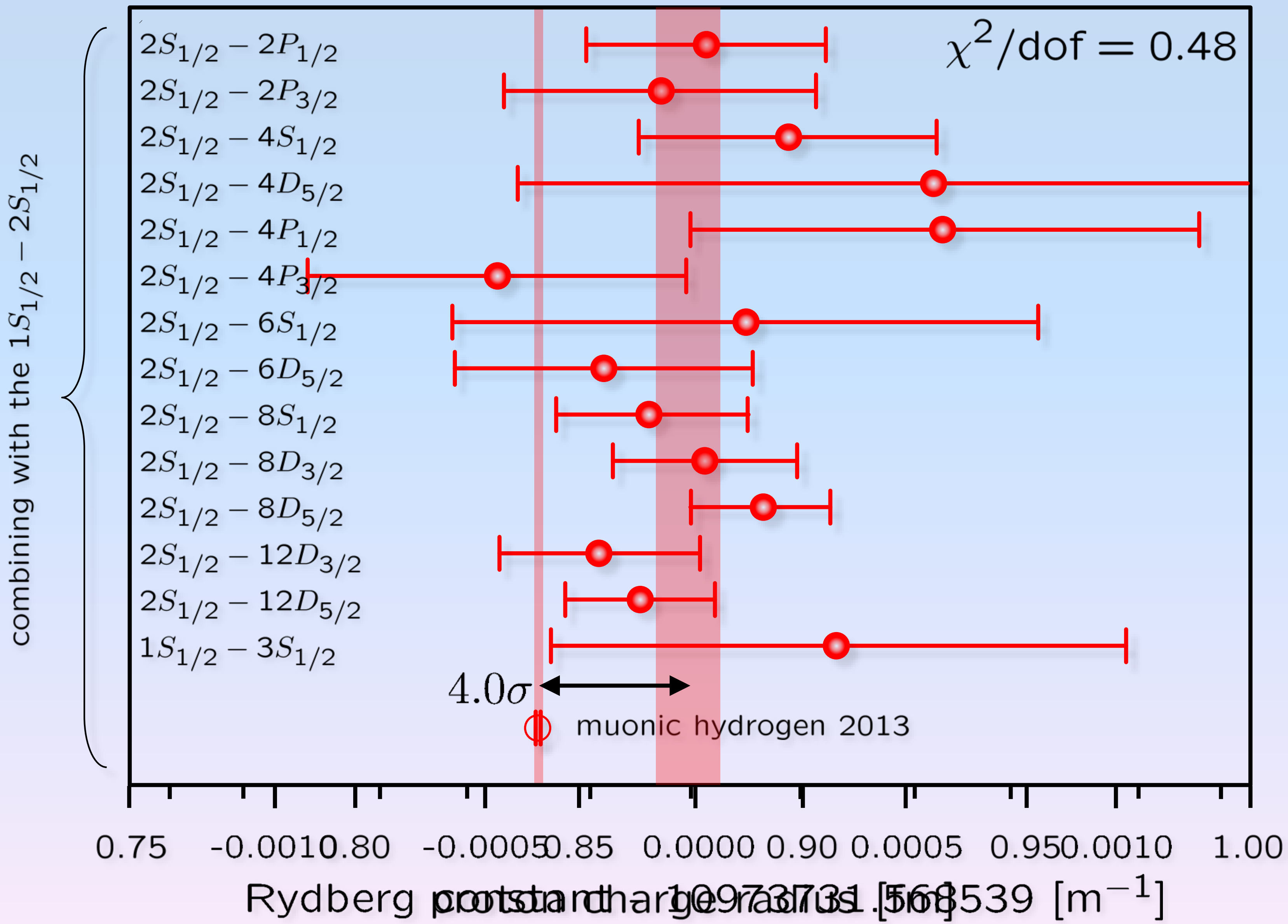


Proton Charge Radius from e-Scattering



Randolf Pohl *et al.* Nature 466, 312 (2010); Aldo Atonigni *et al.* Science 339, 417 (2013)

