

# Nucleon Form Factors

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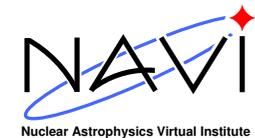
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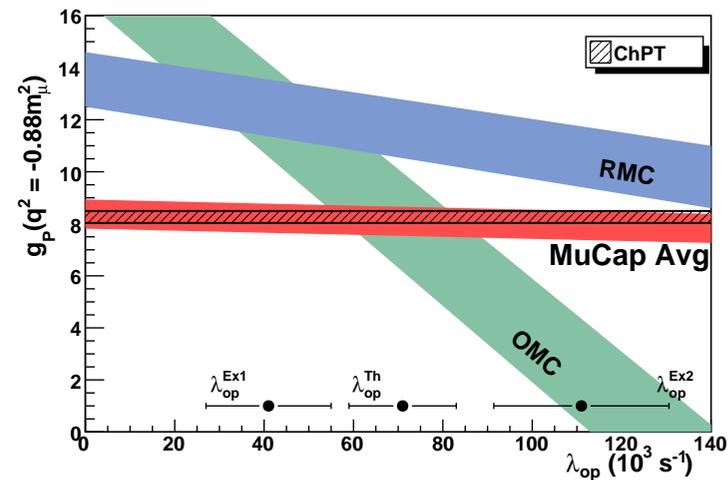
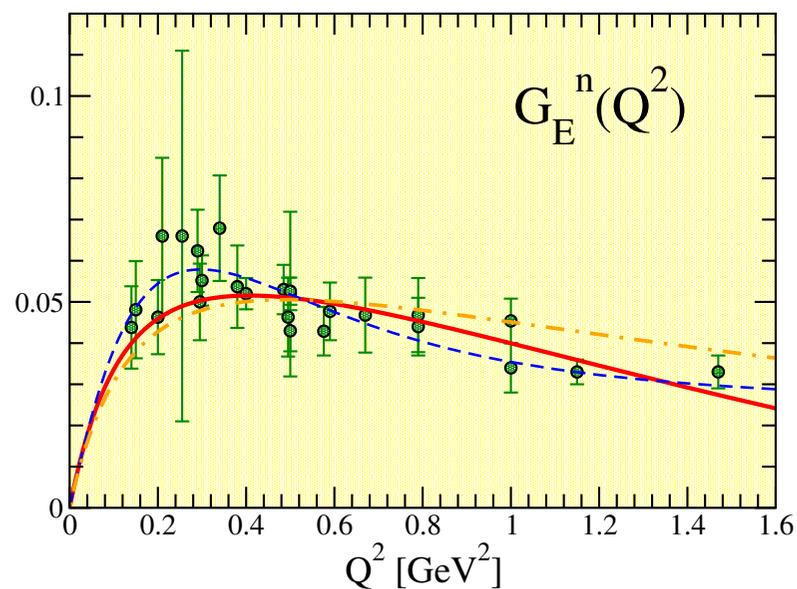
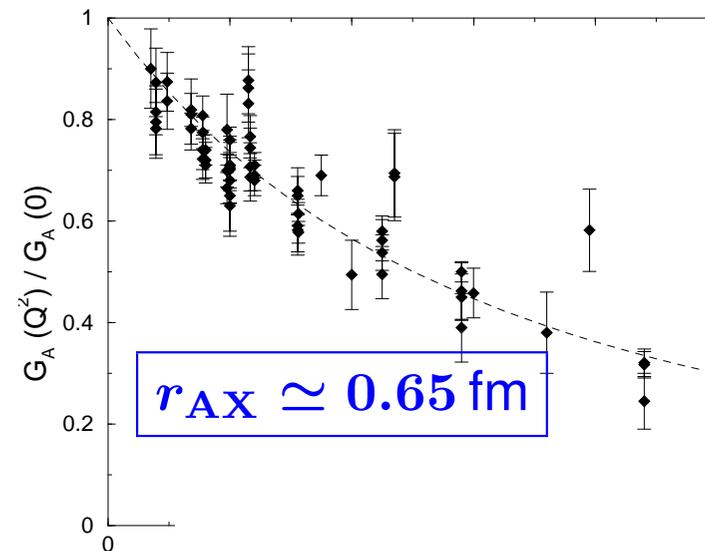
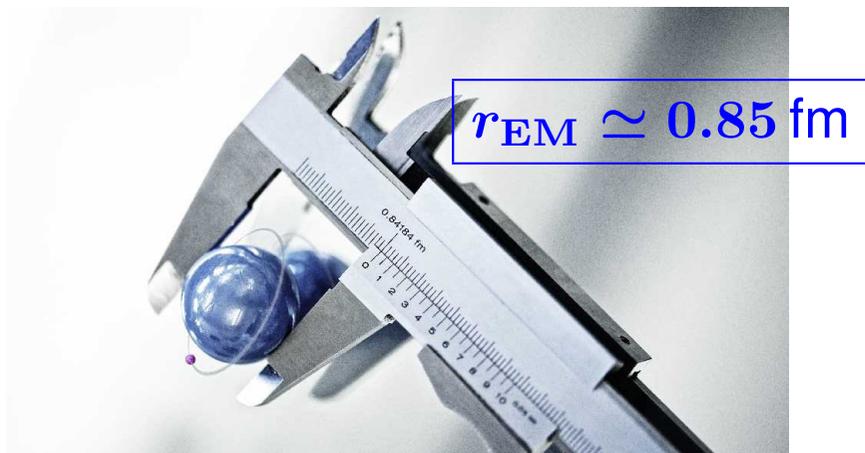
# CONTENTS

- **Introductory remarks**
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- **Results for space- and time-like ffs**
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- **Summary and outlook**

# Introduction

# FACETS of the NUCLEON

- Basic object of QCD in the strong coupling region
- Various probes (electromagnetic, weak) see different facets/scales



# WHY DISPERSION RELATIONS for the NUCLEON FFs ?

- Model-independent approach → important non-perturbative tool to analyze data
- Dispersion relations are based on fundamental principles: **unitarity & analyticity**
- Connect data from small to large momentum transfer  
as well as time- and space-like data
- Allow for a **simultaneous analysis** of all four em form factors
- Spectral functions encode perturbative and non-perturbative physics  
e.g. vector meson couplings, multi-meson continua, pion cloud, ...
- Spectral functions also encode information on the strangeness vector current  
→ sea-quark dynamics, strange matrix elements
- Allow to extract nucleon electric and magnetic radii
- Can be matched to chiral perturbation theory

# Theoretical framework

## BASIC DEFINITIONS

- Nucleon matrix elements of the em vector current  $J_\mu^I$

$$\langle N(p') | J_\mu^I | N(p) \rangle = \bar{u}(p') \left[ F_1^I(t) \gamma_\mu + i \frac{F_2^I(t)}{2m} \sigma_{\mu\nu} q^\nu \right] u(p)$$

- ★ isospin  $I = S, V$  (isoScalar, isoVector) [=  $(p \pm n)/2$ ]
- ★ four-momentum transfer  $t \equiv q^2 = (p' - p)^2 \equiv -Q^2$
- ★  $F_1$  = Dirac form factor,  $F_2$  = Pauli form factor
- ★ Normalizations:  $F_1^V(0) = F_1^S(0) = 1/2$ ,  $F_2^{S,V}(0) = (\kappa_p \pm \kappa_n)/2$
- ★ Sachs form factors:  $G_E = F_1 + \frac{t}{4m^2} F_2$ ,  $G_M = F_1 + F_2$
- ★ Nucleon radii:  $F(t) = F(0) [1 + t \langle r^2 \rangle / 6 + \dots]$  [except for the neutron charge ff]









# CONSTRAINTS ON THE SPECTRAL FUNCTIONS

- Normalizations: electric charges, magnetic moments
- Superconvergence relations  $\cong$  leading pQCD behaviour

$$F_1(t) \sim 1/t^2, F_2(t) \sim 1/t^3 \quad (\text{helicity - flip})$$

Brodsky et al.

$$\Rightarrow \int_{t_0}^{\infty} \text{Im } F_1(t) dt = 0, \quad \int_{t_0}^{\infty} \text{Im } F_2(t) dt = \int_{t_0}^{\infty} \text{Im } F_2(t) t dt = 0$$

- Two ways of implementing the asymptotic QCD behaviour
  - SC relations alone, add broad resonance to generate imag part for  $t \geq 4m^2$
  - Explicit pQCD term in addition to SC relations (smoother interpolation)

$$F_i^{(I, \text{pQCD})} = \frac{a_i^I}{1 - c_i^2 t + b_i^2 (-t)^{i+1}} \quad i = 1, 2, \quad I = S, V$$

# SUMMARY: SPECTRAL & FIT FUNCTIONS

- Representation of the pole contributions: **vector mesons**  
[NB: can be extended for finite width]

$$\text{Im } F_i^V(t) = \sum_v \pi a_i^v \delta(t - M_v^2), \quad a_i^v = \frac{M_v^2}{f_V} g_{vNN} \Rightarrow F_i(t) = \sum_v \frac{a_i^v}{M_v^2 - t}$$

- *Isovector* spectral functions:

$$\text{Im } F_i^V(t) = \text{Im } F_i^{(2\pi)}(t) + \sum_{v=\omega, \phi, \dots} \pi a_i^v \delta(t - M_v^2), \quad (i = 1, 2)$$

- *Isoscalar* spectral functions:

$$\text{Im } F_i^S(t) = \text{Im } F_i^{(K\bar{K})}(t) + \text{Im } F_i^{(\pi\rho)}(t) + \sum_{v=\omega, \phi, s_1, s_2, \dots} \pi a_i^v \delta(t - M_v^2)$$

- Parameters: 2 for the  $\omega, \phi$ , 3 (4) for each other V-mesons **minus # of constraints**

- Ill-posed problem  $\rightarrow$  extra constraint: minimal # of poles to describe the data

# Results

Belushkin, Hammer, M., Phys. Rev. **C 75** (2007) 035202 [hep-ph/0608337]

# GENERAL COMMENTS ON THE FITS

- large MC sampling for initial values, successive improvement by pole reduction, new MCs, ...
- theoretical uncertainty (error bands) from  $\chi_{\min}^2 + 1.04$  [1- $\sigma$  devs.]

→ first time: dispersive analysis w/ error bars !