Proton Charge Radius before 2017 / Rydberg Constant before 2017

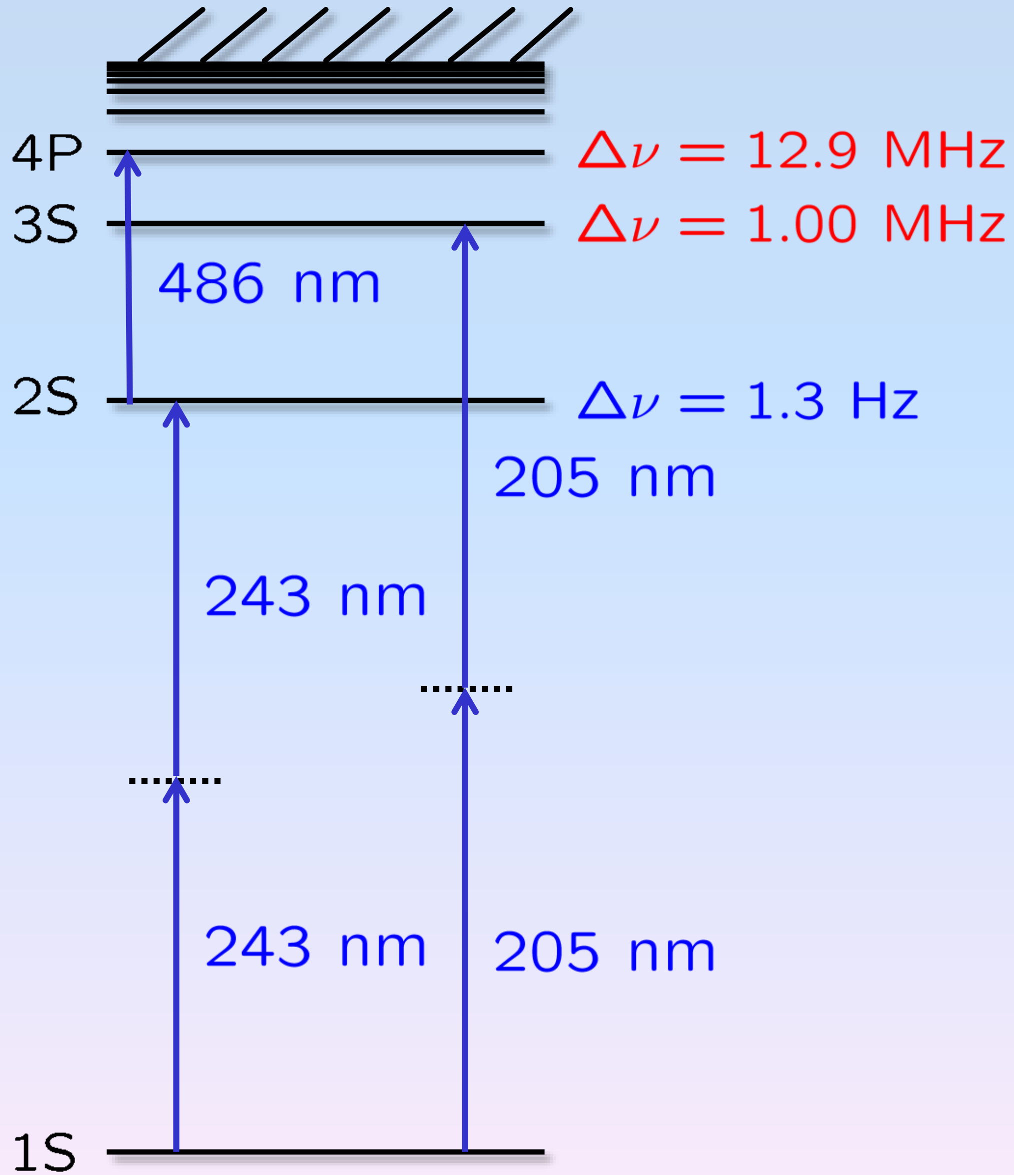


Possible Systematics

Systematic uncertainties that could explain the puzzle:

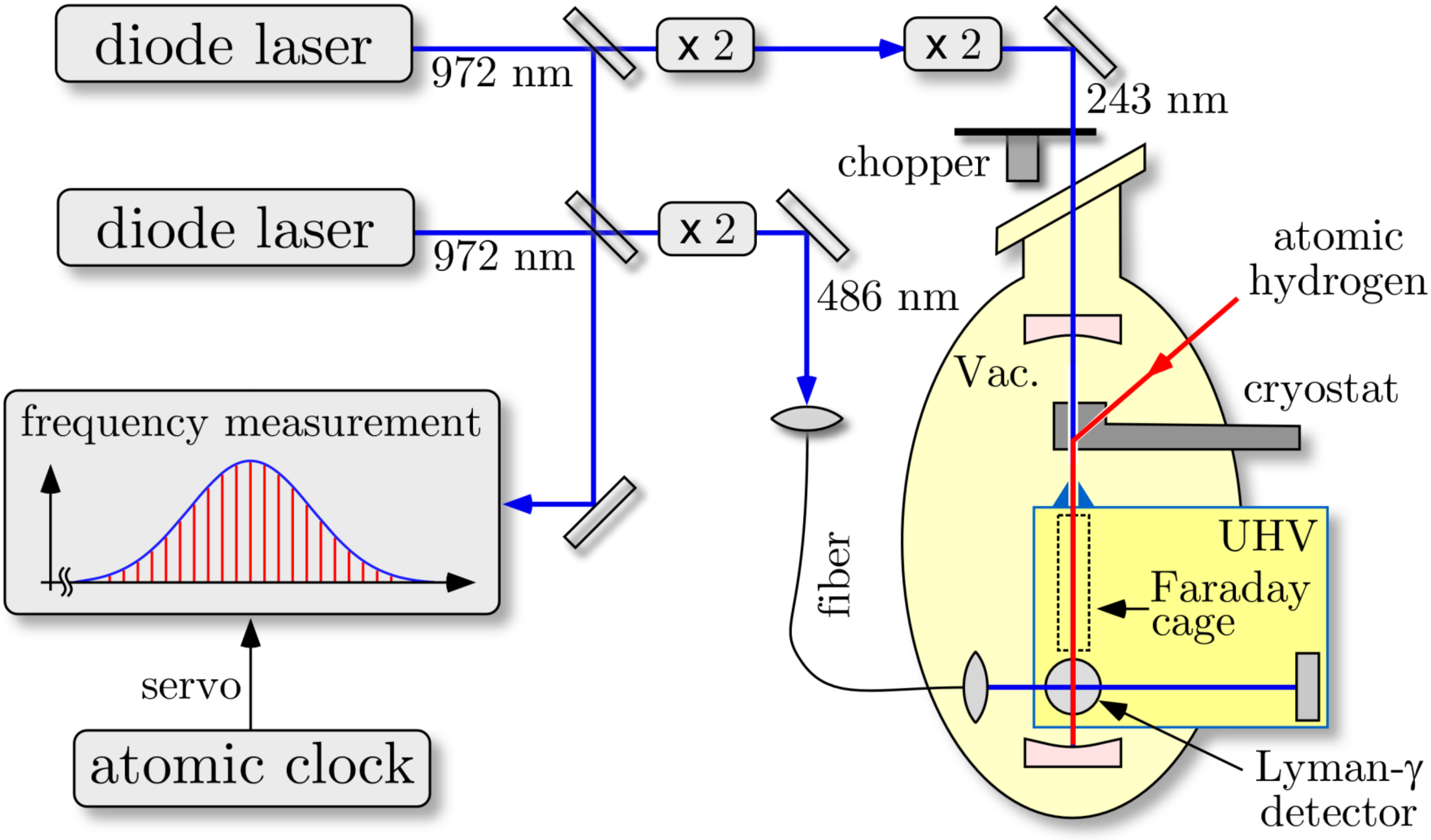
transition	relative uncertainty	relative line widths
H(1s-2s)	4000σ	40
μH(2s-2p)	100σ	4
H(2s-4p)	<1.5σ	7×10^{-4}
H(2s-2p)	<1.2σ	7×10^{-4}