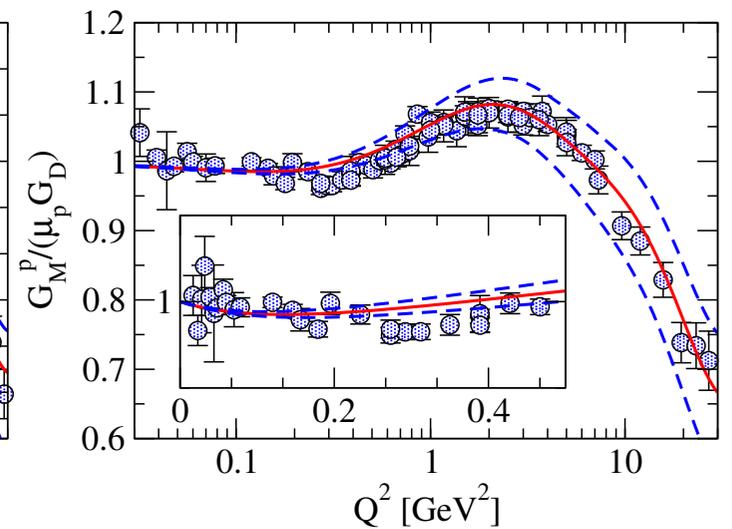
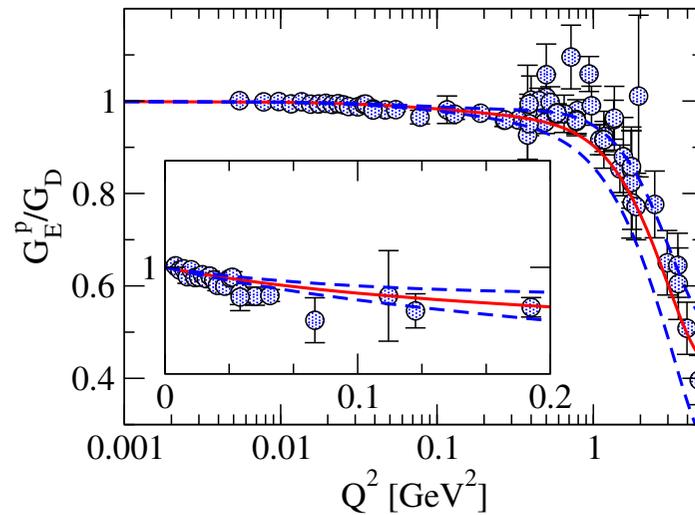
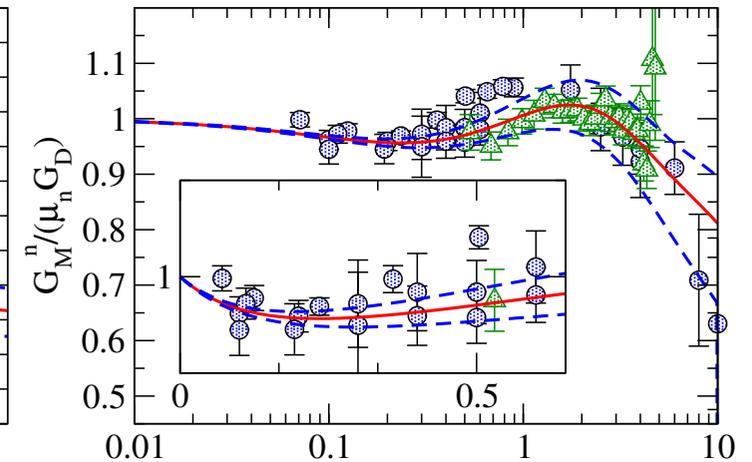
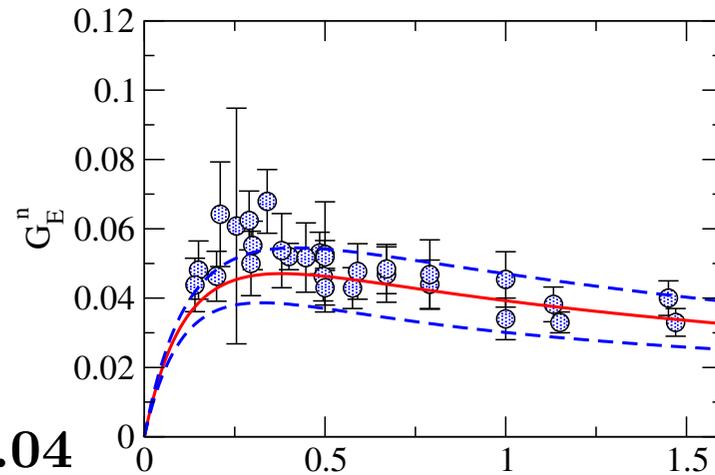
	this work	HM 04	recent determ.
$r_E^p$ [fm]	0.844 (0.840...0.852)	0.848	0.880(15) [1,2,3]
$r_M^p$ [fm]	0.854 (0.849...0.859)	0.857	0.855(35) [4]
$(r_E^n)^2$ [fm <sup>2</sup> ]	-0.117 (-0.11...-0.128)	-0.12	-0.115(4) [5]
$r_M^n$ [fm]	0.862 (0.854...0.871)	0.879	0.873(11) [6]

- [1] Rosenfelder, Phys. Lett. B **479** (2000) 381  
 [2] Sick, private communication  
 [3] Melnikov, van Ritbergen, Phys. Rev. Lett. **84** (2000) 1673  
 [4] Sick, Phys. Lett. B **576** (2003) 62  
 [5] Kopecky et al., Phys. Rev. C **56** (1997) 2229  
 [6] Kubon et al., Phys. Lett. B **524** (2002) 26

- ★ Magnetic radii in good agreement with recent determinations
- ★ Proton electric radius comes out  $\lesssim 0.855$  fm

# SPACE-LIKE FORM FACTORS

- present best fit  
incl. **time-like** data
- $\omega, \phi$  + 2 eff. IS poles
- 5 effective IV poles
- weighted  $\chi^2/\text{dof} = 1.8$   
error bands:  $\chi^2_{\min} + 1.04$



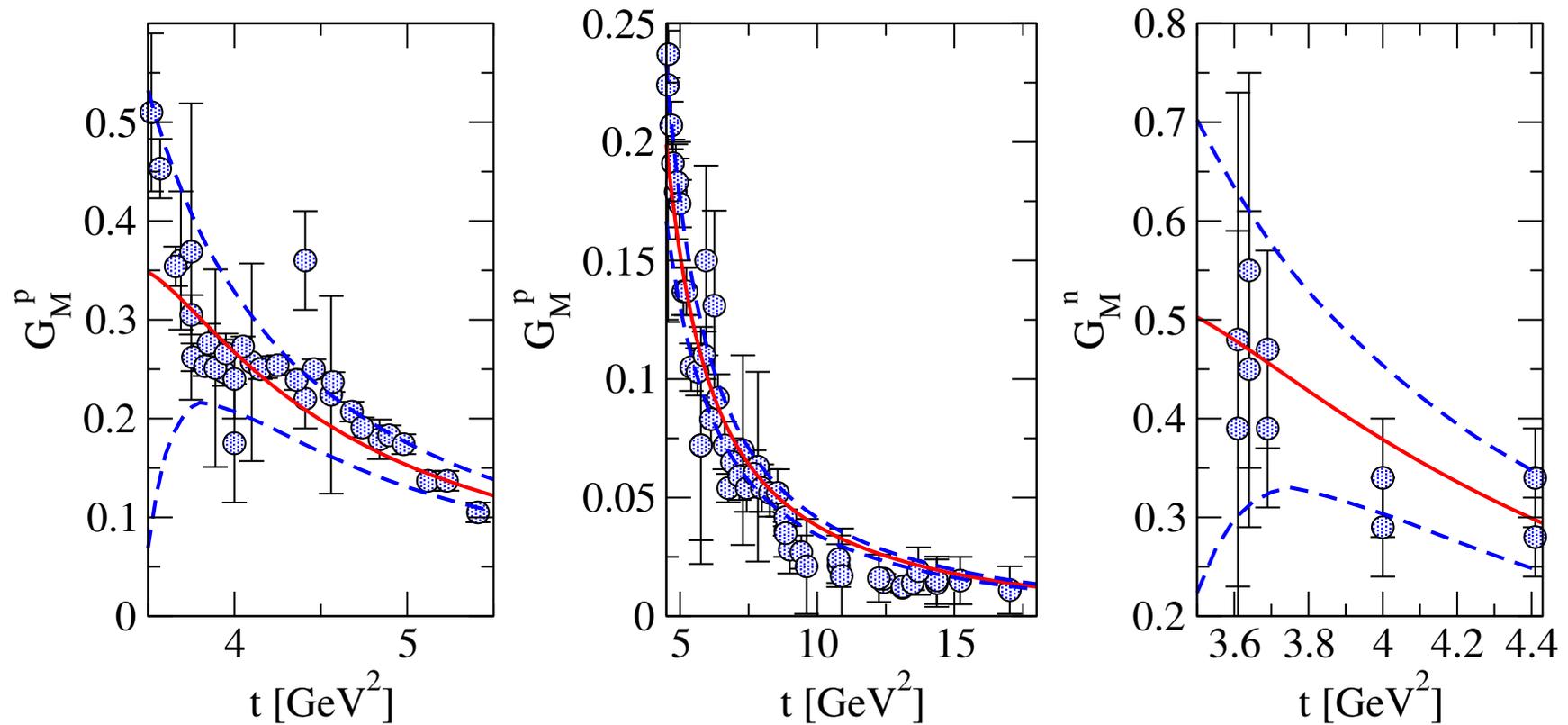
## Improved description

- ★ JLab data described
- ★ higher mass poles  
not at physical values

MMD 96, HMD 96, HM 04

$$G_D(Q^2) = \left(1 + \frac{Q^2}{0.71 \text{ GeV}^2}\right)^{-2}$$

# TIME-LIKE FORM FACTORS



- Only proton data participate in the fits
- All data within one sigma – first time consistent fit w/ space-like ffs

⇒ Need more data on time-like  $G_M^n$  & E/M separation

# The proton radius discrepancy

Lorenz, Hammer, M., Eur. Phys. J. **A 48** (2012) 151 [arXiv:1205.6628 [hep-ph]]

# A SHORT HISTORY

- Dispersion relations always found a “small” radius  
MMD 1995, HM 2004, BHM 2007

- CODATA 2008:  $r_E^p = 0.8768(69)$  fm

- Muonic hydrogen at PSI:  $r_E^p = 0.84184(67)$  fm Pohl et al. 2010  
 $r_E^p = 0.84087(39)$  fm Antognini et al. 2013

- Electron-proton scattering at MAMI:  $r_E^p = 0.876(8)$  fm Bernauer et al. 2010

⇒ zillions of papers, different levels of sophistication, ...

⇒ reanalyze Mainz data including constraints from analyticity and unitarity

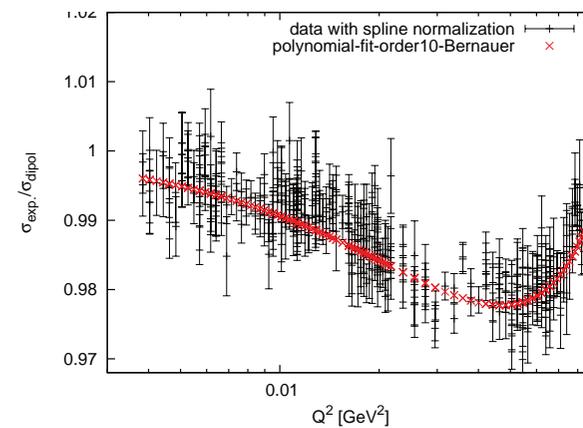
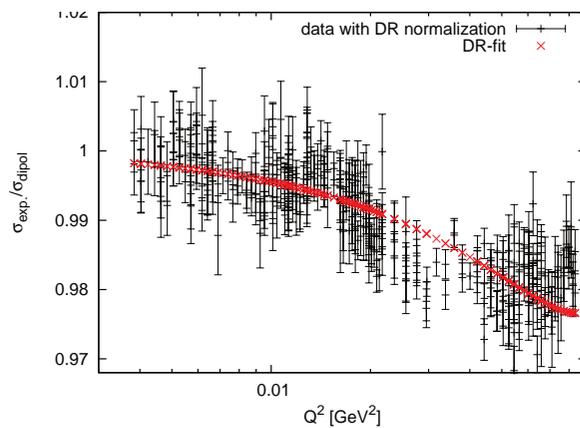
⇒ study various sources of uncertainties ( $2\pi$  continuum, radiative corrections, etc.)



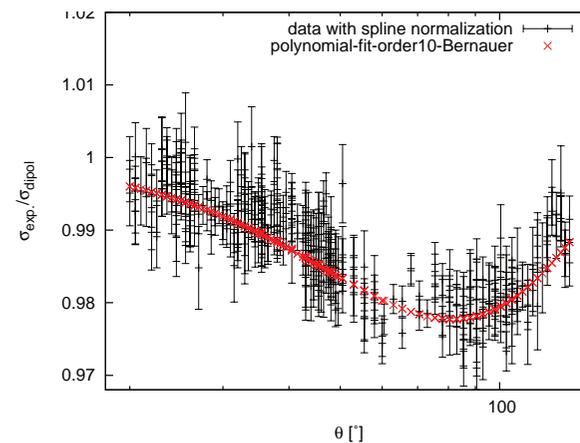
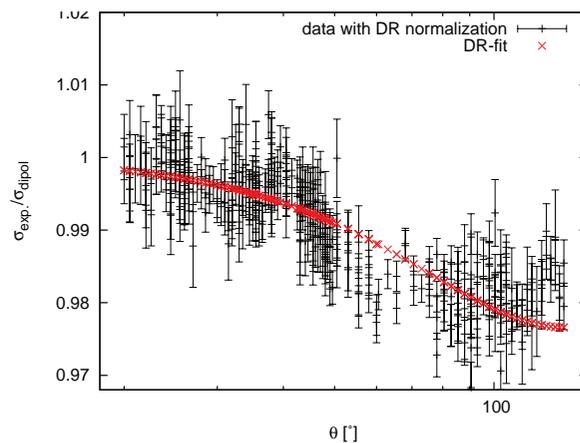
# DIRECT CROSS SECTION FITS: RESULTS

- show data for 180 MeV incident beam energy (floating norms within 4%)

fixed angle:

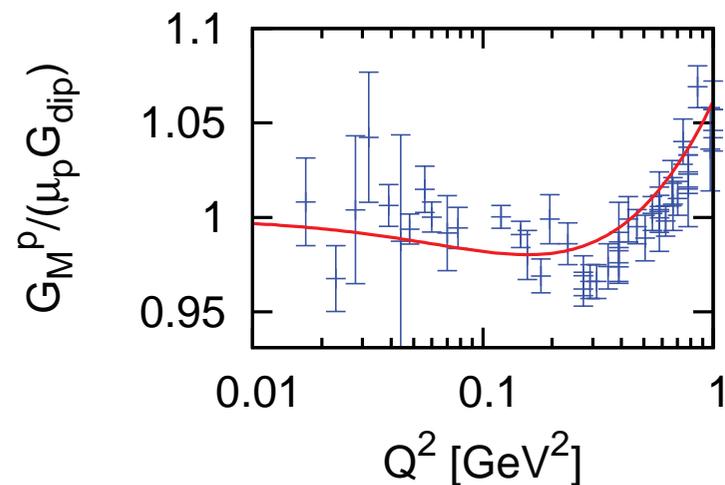
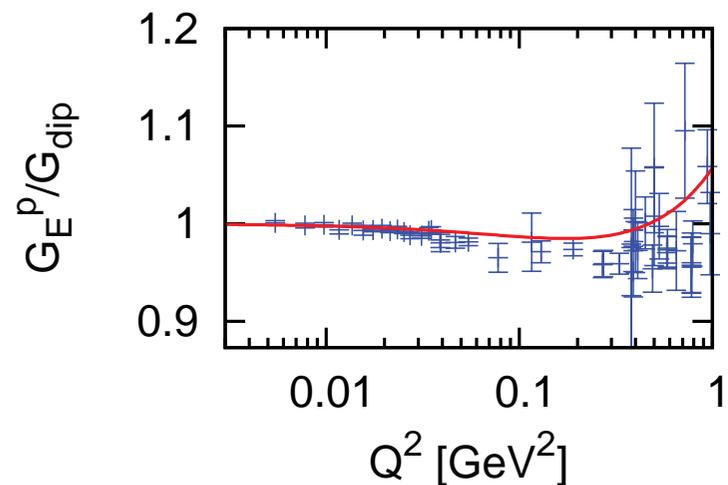
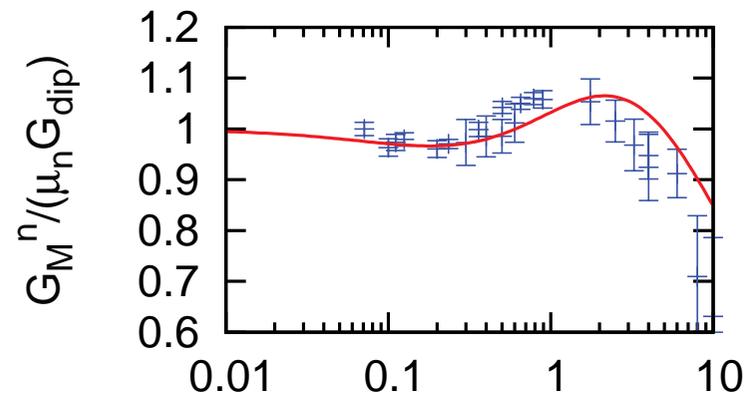
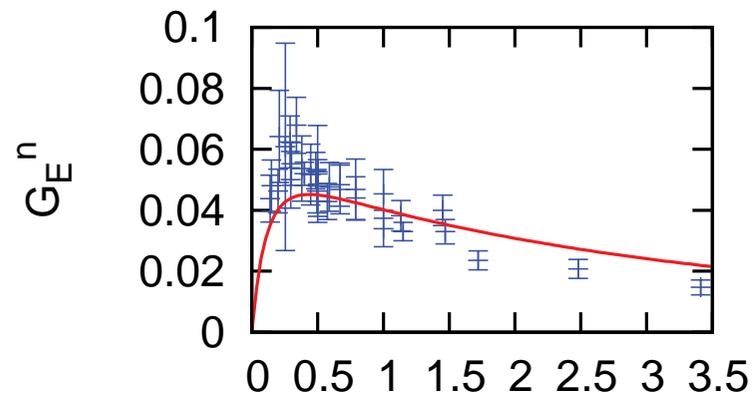


fixed  $Q^2$ :



# DIRECT CROSS SECTION FITS: RESULTS

- compare to FF data basis (proton not fitted)





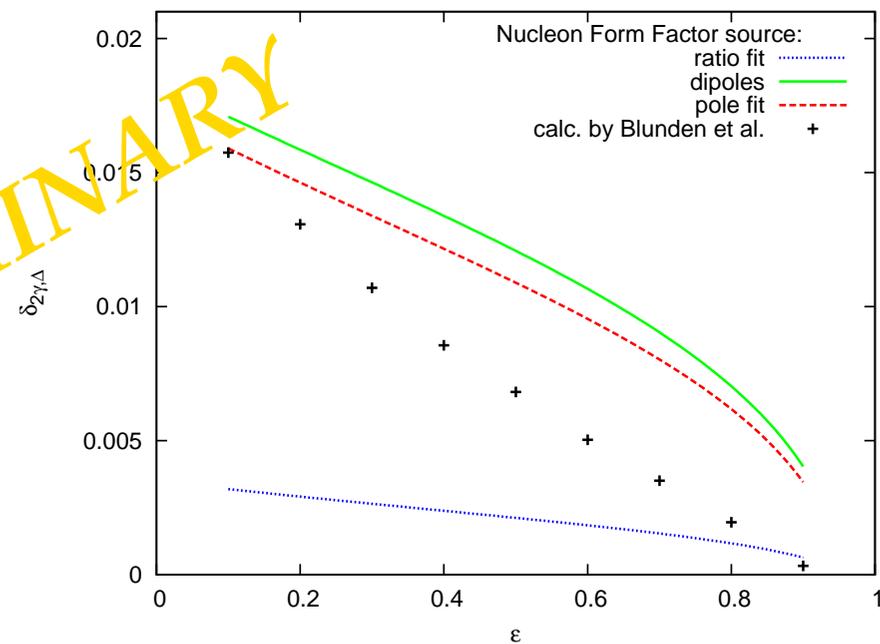
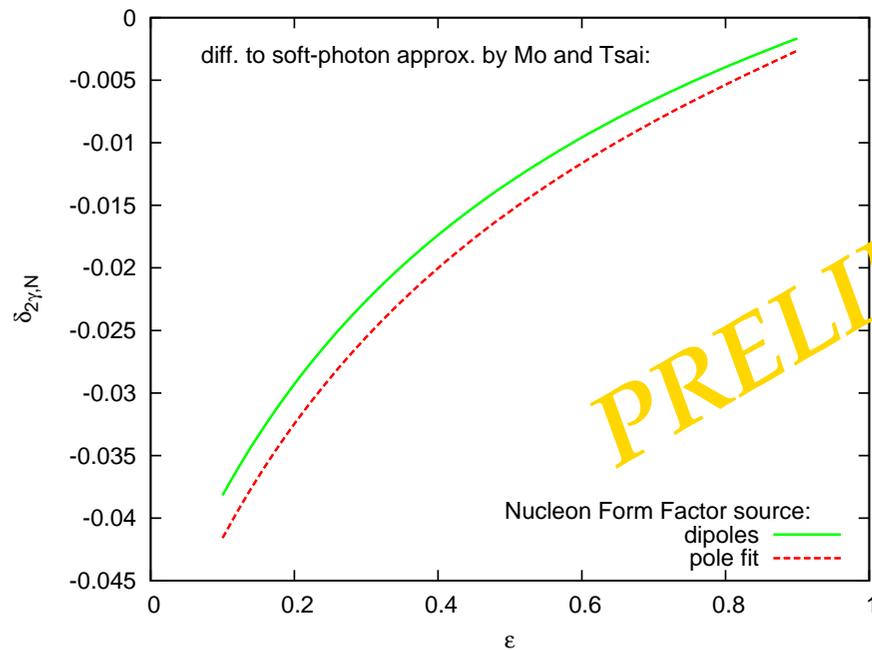
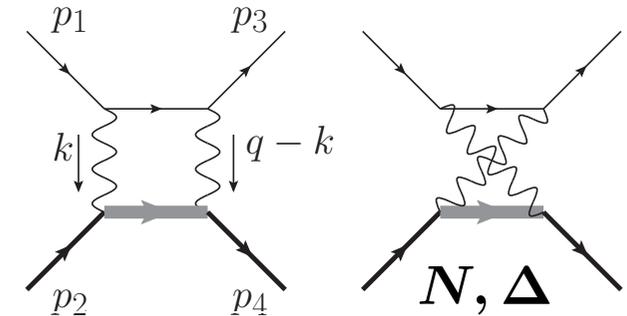
# NEW DEVELOPMENTS

Lorenz, Dong, Hammer, UGM, in prep.