Natural Lifetimes



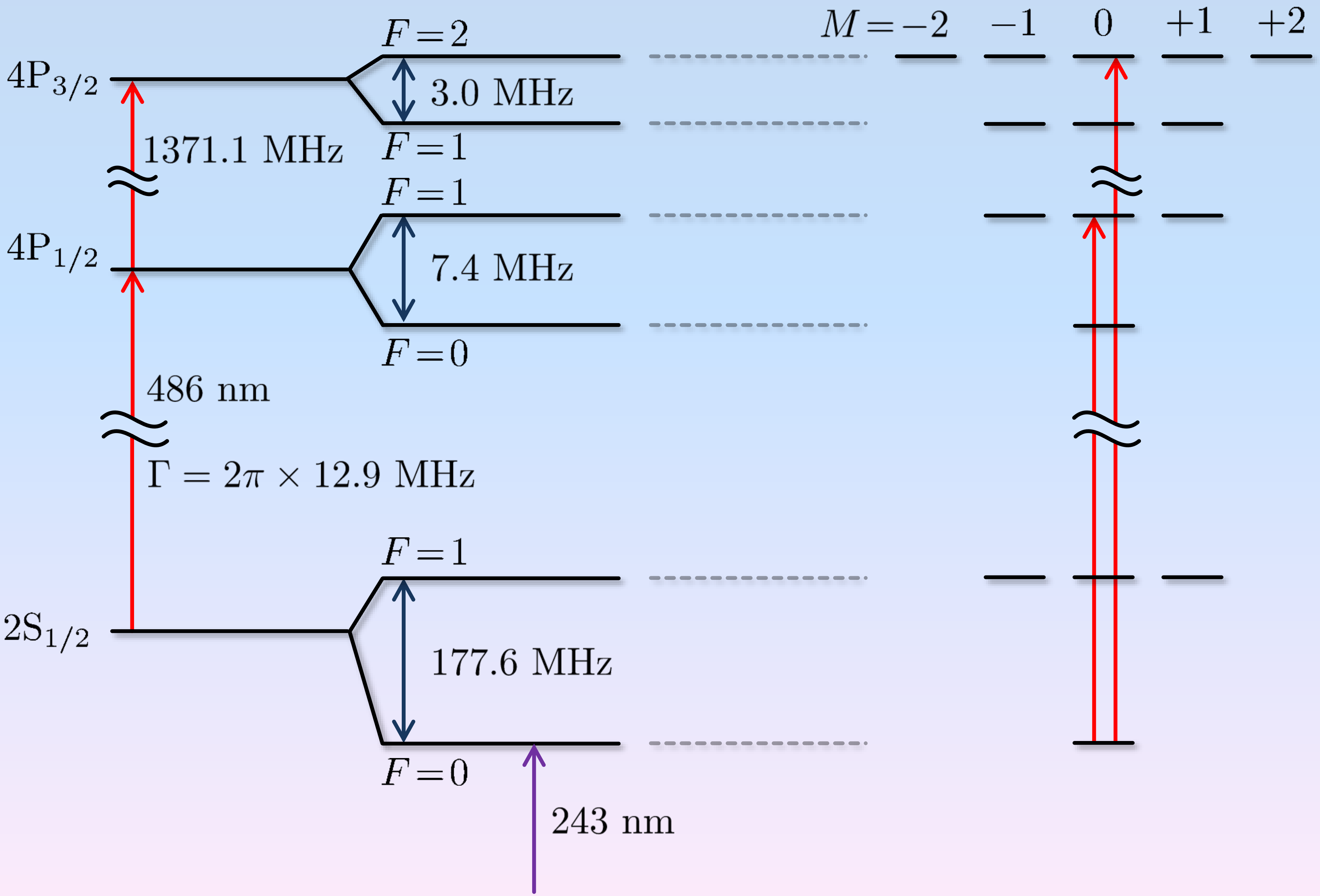
4. More Data

Hydrogen 2S-4P Transition

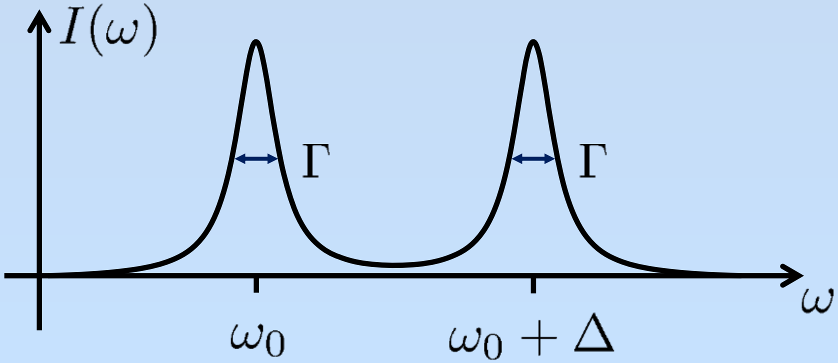


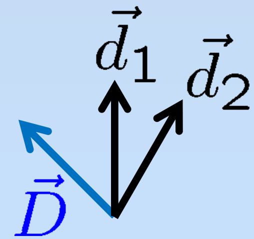
Axel Beyer *et al.* Ann. Phys. (Berlin) 525, 671 (2013)

Hydrogen 2S-4P Components



Cross Damping/Quantum Interference





$\vec{D}$ = detector

$\vec{d}_1, \vec{d}_2$ = dipole moments

$$I(\omega) \propto \left| \frac{\vec{D}\cdot\vec{d}_1}{(\omega-\omega_0)+i\Gamma/2} + \frac{\vec{D}\cdot\vec{d}_2}{(\omega-\omega_0-\Delta)+i\Gamma/2} \right|^2$$

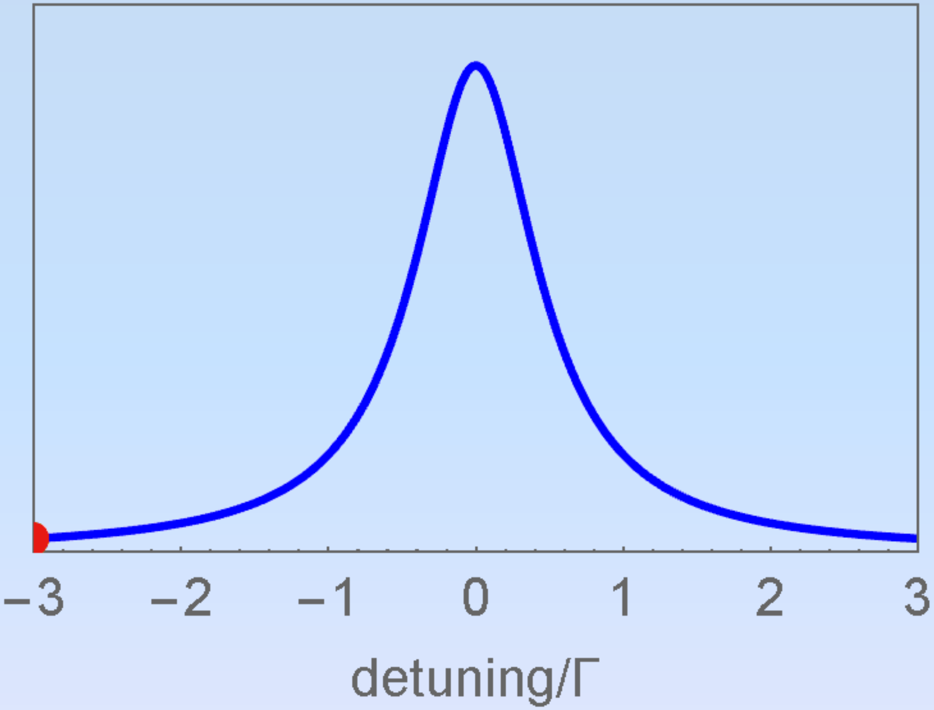
$$= \frac{(\vec{D}\cdot\vec{d}_1)^2}{(\omega-\omega_0)^2+(\Gamma/2)^2} + \frac{(\vec{D}\cdot\vec{d}_2)^2}{(\omega-\omega_0-\Delta)^2+(\Gamma/2)^2}$$