- improved calculation of two-photon exchange effects incl. the  $\Delta$

- TPE correction:  $\delta_{2\gamma} = \delta_{2\gamma,N} + \delta_{2\gamma,\Delta}$

- $\delta_{2\gamma,\Delta}$  more sensitive to ffs than  $\delta_{2\gamma,N}$



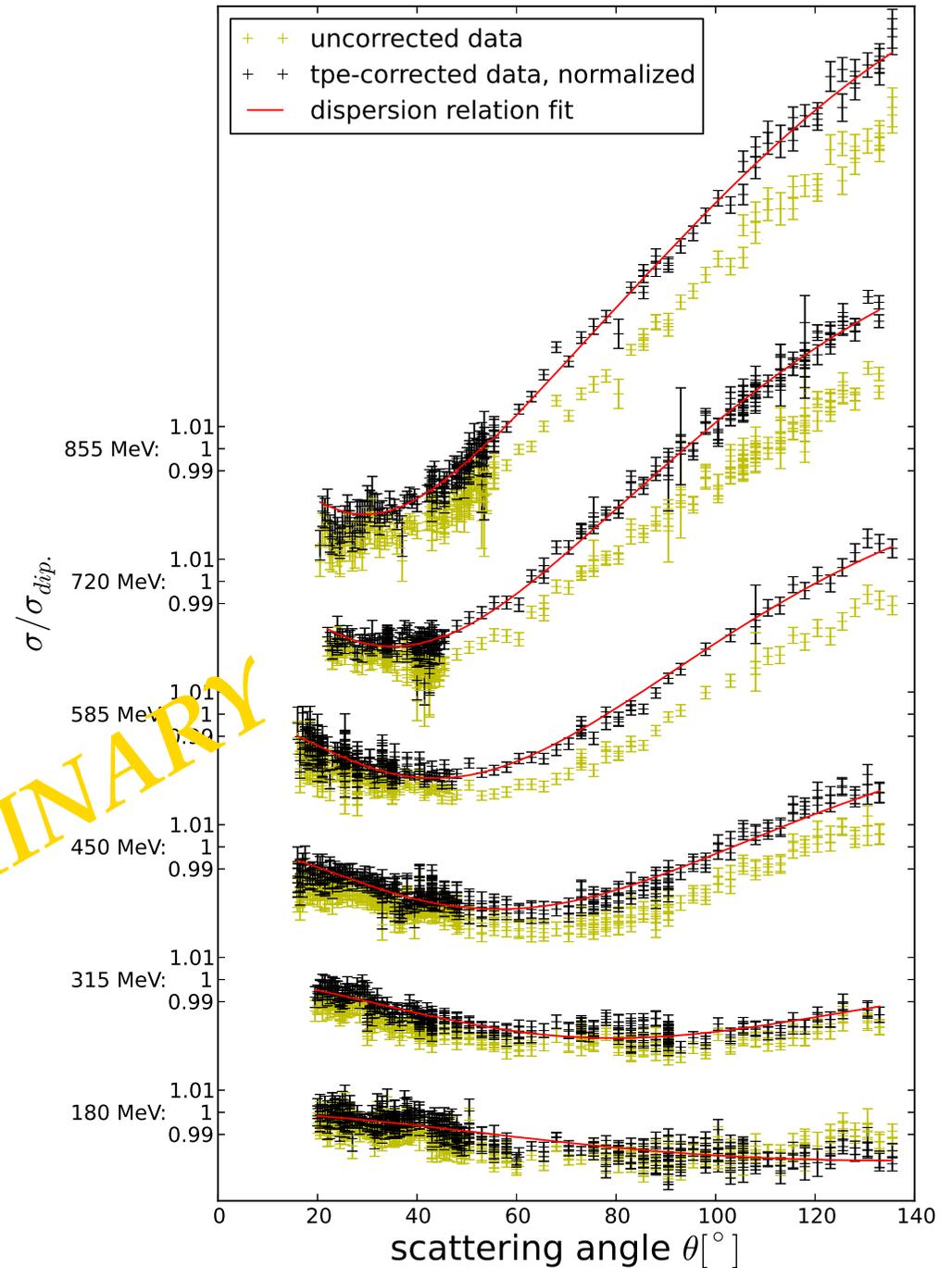
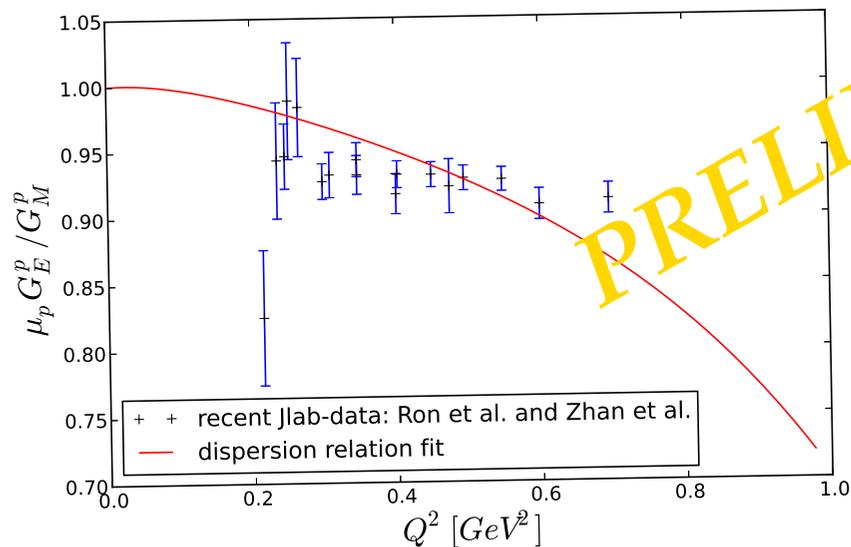
# NEW FITS to MAMI DATA

Lorenz, Dong, Hammer, UGM, in prep.

- floating normalizations
- new TPE corrections

⇒ radii:  $r_{E,p} = 0.843$  fm  
 $r_{M,p} = 0.849$  fm

- prediction of pola. transfer data



PRELIMINARY

# The proton form factors in the timelike region

Haidenbauer, Kang, M., [arXiv:1405.1628 [nucl-th]]

# BASIC DEFINITIONS and FACTS

- differential Xsection in the one-photon approximation

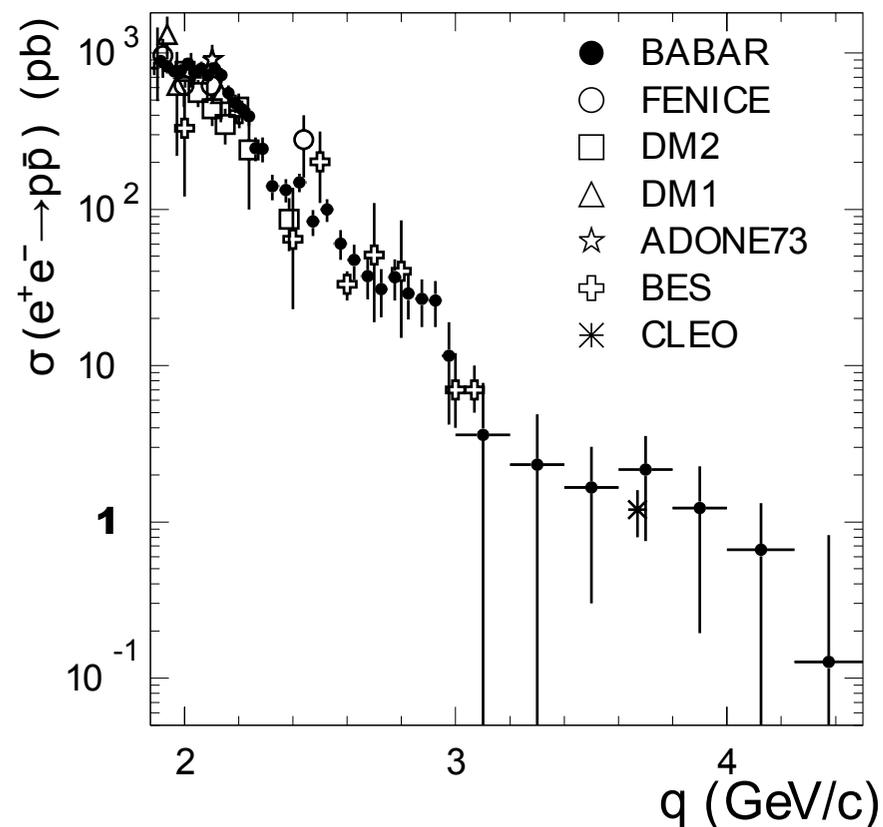
$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \beta}{4s} C_p(s) \left[ |G_M(s)|^2 (1 + \cos^2\theta) + \frac{4M_p^2}{s} |G_E(s)|^2 \sin^2\theta \right]$$

- $G_{E,M}(s)$  are complex for  $s \geq 4M_p^2$
- threshold constraint:  $G_E(4M_p^2) = G_M(4M_p^2)$
- Gamov-Sommerfeld factor:

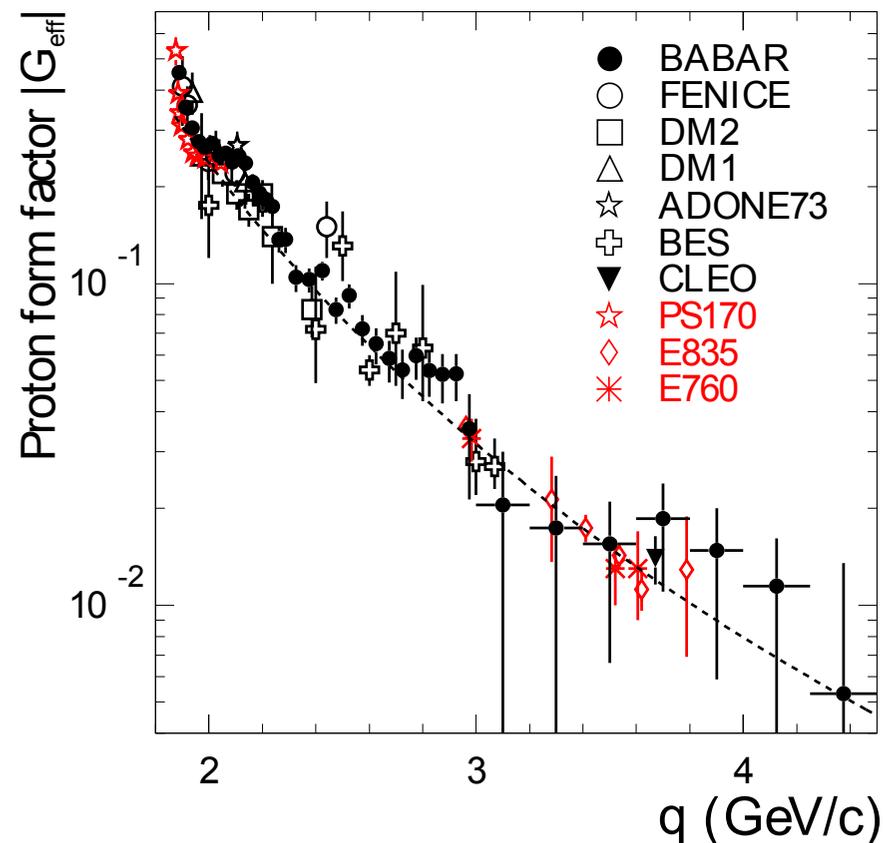
$$C_p = \frac{y}{1 - e^{-y}}, \quad y = \frac{\pi\alpha M_p}{k_p}, \quad \sqrt{s} = 2\sqrt{M_p^2 + k_p^2}$$

- Data from  $e^+e^- \rightarrow \bar{p}p$  and  $\bar{p}p \rightarrow e^+e^-$   
 → extraction of ffs difficult, strong threshold enhancement

- X section



- effective FF ( $\sigma \sim |G_{\text{eff}}|^2$ )