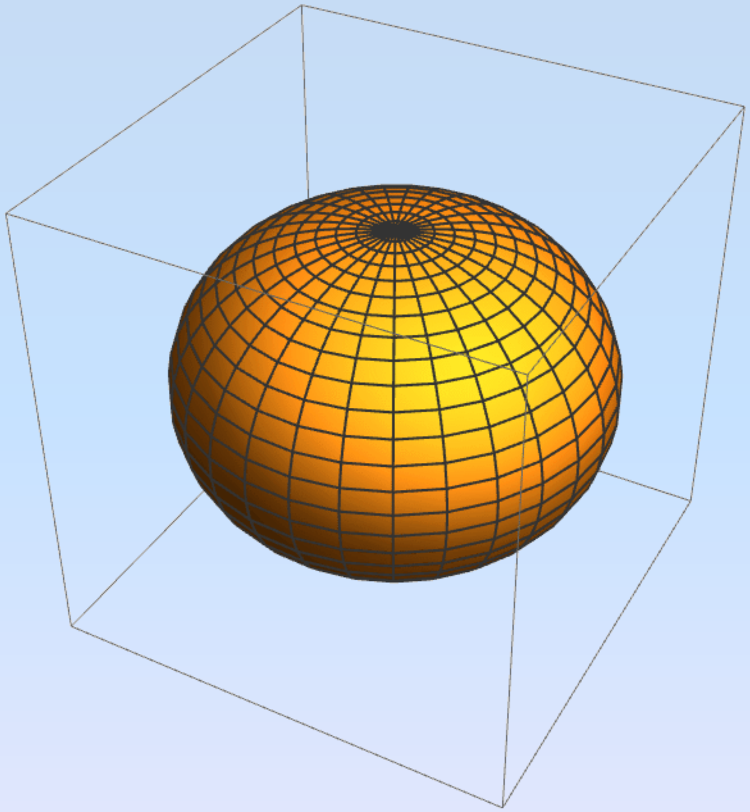
$$+2\mathrm{Re}\left(\frac{(\vec{D}\cdot\vec{d}_1)\ (\vec{D}\cdot\vec{d}_2)}{((\omega-\omega_0)+i\Gamma/2)((\omega-\omega_0-\Delta)-i\Gamma/2)} \right)$$

$$\approx \frac{a_1}{(\omega-\omega_0)^2+(\Gamma/2)^2} + a_2 + a_3(\omega-\omega_0) + \frac{a_4(\omega-\omega_0)}{(\omega-\omega_0)^2+(\Gamma/2)^2}$$

U.D.Jentschura & P.J.Mohr Can. J. Phys. 80, 633 (2002)
M.Horbatsch & E.A.Hessels PRA, 82, 052519 (2010); R.C.Brown *et al.* PRA 87, 032504 (2013)

Cross Damping/Quantum Interference





- line shift vanishes if all atoms/flourescence is detected
- large solid angle improves systematics **and** statistics
- previous 2S-nP/S data not effected
- many additional systematic effects

Lineshape Function

$$I(\omega) \propto \frac{a_1}{(\omega-\omega_0)^2+(\Gamma/2)^2}+a_2+a_3(\omega-\omega_0)+\frac{a_4(\omega-\omega_0)}{(\omega-\omega_0)^2+(\Gamma/2)^2}$$

Convolution with a Gaussian (Doppler): $\frac{1}{\sqrt{\pi}\delta}e^{-\frac{(\omega_0-\omega_0')^2}{\delta^2}}$ $\quad \delta = \sqrt{\frac{2KT}{M}}\frac{\omega_0}{c}$

$$I(\omega) \propto -\frac{2a_1}{\sqrt{\pi}\delta\Gamma}\Im\left[\int_{-\infty}^{+\infty}\frac{e^{-\frac{(\omega_0-\omega_0')^2}{\delta^2}}}{\omega-\omega_0'+i\Gamma/2}d\omega_0'\right]+\frac{a_4}{\sqrt{\pi}\delta}\Re\left[\int_{-\infty}^{+\infty}\frac{e^{-\frac{(\omega_0-\omega_0')^2}{\delta^2}}}{\omega-\omega_0'+i\Gamma/2}d\omega_0'\right]$$

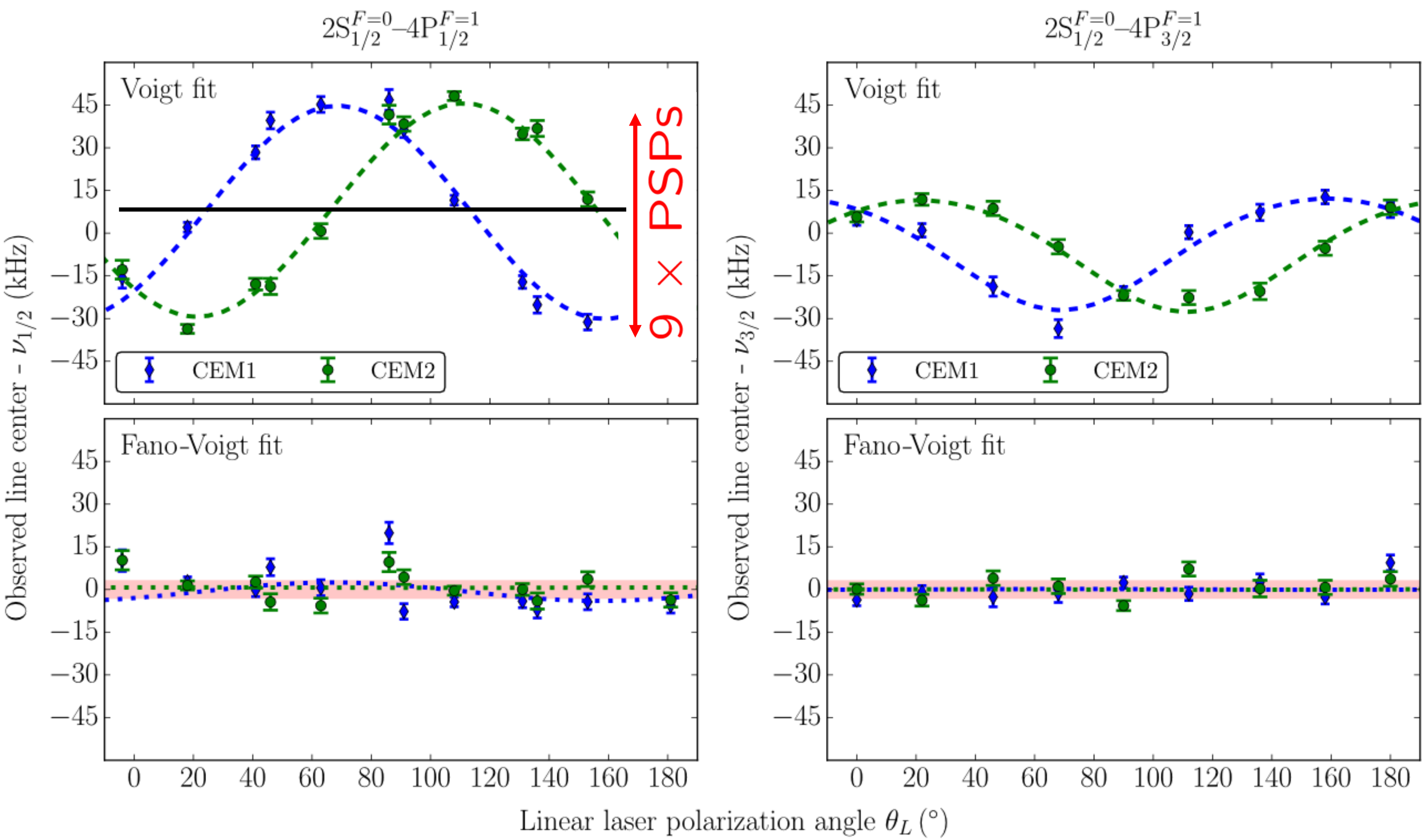
Complex error function (Abramowitz): $w(z) = \frac{i}{\pi}\int_{-\infty}^{+\infty}\frac{e^{-t}}{z-t}dt$

$$I(\omega) \propto \frac{2a_1\sqrt{\pi}}{\delta\Gamma}\Re\left[w(z)\right]+\frac{a_4\sqrt{\pi}}{\delta}\Im\left[w(z)\right] \qquad z \equiv \frac{\omega-\omega_0}{\delta}+i\frac{\Gamma}{2\delta}$$

Lineshape Function

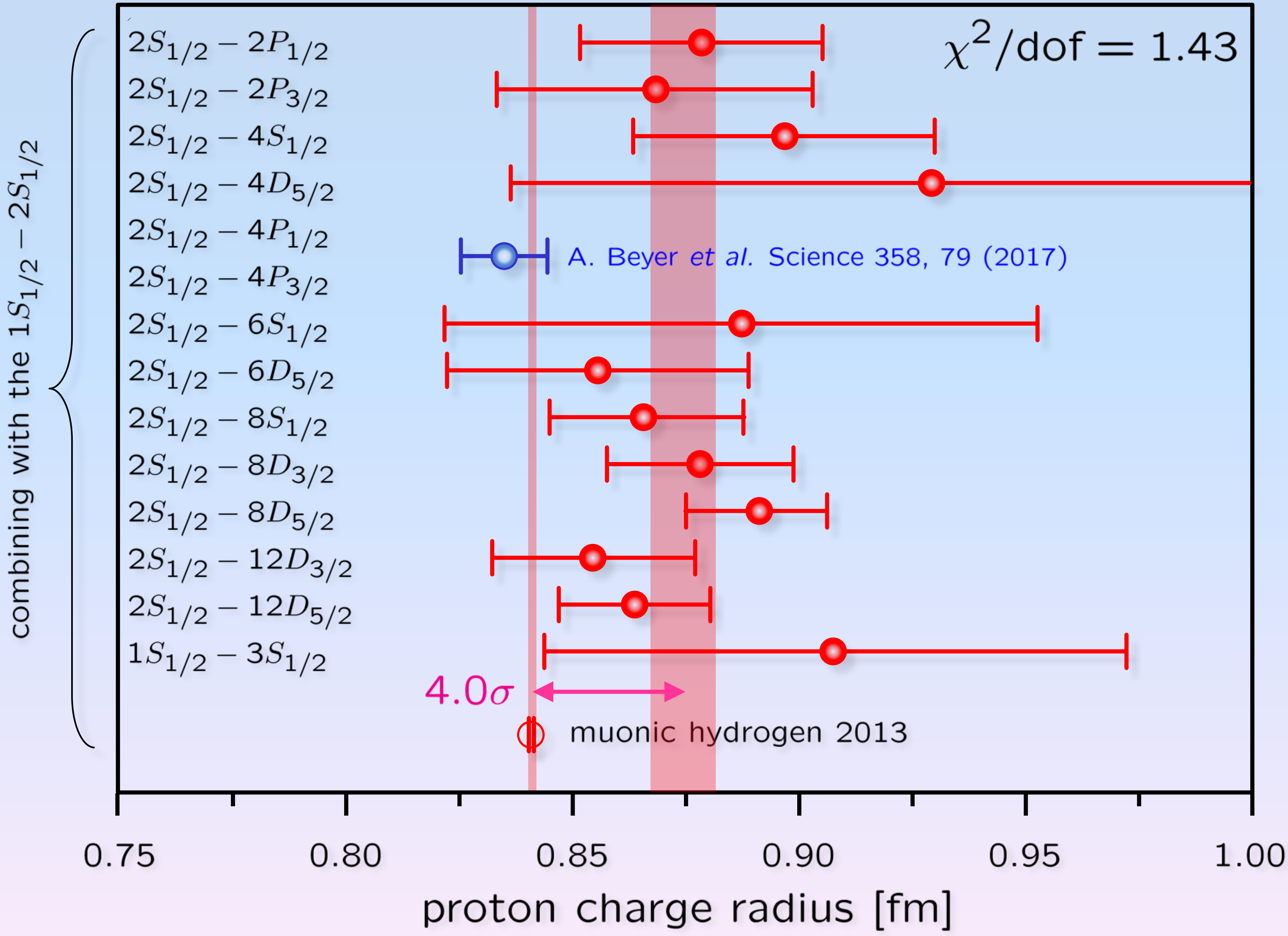
$$I(\omega) = A\left(\Re\left[w(z)\right] + \eta\Im\left[w(z)\right]\right) \qquad \eta = \frac{a_4\Gamma}{2a_1} \ll 1 \qquad z \equiv \frac{\omega - \omega_0}{\delta} + i\frac{\Gamma}{2\delta}$$