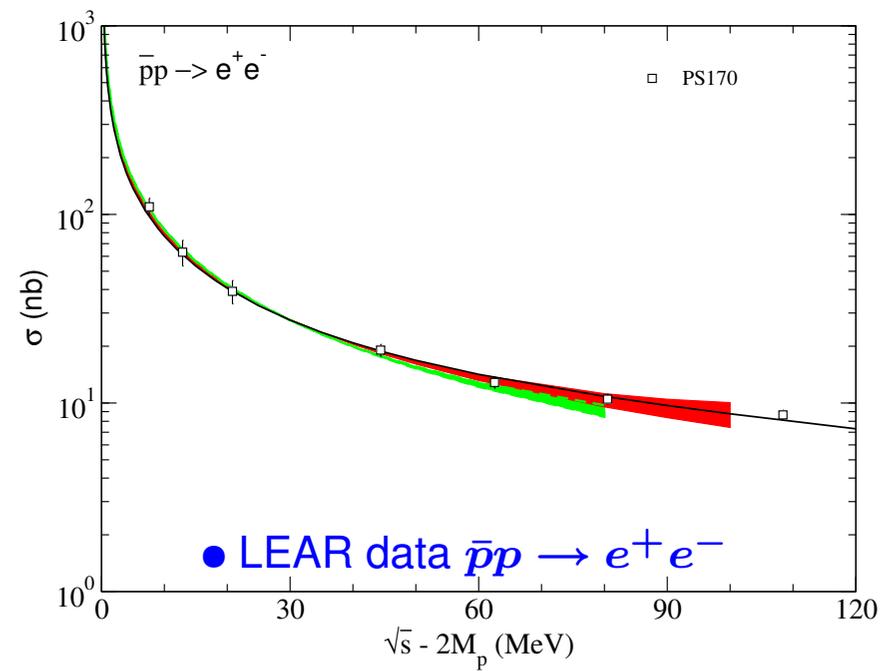
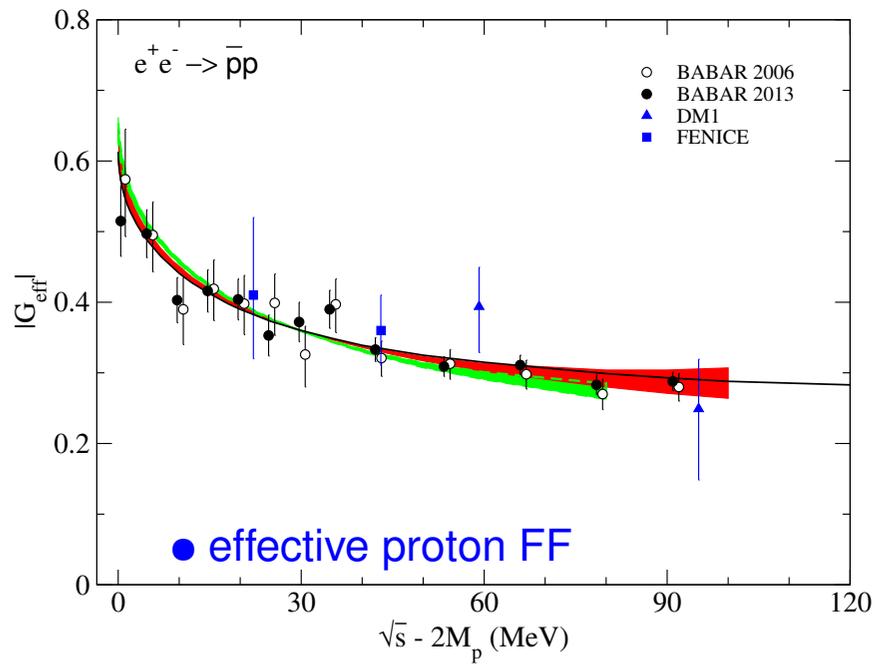
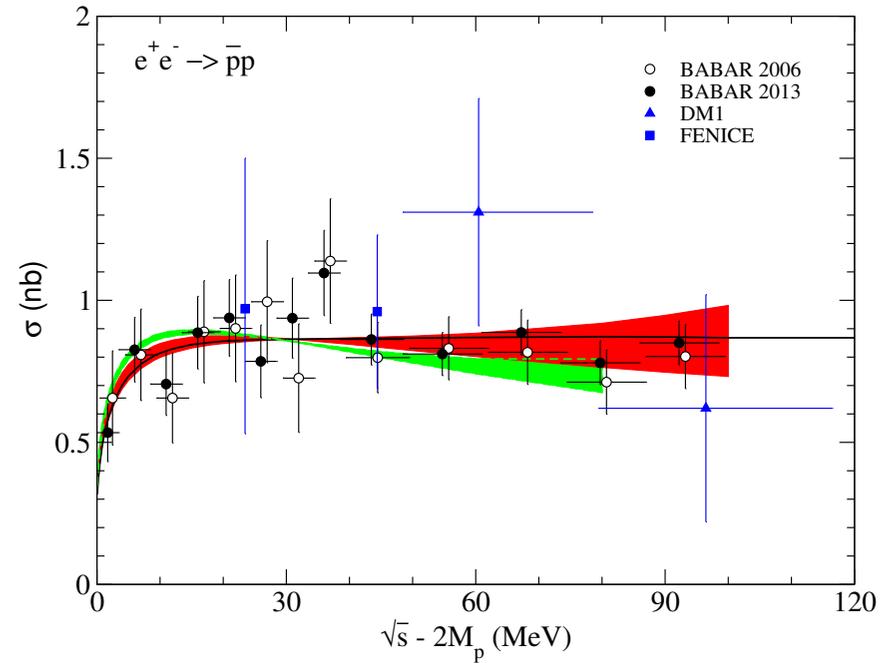
- strong enhancement in the threshold region  $q \sim 2M_p$
- effective form factor consistent with data from  $\bar{p}p \rightarrow e^+e^-$  (mod. norm.)
- threshold enhancement also seen in  $J/\psi \rightarrow \bar{p}p\gamma$ ,  $\psi(3686) \rightarrow \bar{p}p\gamma$ ,  $J/\psi \rightarrow \bar{p}p\omega$ ,  $B^+ \rightarrow \bar{p}pK^+$



# FIT and PREDICTIONS

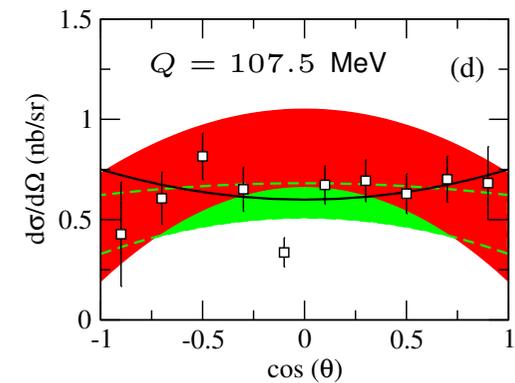
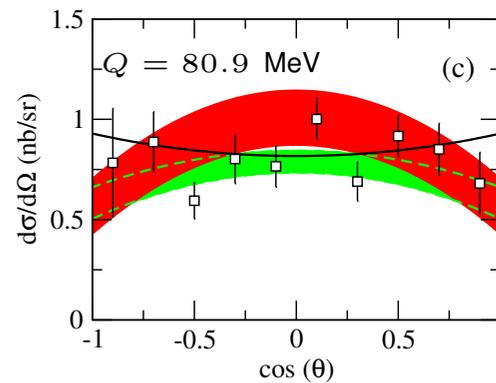
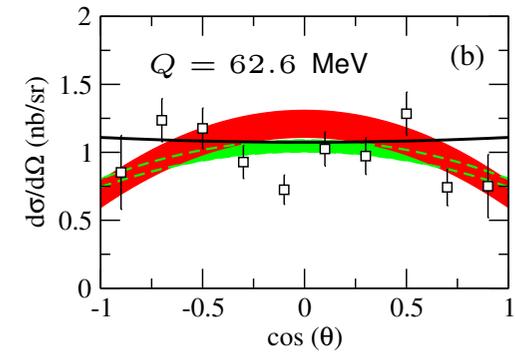
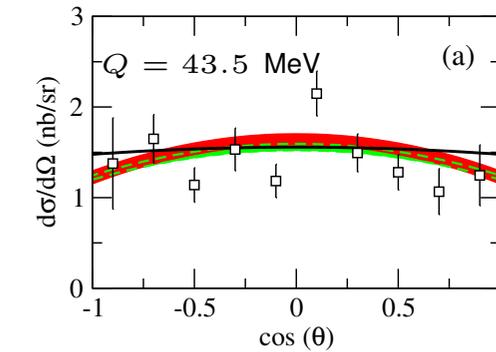
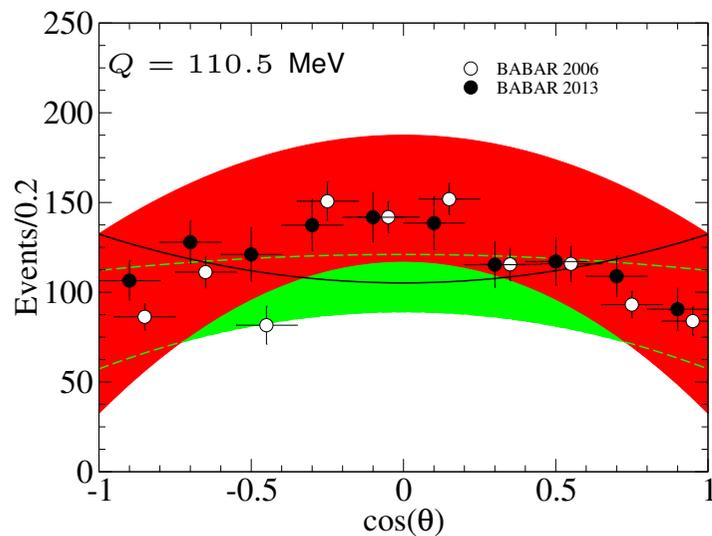
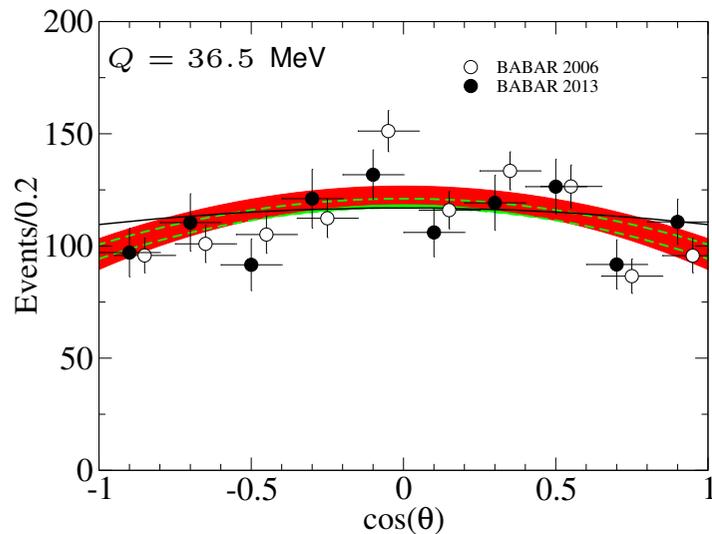
- one parameter fitted for excess energies smaller than 60 MeV

- Predictions:



# PREDICTIONS: DIFFERENTIAL XS

- differential X sections from BaBar and PS170  $[Q = \sqrt{s} - 2M_p]$



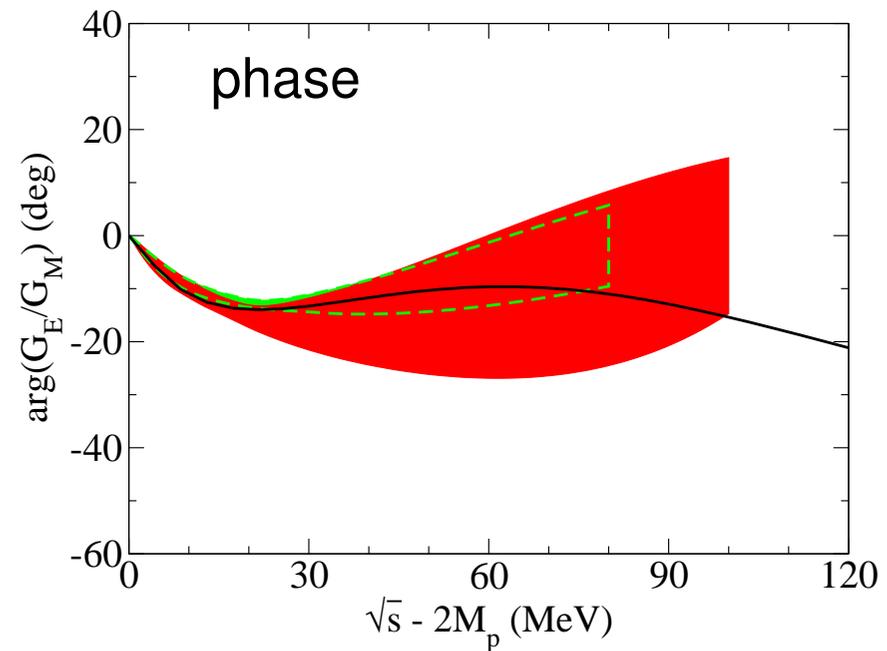
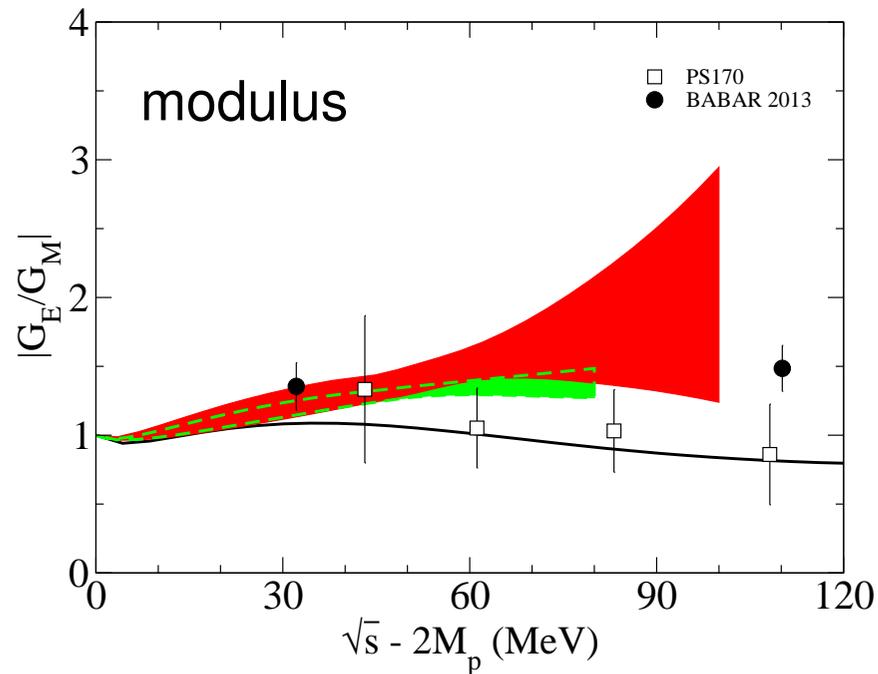
PS170: Bardin et al., Nucl. Phys. B411 (1994) 3

BaBar: Aubert et al., Phys. Rev. D73 (2006) 012005

BaBar: Lees et al., Phys. Rev. D87 (2013) 092005

# FURTHER PREDICTIONS

- complex form factor ratio



- ratio and phase are to be determined w/ PANDA at FAIR
- also predictions for the analyzing power and spin correlation coefficients

↪ see [Haidenbauer, Kang, M., \[arXiv:1405.1628 \[nucl-th\]\]](#)

## SUMMARY & OUTLOOK

- Up-to-date dispersive analysis of the nucleon em form factors
- Improved spectral functions  $\Rightarrow$  **many results**
  - better fits w/ inclusion of time-like form factors
  - theoretical/systematic uncertainty
- Recent interest: radius controversy
  - MAMI  $ep$ -scattering analyzed with DR framework
  - $\Rightarrow$  consistently with earlier works, a **small radius** emerges
  - tbd: further scrutinize the error budget (first results)
  - tbd: extraction of neutron FFs from data on light nuclei
  - tbd: scrutinize the th'y of the electronic Lamb shift & role of  $r_E^p$

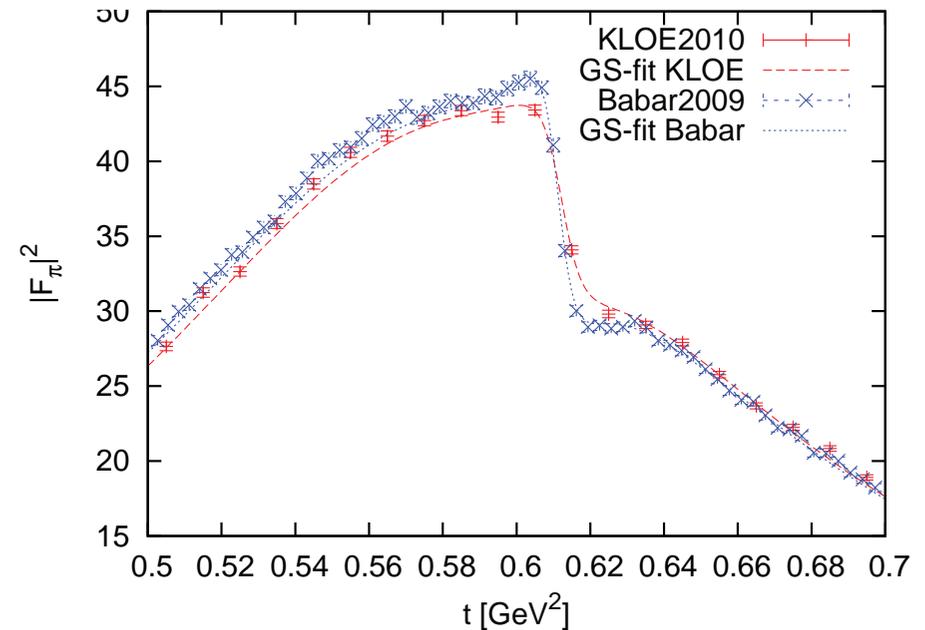
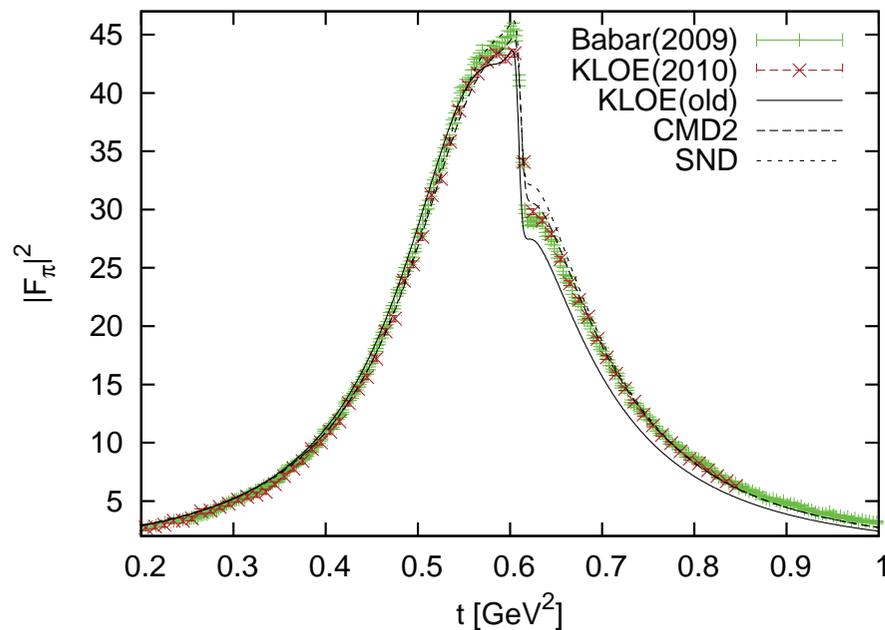
## SUMMARY & OUTLOOK continued

- New analysis of the proton ffs in the timelike region
  - threshold enhancement appears to be a universal phenomenon
  - proton-antiproton FSI can explain the threshold enhancement
  - existing data well described with **one** parameter
  - ⇒ many predictions to be tested at PANDA

# Spares

# UNCERTAINTIES: PION FORM FACTOR UPDATE

- Improved description of the  $2\pi$  continuum via new data for  $F_\pi(Q^2)$

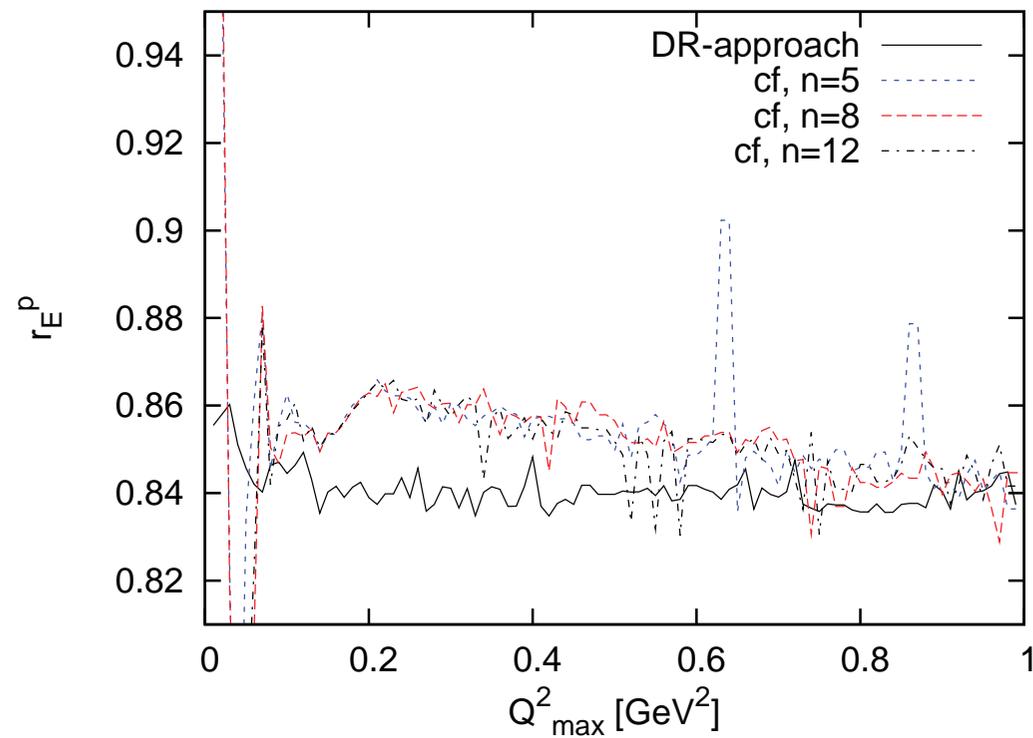


⇒ enters uncertainty of  $r_{E,M}^P$

KLOE, Phys. Lett. B **700** (2011) 102  
BaBar, Phys. Rev. Lett. **103** (2009) 231801

# UNCERTAINTIES: FIT RANGE

- study the dependence on the data range included in the fit



⇒ stable results for  $Q_{\max}^2 \geq 0.15 \text{ GeV}^2$



# TIME-LIKE FORM FACTORS

Haidenbauer, Hammer, M., Sibirtsev, Phys. Lett. B **643** (2006) 29

- fitting also time-like data more complicated

- experimental extraction ambiguous

- E/M separation

- $\bar{N}N$  final-state interactions?