Cross Damping/Quantum Interference

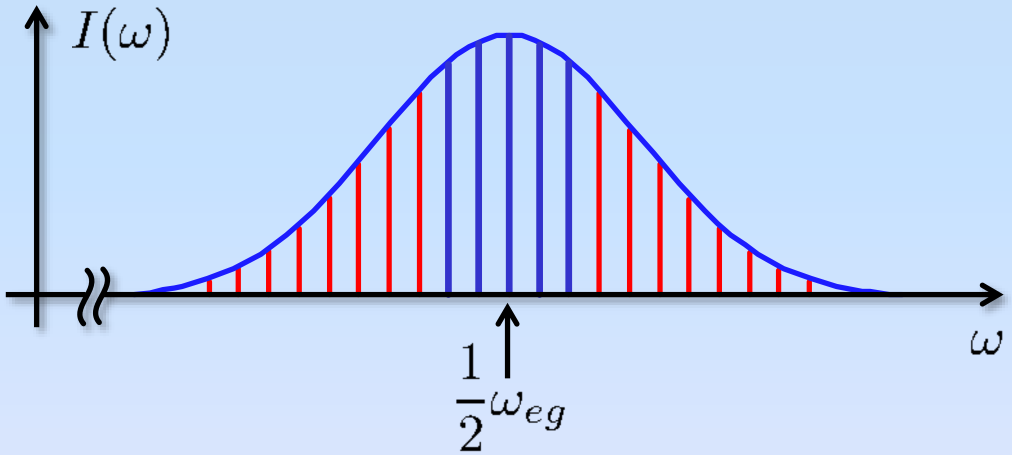


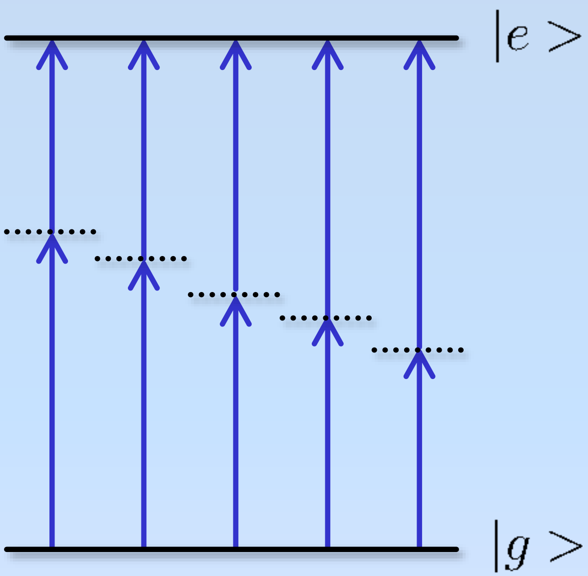
A. Beyer *et al.* Science 358, 79 (2017)

Proton Charge Radius



Two-Photon Comb Spectroscopy

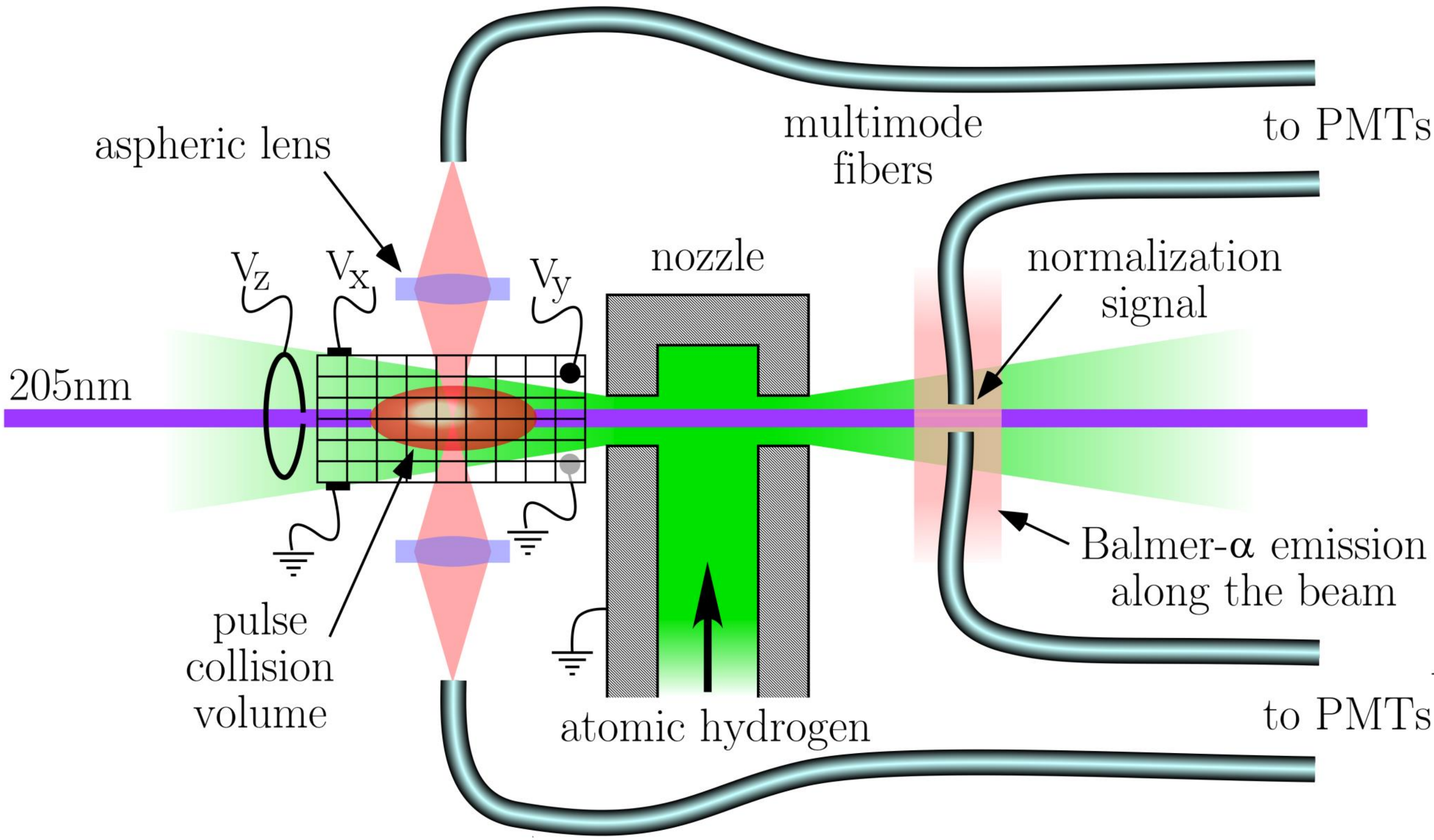




- transition rate power from all modes
- line width of a single laser mode
- ac Stark shift from average intensity