similar to  $J/\psi \rightarrow \gamma \bar{p}p, \omega \bar{p}p$  from BES

Sibirtsev et al., Phys. Rev. D **71** (2005) 054010

Haidenbauer et al., arXiv:0804.1469 [hep-ph]

similar to  $B^+ \rightarrow \bar{p}pK^+$  from BaBar

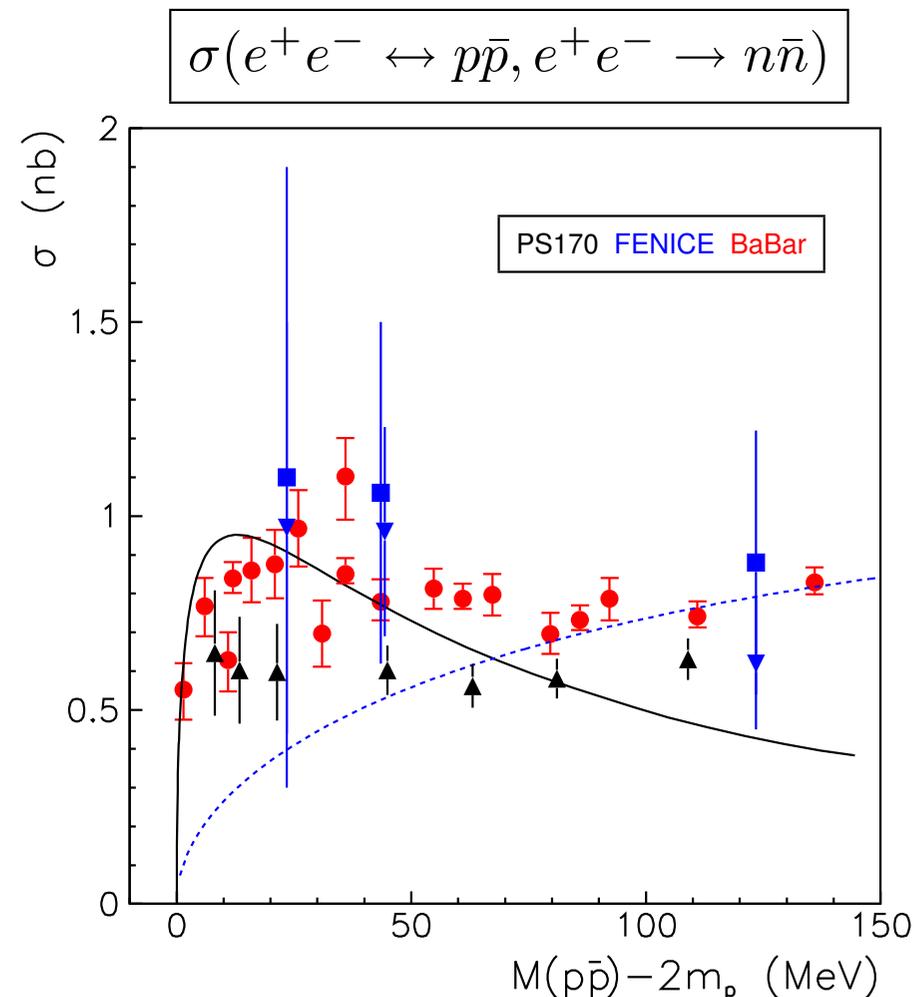
Haidenbauer et al., Phys. Rev. D **74** (2006) 017501

- subthreshold resonance ? (or FSI ?)

Antonelli et al., Nucl. Phys. B **517** (1998) 3

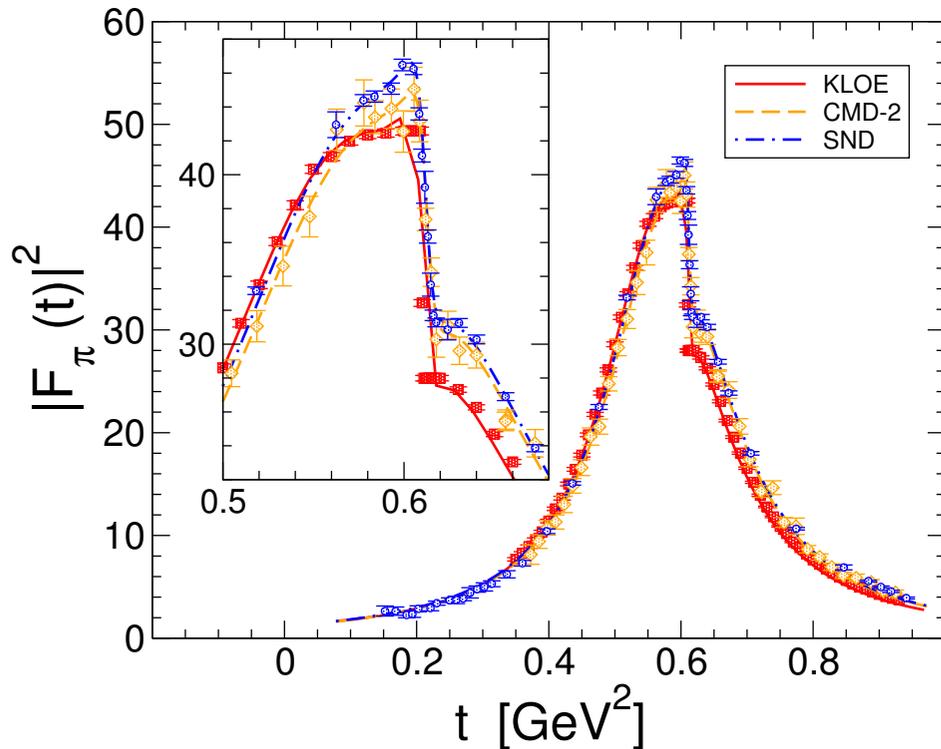
- many new proton data (radiative return)

BES, CLEO, BaBar



Belushkin, Hammer, M., Phys. Lett. B **633** (2006) 507 [arXiv:hep-ph/0510382].