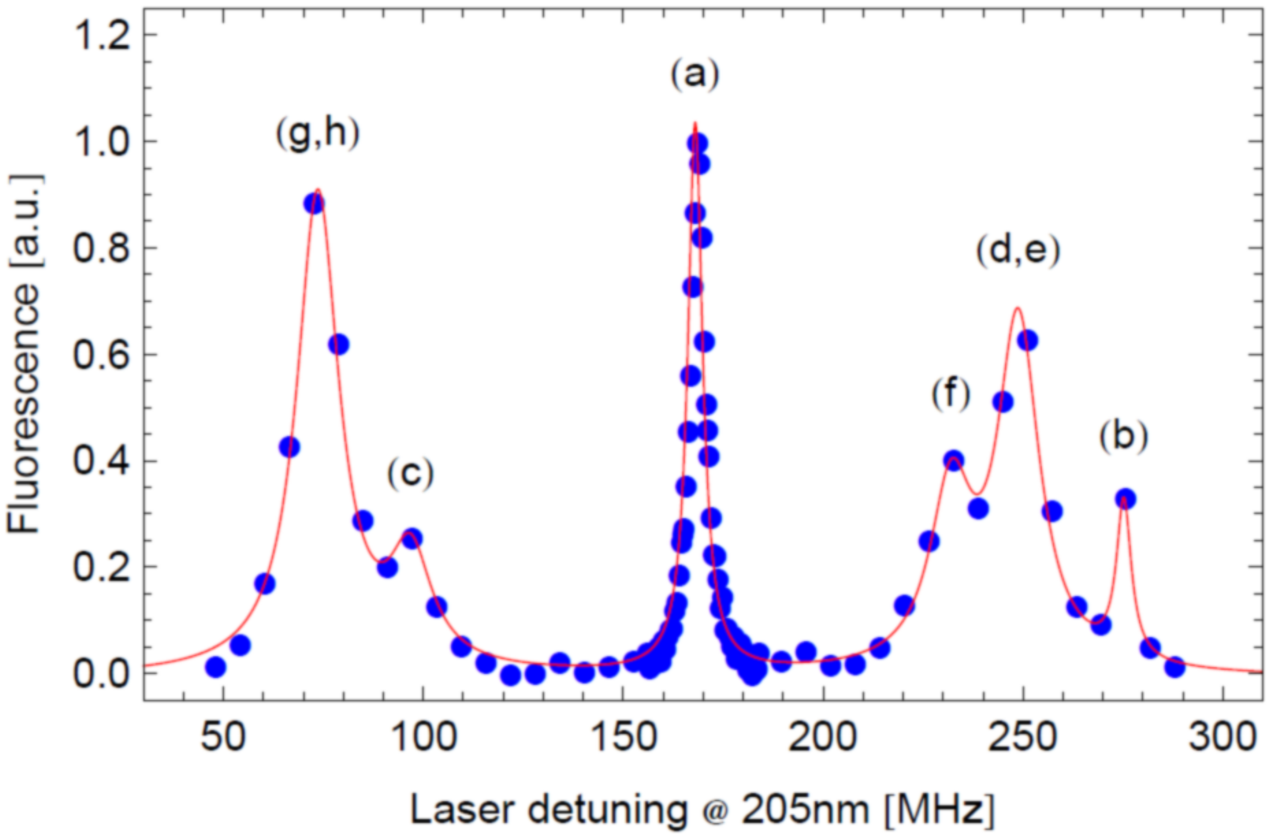
Pionieered by: Ye.V.Baklanov and V.P.Chebotayev, Appl. Phys 12, 97 (1977)

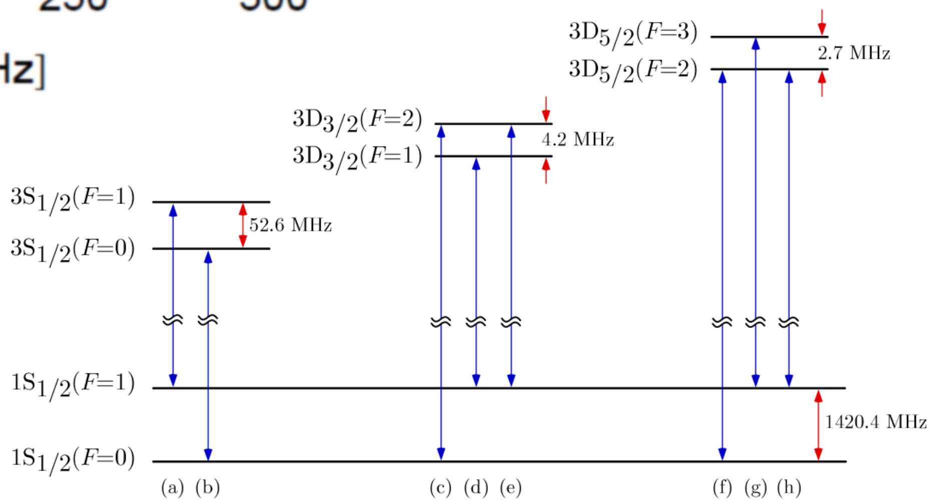
Hydrogen 1S-3S Transition



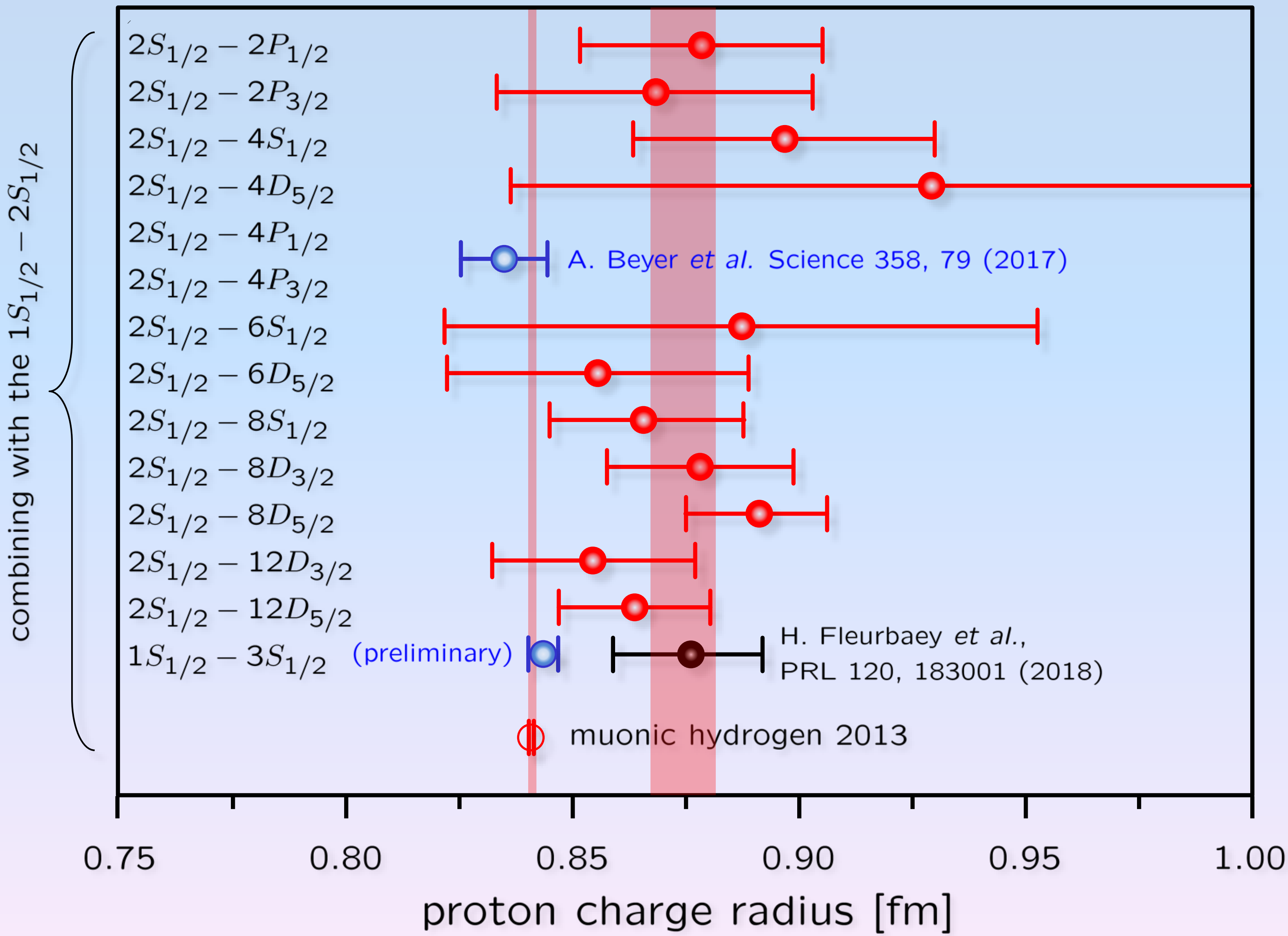
Phys. Rev. A 93, 042509 (2016) and Ann. Phys. 525, L29 (2013)

Hydrogen 1S-3S Transition





Proton Charge Radius



Summary / Outlook

- two measurements deterimine r_p **and** R_∞ (if QED is correct)
- $1S - 2S$ the only (metrologically relevant) narrow line
- additional measurements test QED by comparing parameters

- Next:
  - finish $1S - 3S$ transition frequency
  - measure the $2S - 6P$ transition
  - improve $2S - 2P$ Lamb shift (Hessels *et al.*, York)
  - three body systems: H_2^+, He, Li^+ ...
  - measure $1S - 2S$ transition frequency in He^+ (60.8 nm)

see also: Th. Udem Nat. Phys. 14, 632 (2018)

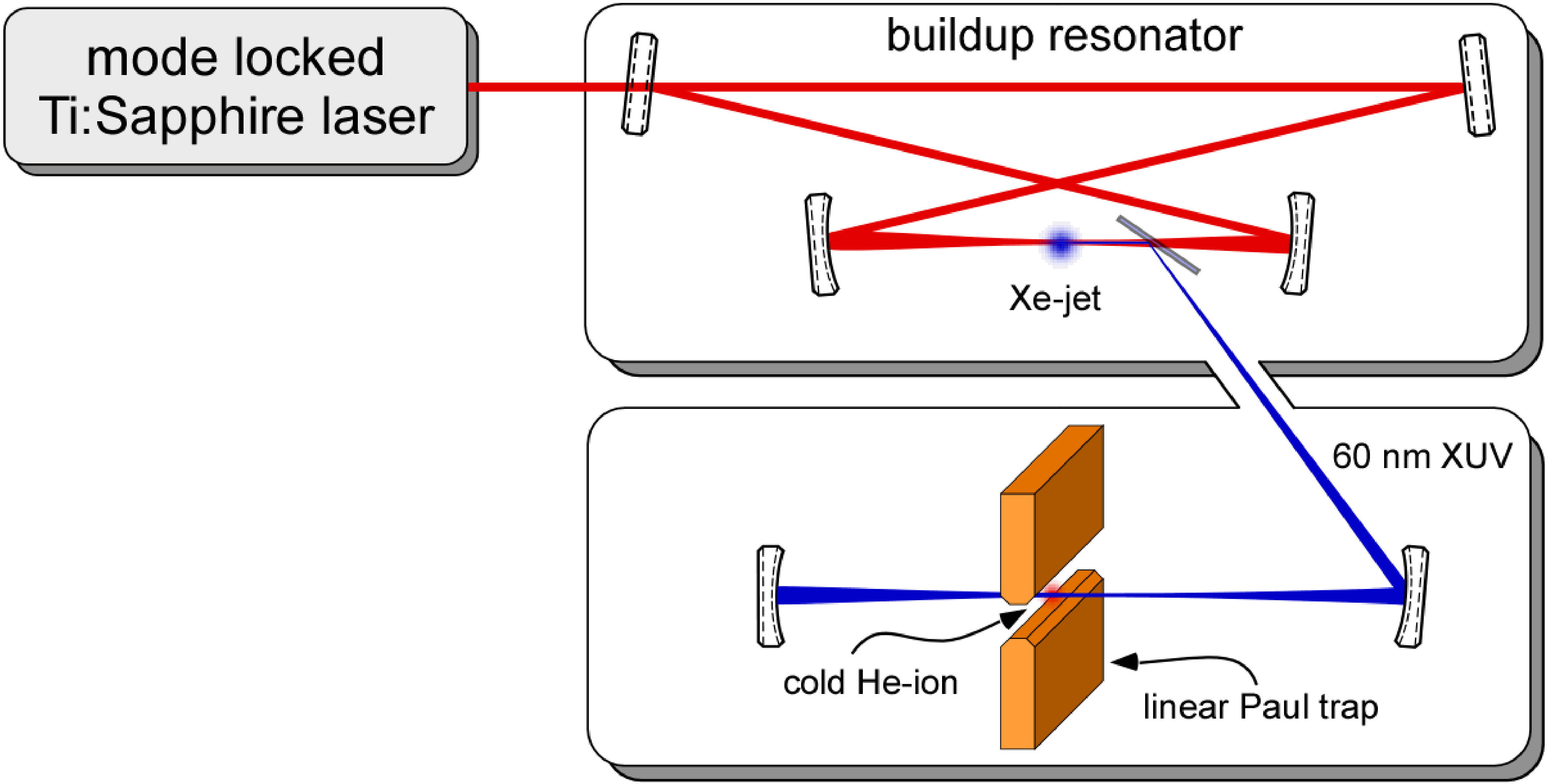
The Better Hydrogen: He^+

- Can be trapped and (sympathetically) cooled
- Nuclear size is beeing measured with monic He^+
- More sensitive to high Z QED contributions
- No HFS ($^4He^+$)
- Systematics (with reasonable conditions*):
    - No quadrupole shift
    - 2nd order Doppler 2×10^{-17}
    - dc Stark shift 3×10^{-16}
    - ac Stark shift 10^{-14}
    - Zeeman effect 8×10^{-17}

➡ But: 60.8 nm laser of sufficient power required !

*) see proposal in Phys. Rev. A 79, 052505 (2009)

He+ Spectroscopy



M. Herrmann et al. PRA 79, 052505 (2009)

Thank you for your Attention