## • Pion FF from KLOE/CMD-2/SND



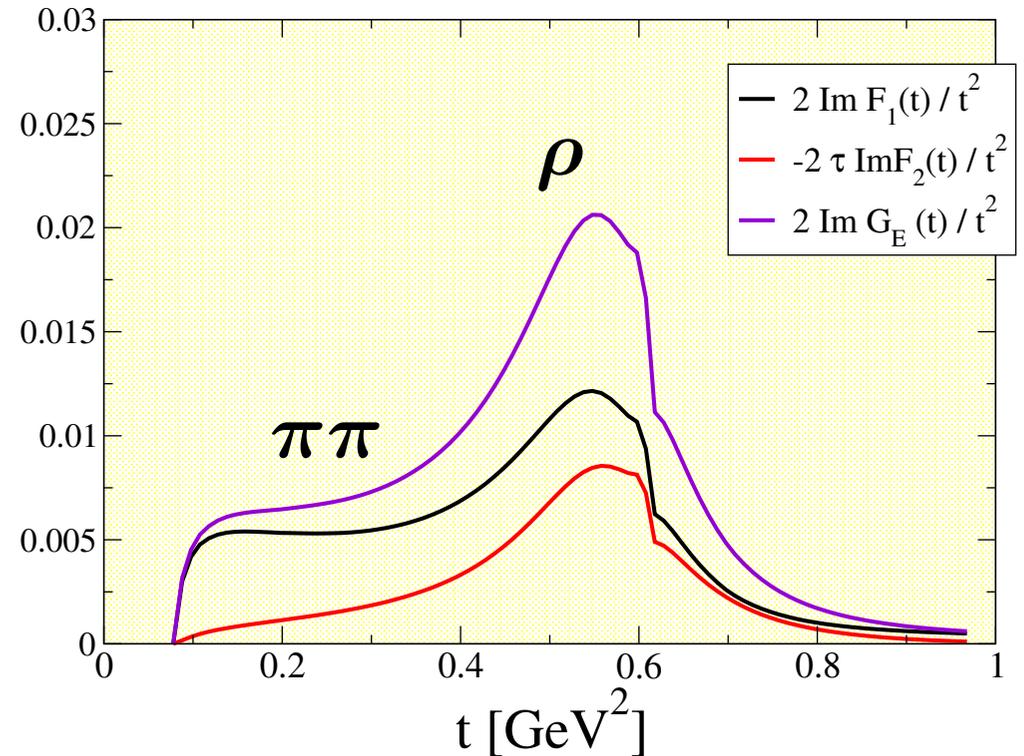
★ pronounced  $\rho - \omega$  mixing

KLOE Coll., Phys. Lett. B **606** (2005) 12

CMD-2 Coll., Phys. Lett. B **578** (2004) 285

SND Coll., J. Exp. Theor. Phys. **101** (2005) 1053

## • Nucleon isovector spectral functions

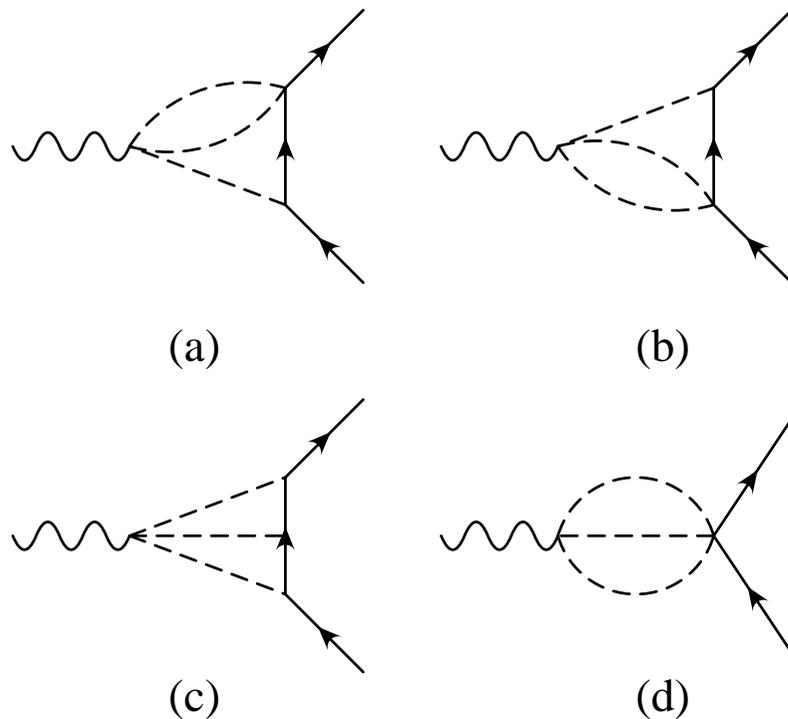


★ pronounced  $\rho$  peak

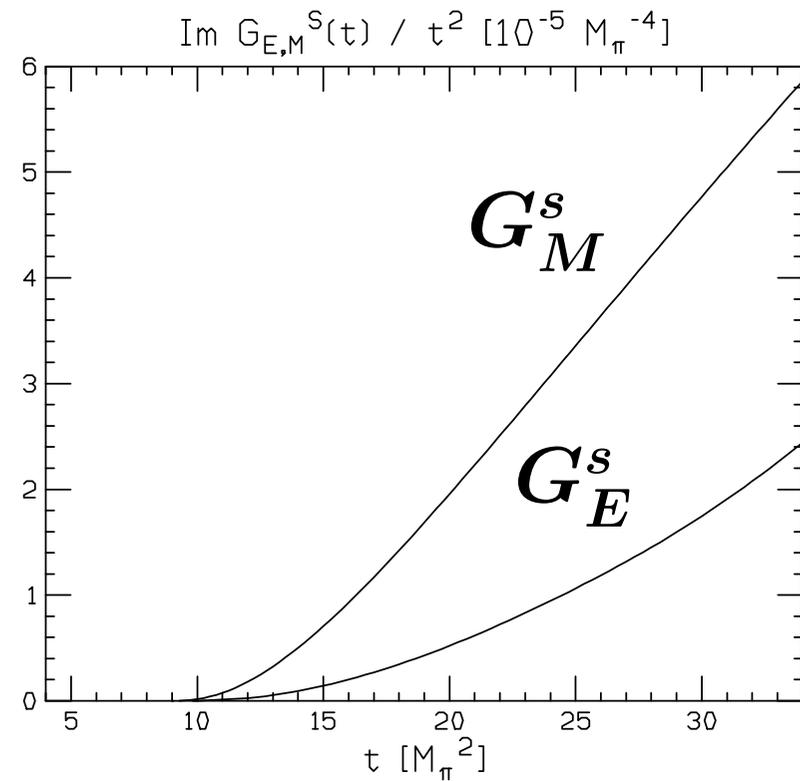
★ strong shoulder on the left wing

⇒ isovector radii

- Two-loop CHPT calculation



- Electric/magnetic spectral fcts



- ★ **no** shoulder on the left wing
- ★ **clean** omega-pol dominance

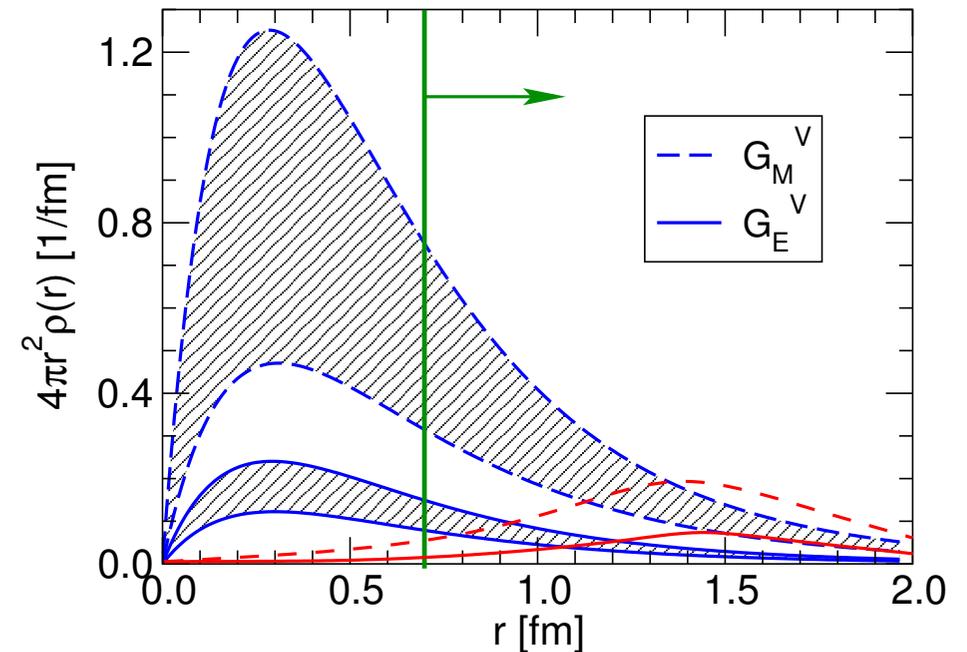
# ON THE PION CLOUD OF THE NUCLEON

Hammer, M., Drechsel, Phys. Lett. B **586** (2004) 291

- FW find a very long-ranged contribution of the pion cloud,  $r \simeq 2$  fm

Friedrich, Walcher, EPJ A **17** (2003) 607

- longest range component can be extracted from the isovector spectral function
  - separation of the  $\rho$ -contribution
  - three methods applied to do this
  - theoretical band



$$\rho_i^V(r) = \frac{1}{4\pi^2} \int_{4M_\pi^2}^{40M_\pi^2} dt \operatorname{Im} G_i^V(t) \frac{e^{-r\sqrt{t}}}{r} \quad (i = E, M)$$

- much smaller pion cloud contribution for  $r \geq 1$  fm compared to FW
- results independent of the contributions from  $t > 40M_\pi^2$



# Two-photon corrections

Belushkin, Hammer, M., Phys. Lett. **B 658** (2008) 138 [arXiv:0705.3385 [hep-ph]]



# ANALYSIS of TWO-PHOTON CORRECTIONS

- Hybrid analysis: FF data for the neutron, cross sections for the proton
- Easiest to compare at cross section level

⇒ reconstruct “PT cross section” from FF data (A = SC, pQCD)

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{Ros,A}} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{A}} (1 + \delta^{2\gamma})$$

- Comparison with direct calculation (Blunden et al.)

→ add in Coulomb correction:  $\Delta^{2\gamma} = \delta^{2\gamma} + \delta^C$

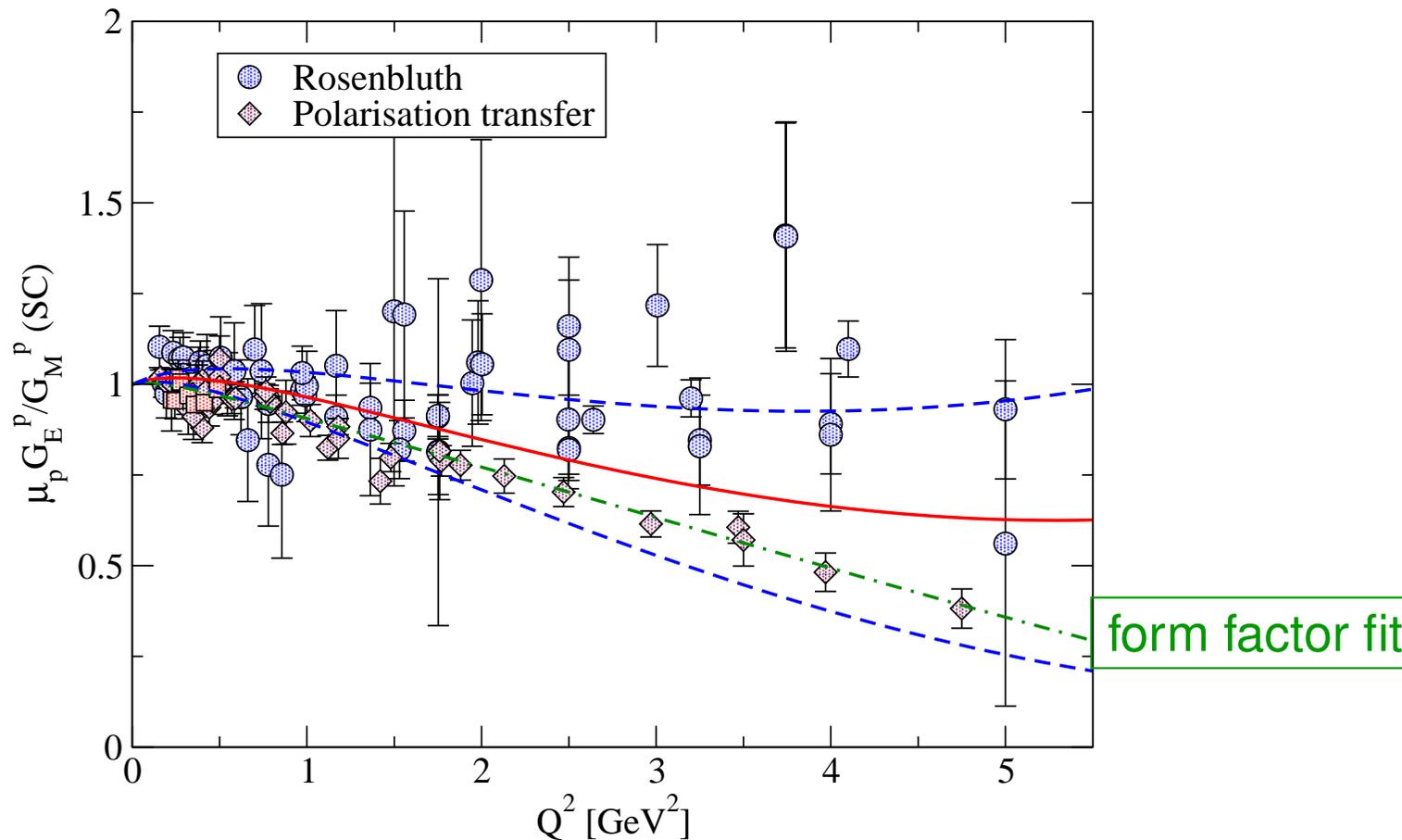
[Coulomb corrections from Arrington and Sick, Phys. Rev. C **70** (2004) 028203]





# COMPARISON w/ FORM FACTOR RATIOS

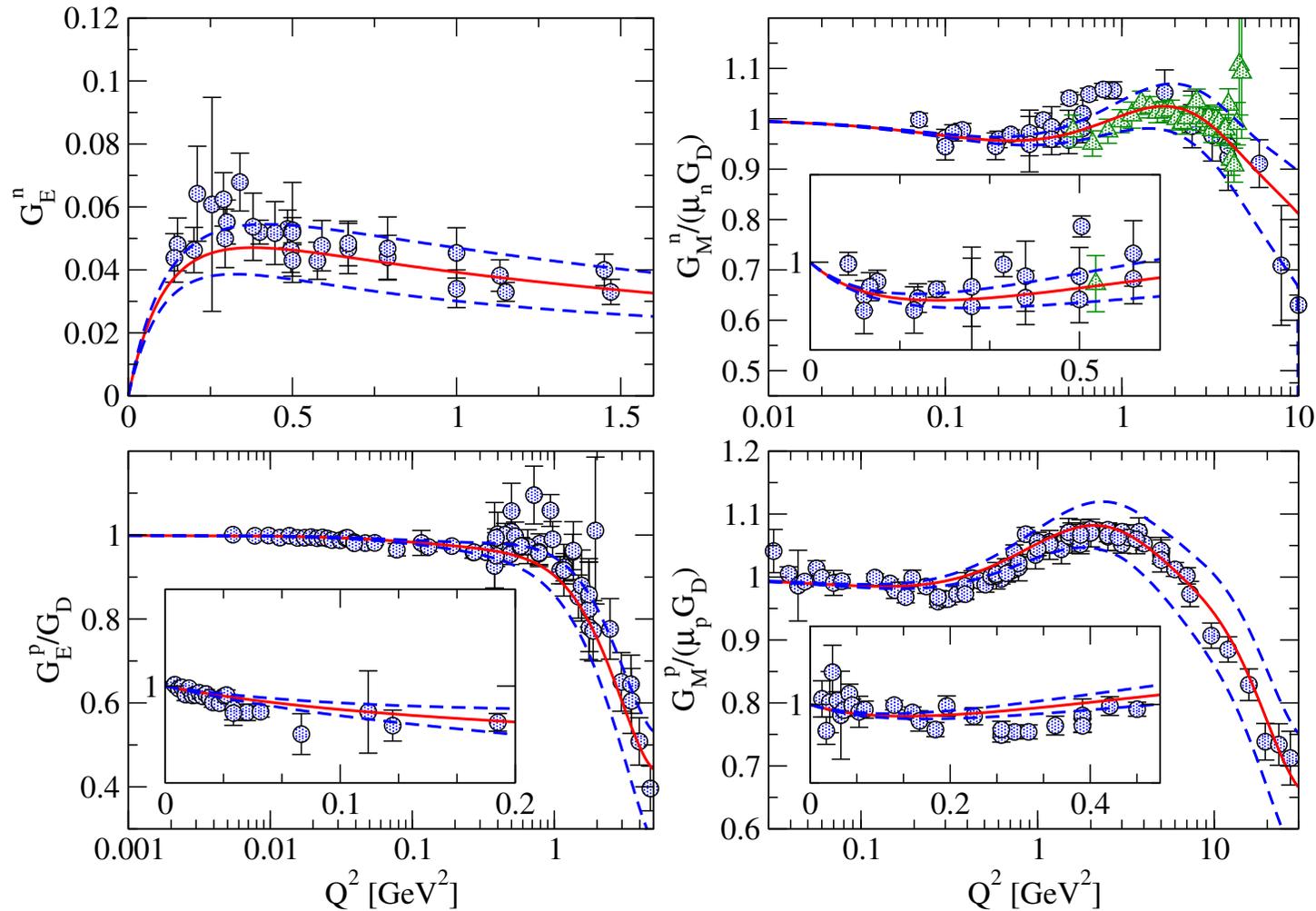
- FF ratio from X sec analysis (SC) compared to PT data



- consistent within error bands
- form factor data not included in the analysis

# SPACE-LIKE FORM FACTORS: NEW CLAS DATA

CLAS collaboration, Phys. Rev. Lett. 102, 192001 (2009) [Ph.D. thesis Jeff Lachniet, CMU, 2005]



→ apparent discrepancy to be resolved

